

Changing the Equation: After-School Math Curriculum



Based on *After-School Math PLUS*
from the Educational Equity Center at
Academy for Educational Development



Funding provided by



Introduction

Changing the Equation: After-School Math Curriculum

In this packet, you will find four different sets of themed math lessons, each containing an introduction, materials list, activity plans, discussion prompts, and spotlights on employees from Sandia National Laboratories who rely on math in their careers.

These four curricula include short, easy-to-follow after-school activities that support and enhance math learning. They offer math activities in a real-world context and include highlights of math's vital role in careers.

Table of Contents

Curriculum

Pages

Jump Up for Math

Investigate the math in jumping rope.
Compare powerful animal jumpers!

Pages JU 1 - JU 16

Art Math

Explore tessellations and fractals.
Build a telescope!

Pages AM 1 - AM 17

The Built Environment Math

Learn about measurement, proportion, and scale.
Design your ideal community!

Pages BE 1 - BE 13

Music Math

Learn the difference between rhythm and beat.
Design your own musical patterns!

Pages MM 1 - MM 15

Changing the Equation: After-School Math Curriculum

JUMP UP FOR MATH

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Sandia National Laboratories

KIT: JUMP UP FOR MATH

This After-School Math Kit includes engaging activities that are fun for both students and after-school staff. Students learn math while working in cooperative, supportive groups facilitated by staff members. Even better, after-school staff don't need to be math experts!

Through these activities, students and staff hone math skills, gain confidence in math, and increase their enjoyment of math.

THIS KIT INCLUDES

■ **Instructions to facilitate 3 activities.** These activities are simplified summaries and extensions of the “After-School Math PLUS: Jump Rope Math” curriculum created by the Educational Equity Center at Academy for Educational Development (AED).

The 3 activities are:

Part 1. How Far Can You Go?

Part 2. Going to Great Lengths

Part 3. Legendary Leapers

■ **Scientist Spotlight** Ask your students “who is a scientist?” and you'll typically get answers that include white lab coats, microscopes, and bubbling beakers. All of these images reflect some aspect of science and STEM (Science, Technology, Engineering, Math), but they don't provide a full picture. We include stories of two STEM professionals that work at Sandia National Laboratories. Read these with your students, and together list some of the activities, skills, and experiences from the stories. What surprises your students? What was unexpected? Does this change how they think of scientists and engineers?

TIPS FOR LEADING ACTIVITIES

- Give students opportunities to share their ideas with you and with each other.
- Make sure to introduce each activity. Learning happens best when learners know what's coming up and why it matters to them.
- Ask open-ended questions, rather than those that have a “yes” or “no” answer.
- Ask questions that inspire the learner to thoughtfully analyze a situation and consider consequences, such as, “What do you think will happen if you do this?”
- Give the learner time to answer the question. Ask the question, then wait. A while. Trust us: thoughtful answers take time.
- When a learner tells you what they think, respond by repeating and paraphrasing what they have said without criticism.
- Don't give too much praise or reject ideas. Telling a learner they are right or wrong can discourage them from generating additional ideas or pursuing deeper exploration.

Part 1 - How Far Can You Go? Main Idea: How far do you think you can skip rope in a minute? For this activity, instead of jumping rope in place, students will be skipping rope. They will make predictions and measure how far a person travels with each turn of the rope.

INTRODUCTION

When introducing this activity, let students know that this is not a competition. Reassure students that their jump-roping skills are not essential; their math skills are! Students will jump rope, collect data, and construct a line graph that can be used to predict change in distance over time.

MATERIALS

- Jump Ropes (1 per group)
- Pencils
- Measuring tape (1 per group)
- Chart paper
- Markers
- Rulers
- *How Far Can You Go? Data Collection Sheet* (1 per group, found on page JU 11)
- Graph paper

ACTIVITY: HOW FAR CAN YOU GO?

STEP 1: DISCUSS (10 MINUTES)

As a group, ask students, “Can you think of examples in sports where the athletes are timed?” Listen to responses. Introduce the activity by telling students they will skip rope (moving forward while jumping rope) and try to predict how far they will go in a given time and then measure it.

Discussion: What is meant by “given time?” How can we measure distance? How can we measure time? Do you think you will skip exactly the same distance each time? Do you think the number of times a person skips rope might vary?

STEP 2: SKIPPING ROPE (20 MINUTES)

Divide students into pairs or groups of three and let each group select a jump rope. Explain that each member of the group will take a turn skipping rope while the others keep time and record the distance jumped on the *How Far Did You Go? Data Collection Sheet*. The groups will measure the distance covered for three different times (e.g., 10 seconds, 30 seconds, 1 minute).

Give students 20 minutes to take turns skipping rope. Measuring tapes can be used to measure distance and a stop-watch (commonly found on most cell phones) can be used to measure time. For younger children, you can mark a set distance on the floor with tape and have the students time how long it takes to cover that distance.

STEP 3: CREATING LINE GRAPHS (10 MINUTES)

Distribute graph paper and tell students that each group will create a line graph to chart the data they collected. Help students label the axes. The x-axis (horizontal) should be time in seconds (1 second, 2 seconds, etc.), and the y-axis (vertical) should be distance in feet or yards (1 foot, 2 feet, etc.). Ask students what numbers they want to put on the x-axis. Can they fit a whole minute on the graph if they include every number from 1 to 60 seconds? Would it be easier to count by multiples (e.g., count by threes or fives)? Let students decide, in their groups, what multiples they'd like to use.

Draw a sample line graph on chart paper. As a class, review and discuss how each point should be added to the graph. Start by finding the x-axis value, then go up from the axis until you hit the y-axis value. It may be helpful to demonstrate using two fingers (one on each axis) and moving them both at the same speed from the axes until they meet; that's where the point on the graph should go. Have students fill in their graphs using their personal data; they should end up with three points on the graph. Have rulers available to help students place their points and connect them. The three points should be connected with straight lines.

Discussion: Ask groups to explain their graphs to the class. Does your line go upward or downward? What does that mean? Did they skip rope further when given more time? How does the graph show this? Give students time to understand and discuss their graphs.

STEP 4: TESTING HYPOTHESES AND MAKING PREDICTIONS (10 MINUTES)

Have students look at their graphs again. Ask them to predict how far they would skip rope in another time on the x-axis (e.g., 25 seconds or 45 seconds). Help each student come up with a prediction based on his or her graph. Then, in their groups again, have students skip rope for the new time and measure their distance to test their predictions. Was their distance prediction accurate? Can they predict how long it takes to jump a certain distance? How can they test this prediction?

Discussion: Have each group share their findings with the class. Did they get the results they predicted? If not, what was different? What are some possible explanations for the difference(s)? Did anyone try to jump farther, faster, or slower? How could that affect their predictions from the graph?

Part 2 - Going to Great Lengths Main Idea: Does the length of jump rope that works for you change depending on your height? Students will test different jump rope lengths to find the length that works best for them. They will measure and record the length of their favorite jump rope, measure and record their height, use the data to create a graph, and look for a pattern. They will then use their graphs to make predictions.

INTRODUCTION

When introducing this activity, let students know that this is not a competition. Reassure students that their jump-roping skills are not essential. Students will use standard and nonstandard units of measure to record the lengths of jump ropes and the height of jumpers. They will compare the preferred length of rope for each jumper to his/her height and look for a mathematical relationship between the two values. They will calculate ratios and determine if there is a consistent ratio between jump rope length and jumper height. Finally, students will make scatter graphs with the class data and use the graphs to determine a ratio and make predictions about the length of rope needed for different heights.

MATERIALS

- Cotton clothesline or other type of rope cut into 10-foot lengths (1 per group)
- Standard measurement tools (e.g., meter stick/yard stick, tape measure)
- Nonstandard measurement tools (e.g., feet, sneakers, hands)
- Graph paper
- Markers
- Colored dots
- Pencils
- *Going to Great Lengths Data Collection Sheet* (1 per group, found on page JU 12)
- *Standard Unit Data Chart* and/or chart paper (1 per group, found on page JU 13)

ACTIVITY: GOING TO GREAT LENGTHS

STEP 1: DISCUSS (25 MINUTES)

Many people jump rope for exercise and for fun. What kinds of jump rope games have you played?

Divide students into groups of four to six people. Give each group a 10-foot length of clothesline and have each student take a turn using it to jump rope. (Students may naturally begin to shorten the length of the rope by wrapping the ends around their hands.)



Discussion: What is the “best” length for a jump rope? What do we mean by “best?” Does one length of jump rope work for everyone? How would you determine the best length? Some example responses may be: “The best length would let me jump ten times without missing.” Or, “The best length would let me jump forever.” Encourage students to discuss their ideas fully. Does a jump rope need to be longer or shorter than the person jumping? You may want to demonstrate a rope that is too short or too long.

As a class, determine which standard unit of measure you should pick to measure jump rope length and student height (e.g., centimeters, meters, inches, feet). If we didn’t have rulers or measuring tapes, what else could we use to measure? Record all suggestions and review the list (e.g., shoe-lengths, actual feet lengths, hand-widths, floor tiles). Explain that these are nonstandard measurements, and have each group pick one to use to compare to the standard unit the class picked earlier.

Distribute the *Going to Great Lengths Data Collection Sheet* to each group. Ask students to measure the height of each jumper using both the standard and nonstandard units of measure and record on their data sheets. Encourage students to take turns so that each student is measured and has a turn to measure and record the data.

Ask each student to jump rope with a piece of clothesline, adjusting the length until it is best for him or her (by wrapping the ends around his/her hands). Have students measure this “best” length of the rope (excluding any excess that was wrapped around his/her hands) with both the standard and nonstandard units and record it on the data sheet next to the student’s height.

STEP 3: SHARE FINDINGS (10 MINUTES)

Discussion: Ask each group to share their findings. What nonstandard measures did they use? How did it compare to the standard unit used? Which unit is easier to use? Why do you think there are standard measures?

Explain that nonstandard units can change over time so you can get different measurements of the same thing. For instance, as you grow, your foot and your hand grow too, so using those to measure isn’t consistent.

Ask students if they can organize their data in a way that might show a pattern. For instance, by arranging all of the student data by height—from tallest to shortest or vice versa—it might be possible to detect if there is a relationship between the height of a person and the length of the jump rope they need. Using the *Standard Unit Data Chart* or chart paper, create a table by recording each student’s data starting with the shortest student and ending with the tallest. Do students see any patterns in the class data? Do the tallest students need the longest jump ropes?

STEP 4: CREATE A SCATTER GRAPH (15 MINUTES)

Explain to students that the data on the chart paper is organized in a data table. That's one way to organize data to help find patterns and relationships. Another way to organize data to help visualize patterns is to make a graph. On a large piece of chart paper, label the x-axis (horizontal) "Student Height" and the y-axis (vertical) "Jump Rope Length." Help students label the chart with the unit they used for their standard measure. Have each student place a colored dot on the chart where his or her height and jump rope length meet.

When the graph is complete, ask students if they see a pattern in the dots. What does the graph show? Did the shorter students use shorter jump ropes? Did taller students use longer jump ropes? Most likely, the dots don't all fall on a straight line. They are probably scattered around an imaginary line from the shortest students' dots to the tallest. This invisible line is called a **trend line**, and this type of graph is called a **scatter graph**. Have a student or two help you draw a straight line on the graph (in pencil) from the shortest jumper's dot to the tallest jumper's dot. Are most of the other dots near the line? If not, can a student help draw a better trend line that most of the dots will be near? The trend line does not necessarily need to intersect (or go through) any of the individual dots; it is a generalization showing a trend in the data.

Are there some dots that are far away from the trend line? These are called **outliers** and represent jumpers who don't fit the trend or pattern of everyone else. If there are outliers on the graph, what makes them unique? Did someone short like a really long jump rope? Why? What else might cause someone to be an outlier?

Discussion: Ask the students to describe the relationship between student height and jump rope length. Can students use the information in their scatter graph to predict what length of rope a 6-foot tall jumper would need?

Part 3 - Legendary Leapers **Main Idea:** What animal can jump the farthest? The highest? For this activity, students will compare their jumping distances with those of other animals and learn about the adaptations that help these impressive leapers.

INTRODUCTION

Animals move to find food, to escape predators, or to find a safe place to live or rest. Jumping is a type of locomotion that requires special adaptations such as long legs, large muscles, modified foot and ankle bones, or spring-like tails. Jumping spiders use a hydraulic system that transfers body fluids to their legs, allowing them to leap several times their own body length. Jumping spiders jump to catch their prey; frogs will jump to escape predators; and a kangaroo uses jumping as its primary mode of locomotion.

Students will first compare how far they can jump with the table showing distances that various animals can jump. Next, students will reflect on the relationship between body size and the distance an animal can jump.

In preparation, create a 10-foot-long jumping track by placing masking tape on the floor in a straight line; then place a 5-foot piece of masking tape perpendicular to this, creating a “T” shape on the floor. Mark off 1-foot increments along the 10-foot strip.

MATERIALS

- Tape measure or yard stick
- Masking tape
- Graph paper
- Chart paper
- Adequate flat space to jump
- *Animal Jumping Distance Table* (1 per student, found on page JU 14)
- *Animal Body Length Jumping Table* (1 per student, found on page JU 15)

ACTIVITY: LEGENDARY LEAPERS

STEP 1: DISCUSS (5 MINUTES)

Have students list some ways that animals move: run, swim, fly, soar, glide, jump, slither, crawl, walk, etc. Today we are going to learn about animals that jump. Ask: What animals can you think of that are good jumpers? How do you think you might compare?

**STEP 2: THE LONG JUMP (20 MINUTES)**

Have students line up along the 5-foot strip of masking tape. This will be the start line. Ask two students to be spotters along the 10-foot strip of masking tape. Each of the spotters will have paper, pencil, and a ruler. Have students take turns jumping as far as they can, starting from a crouching position. The spotters will record how far each student jumps (use the ruler to measure to the nearest marked increment). Record distances either by using a sheet of paper or by placing tape at the spot of the jump labeled with each student's initials. Make sure the spotters swap with two other students so that everyone gets a turn.

Have each student compare their distance with the *Animal Jumping Distance Table*. Which animals could they out-jump? Compare the results.

STEP 3: BODY LENGTH RATIOS (45 MINUTES)

Ask the students: "Do you think how tall you are affects how far you can jump?" Test this idea. Divide the students into pairs, and have each pair use the tape measure to measure their height in feet. Write this number down.

Create a scatter graph. On a large piece of chart paper, label the x-axis (horizontal) "Student Height (feet)" and the y-axis (vertical) "Long Jump Distance (feet)." Help students label the chart and have each student place a colored dot on the chart where his or her height and long jump distance meet. Is there a relationship? How many students were able to jump farther than their body length? Was anybody able to jump twice as far as they are tall (that is, 2 times their body length)?

Do you think there is a relationship between how big an animal is and how far it can jump? For example, if a grasshopper were the same height as a person, would it be able to jump farther than 2.5 feet?

Hand out the *Animal Body Length Jumping Table*. Demonstrate the following calculation with the entire group: A football field is 100 yards = 300 feet = 3,600 inches long. A grasshopper is about 1 inch long. Compare this to a grown man who is 6 feet 2 inches, which is 74 inches, or 74 times taller than a grasshopper. To compare the jump of the grasshopper to this man, you would multiply 74×30 inches = 2,220 inches = 61 yards. In other words, a person-sized grasshopper would be able to jump more than half a football field in a single bound.

Similarly, a rocket frog that can leap 6.5 feet (35 times its own length) is equivalent of a 6-foot human making a 105-foot-long jump. What other comparisons can you make? Have students work in small groups to try to calculate human-sized long jumps for different animals.

CHANGING THE EQUATION: JUMP UP FOR MATH PART 3



explora!

Ideas You Can Touch
Ideas que puedes tocar

Discussion: Have each group share their findings with the class. Was it difficult to make the calculations? Have each student explain their reasoning and how they worked out their calculations. It's OK if the student didn't finish the calculation or wasn't able to do it correctly; let them tell the group what they tried, and ask the group to listen respectfully. Did each group approach the problem in the same way? What kinds of answers did the group get? Which answers make the most sense and why? Which animal was the most surprising?

CHANGING THE EQUATION: JUMP UP FOR MATH PART 1



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Ideas You Can Touch
Ideas que puedes tocar

How Far Can You Go?

Data Collection Sheet

Jumper:	
Time	Distance

Jumper:	
Time	Distance

Jumper:	
Time	Distance

CHANGING THE EQUATION: JUMP UP FOR MATH PART 2



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Ideas You Can Touch
Ideas que puedes tocar

Going to Great Lengths

Data Collection Sheet

Jumper:		
Height of Jumper	Jump Rope Length (in standard units)	Jump Rope Length (in non-standard units)

Jumper:		
Height of Jumper	Jump Rope Length (in standard units)	Jump Rope Length (in non-standard units)

Jumper:		
Height of Jumper	Jump Rope Length (in standard units)	Jump Rope Length (in non-standard units)



Animal Jumping Distance Table

Animal	Jumping Distance (feet)
Elephant	0
Flea	1
Grasshopper	2.5
Meadow Jumping Mouse	3
Kangaroo Rat	9
Hare	12
Lemur	25
Red Kangaroo	30
Mountain Lion	40

Animal Body Length Jumping Table

How many times their actual body length can these animals actually jump?

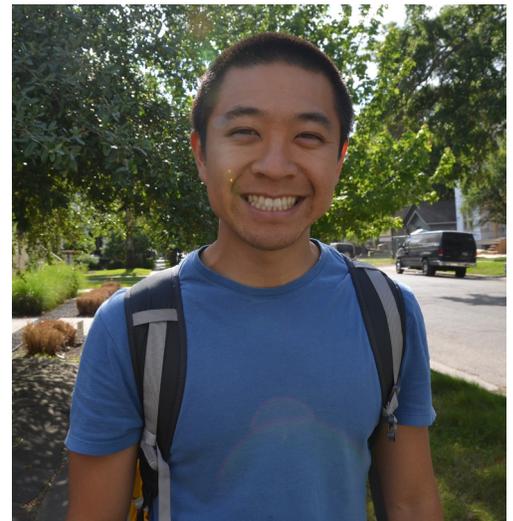
Animal	Body Length (feet or inches)	Ratio of the body length compared to length of jump
Elephant	13 feet	0
Olympic long jumper Michael Powell	6 feet 2 inches	4.76
Impala antelope	4 feet 6 inches	6
Red Kangaroo	4 feet 6 inches	10
Kangaroo Rat	6 inches	12
Meadow Jumping Mouse	5 inches	7
Grasshopper	1 inch	20
Rocket frog	2 inches (can jump 13 feet)	35
Froghopper Insect	.2 inch (can jump 28 inches)	100
Flea	.05 inch	160

I work at Sandia National Laboratories as a solar energy engineer. My team and I work to invent new technologies to provide electricity to more people, for less money, and in a clean way that doesn't harm the environment. Every project starts with the engineering design process, where we plan out what we need to build. I need math to help me measure components and calculate the amount of materials I need. Math also allows us to calculate how efficient our technologies are at turning the sun's energy into electricity. The math I learned in school provided me with a foundation that helped me better understand science and engineering. Now I get to travel the world and make people's lives better through solar technologies. Sometimes even simple math problems can turn into extraordinary technologies and lead to incredible adventures for whoever dares to solve them!



KENNETH ARMIJO
Engineer

I am a mathematician at Sandia National Laboratories where I am part of a Modeling and Simulation team. We develop computer programs that simulate real events to understand what happens to the electronics. For example, we can study what happens to the body of a car in a collision with modeling instead of having to crash a car. Our simulation helps engineers make safer cars. I use a lot of mathematics in my job! Math is a key component of all computer codes and programs. The math I learned serves as a foundation that lets me create and apply ideas to many fields in science and engineering. I also use math for my hobbies outside of work! I like to play racquetball where I constantly have to calculate the trajectory of the ball. When I hit it, where is it going to go? Will I win the point? I use math to help me win! I prefer to think of all math like a puzzle or a game; once I understand the rules about how to play the game, it becomes fun to try to win! Math homework is the same way; once you understand the ideas behind the numbers, and if you keep practicing, you will become a pro!



ANDY HUANG
Mathematician

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KIT: ART MATH

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Through the activities in this Kit, students and staff hone math skills, gain confidence in math, and increase their enjoyment of math.

THIS KIT INCLUDES

■ **Instructions to facilitate 3 activities.** These activities are simplified summaries and extensions of the “After-School Math PLUS: Art Math” curriculum created by the Educational Equity Center at Academy for Educational Development (AED).

The 3 activities are:

- Part 1. Creating a Kaleidoscope
- Part 2. Fun with Geometry
- Part 3. Fractal Art

■ **Scientist Spotlight** Ask your students “who is a scientist?” and you'll typically get answers that include white lab coats, microscopes, and bubbling beakers. All of these images reflect some aspect of science and STEM (Science, Technology, Engineering, Math), but they don't provide a full picture. We include stories of two STEM professionals that work at Sandia National Laboratories. Read these with your students, and together list some of the activities, skills, and experiences from the stories. What surprises your students? What was unexpected? Does this change how they think of scientists and engineers?

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- Don't give too much praise or reject ideas. Telling a learner they are right or wrong can discourage them from generating additional ideas or pursuing deeper exploration.



Part 1 - Creating A Kaleidoscope **Main Idea:** Students will use rulers, compasses, and protractors to place three mirrored surfaces at specific angles to create complex, symmetrical reflections of colorful patterns in a kaleidoscope. They will then explore the math involved in the construction and in the reflected patterns.

INTRODUCTION

When light enters a kaleidoscope, the mirrored surfaces bounce the light back and forth (think of a ball bouncing off a wall), creating multiple images of the beads, sequins, and other objects inside. Students will discover the relationship between the number of images they see and the position of the mirrors. For instance, a two-mirror kaleidoscope with mirrors positioned in a V-shape with a 45-degree angle will create eight reflections, or a four-pointed star. However, if the two mirrors are situated at 90 degrees, then four images, or a two-pointed star, will appear. A 30-degree position will create twelve images, or a six-pointed star.

MATERIALS

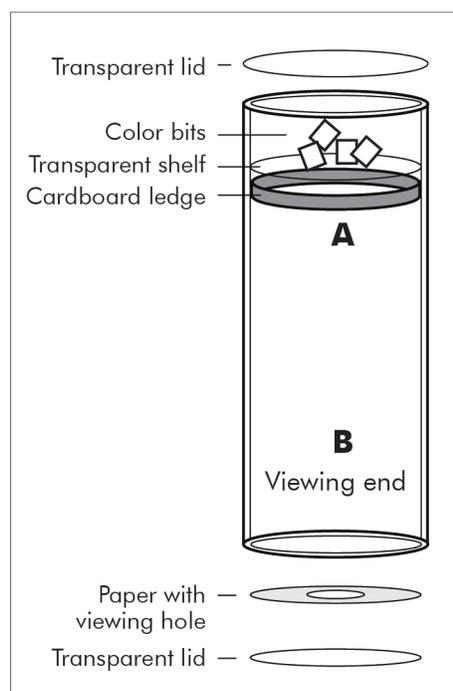
For each student:

- 1 empty can with both ends removed - this tube will be the body of the kaleidoscope
- Construction paper
- 1 sheet cardboard mirror
- 1 sheet transparency film
- Clear colored beads or plastic chips
- 1 piece of corrugated cardboard

For each small group:

- Markers
- Ruler
- Compass
- Protractor
- Clear tape
- Scissors
- Glue sticks
- 4 Plexiglass mirrors

Tube construction



ACTIVITY: CREATING A KALEIDOSCOPE

Note: In preparation, make a kaleidoscope before the class for demonstration and comparison. Make this kaleidoscope with two mirrors rather than three. Initially, put three mirrors together in a triangle as per the instructions below, but blacken one of the mirrors or simply turn it around, so that only two of the mirrors can reflect the image.

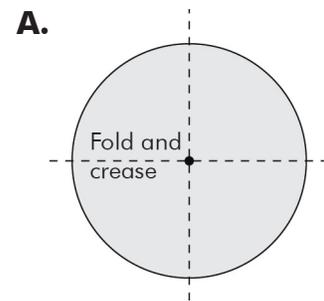
STEP 1: PREPARE THE PIECES

Preparing the tube: Have students mark one end of their can side A and the other end B.

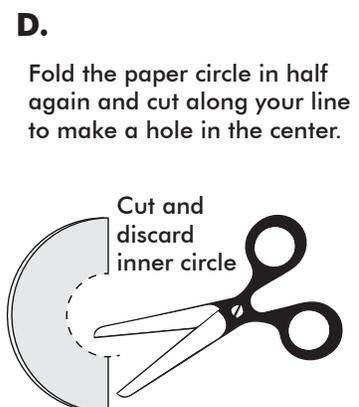
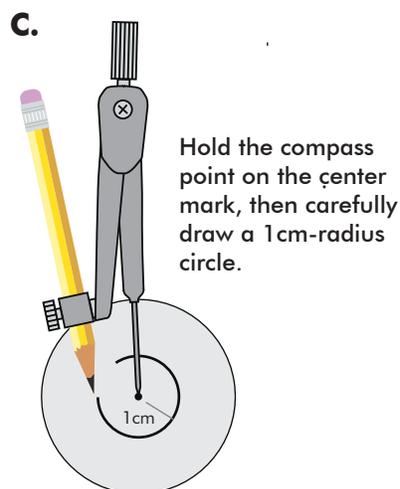
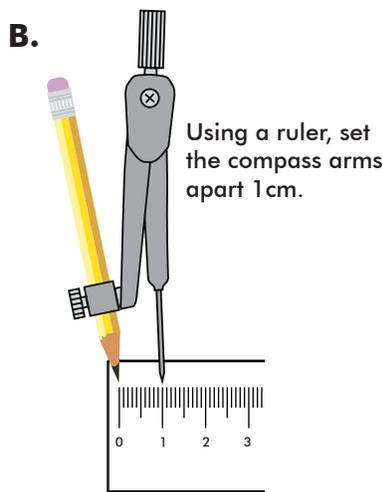
Paper circle: Set one end of the can on a piece of construction paper and trace the end with a pencil to make a circle. Cut out the circle. This is now a template.

Large transparent circles: Use this template to cut two circles out of the transparency film. To ensure the paper circle template doesn't move, use tape to secure the center of the paper circle to the transparency film, then cut out two circles this size.

- A.** *Paper with viewing hole:* Fold the paper circle template in half. The crease at the center of the circle is the diameter. Use a ruler to measure the crease and put a mark at the center. Now unfold it.
- B.** Set the compass at 1 cm.
- C.** Hold the compass point on the center mark, then carefully draw a 1 cm-radius circle.
- D.** Fold the paper circle in half again and cut along your line to make a hole in the center.



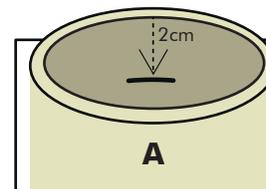
Small transparent circle: Place the ends of the compass just inside the end of the can tube and adjust it so that the pencil and metal point just barely touch the inside edges of the tube. Then, without moving the ends, transfer the compass to a piece of paper. Press firmly with the metal point and make a small mark with a pencil on the paper to leave two marks. Use a ruler to draw a line between the two marks. This is the diameter of a new circle. Find the radius by cutting the diameter in half. Put a mark on the diameter line in the middle. Place the point of the compass on the center mark, put the pencil at the end of the radius, and draw a circle. Cut out this circle, tape it on the transparency film, and cut out the circle in the film. This circle will fit inside the tube.



Cardboard strip: Cut a strip of corrugated cardboard 20-30cm long and 1cm wide. Make two of these. In some cases, you may need to make the strip longer depending on the circumference of your can.

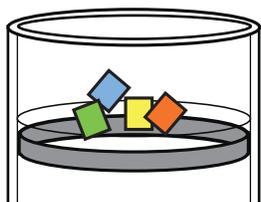
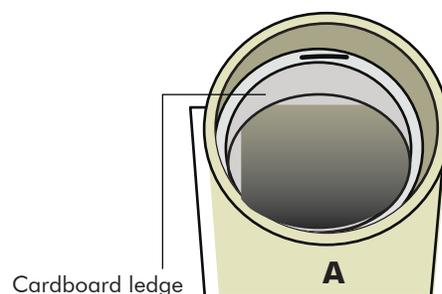
STEP 2: ASSEMBLY

Tube assembly: Place a ruler 2cm into end A of the tube. Make a mark on the tube at the end of the ruler. Continue to make five more marks, all 2cm from the end of the tube.



Roll the cardboard strip into a loose circle and insert it into the end of the tube marked A. Push the strip up against the tube ensuring the cardboard ring is snug and touches the can all the way around. The ends will overlap; mark the strip where the edges overlap and cut off the extra.

Apply glue just under the pencil marks inside end A. Insert the cardboard strip so that it covers the glue. Adjust the strip so that the top edge just touches the pencil marks and press into place.

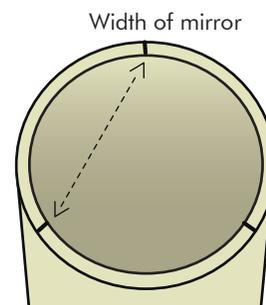


Take the small transparent circle, insert it into end A, and let it rest on the cardboard ring. Place colored transparent beads and plastic chips on the transparent circle.

Apply glue to the ridge of end A and press one of the large transparent circles firmly in place on the outside of the tube. Instead of glue you could tape this circle into place.

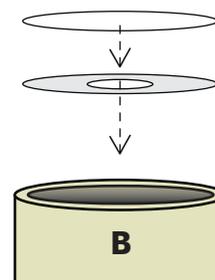
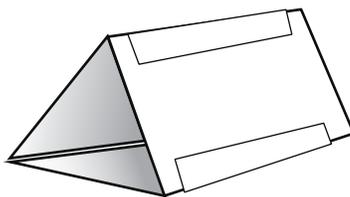
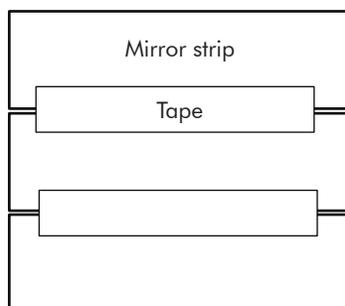
STEP 3: THE MIRRORS

Cardboard mirror strips: Use the other cardboard strip and wrap it around the outside of the tube. Mark where the strip meets itself and cut off the excess. Measure the length of the strip and divide this length by three. Use this number to mark the strip to make three equal parts. Wrap the strip around tube end B, hold it in place, and make marks on the end of the tube that correspond to the marks on the strip. Make a mark where the ends of the cardboard strip meet. Using these marks, measure the distance from one mark to another; the distances between any two marks should be equal. This distance will be the width of the mirror strips.



Insert a ruler inside end B until it touches the cardboard ring at the other end. Read this distance on the ruler. This distance will be the length of the mirror strips.

Using the measured width and length, measure and cut out three mirror strips. Place the strips mirrored-side down and join them together lengthwise and tape them together. Stand the strips up and fold them in so that the mirrored sides form the inside of a triangle. Insert the triangle into the tube from end B until it touches the ring at the other end.



Apply glue to the ridge at end B, attach the paper circle with the hole in the center, and cover it with the last large transparent circle.

Your kaleidoscope is ready to use!

ACTIVITY: EXPLORING WITH THE KALEIDOSCOPE AND MIRRORS

STEP 1: EXPLORE THE KALEIDOSCOPE (20 MINUTES)

Exploring with the kaleidoscope: Have students look through their kaleidoscopes. What do they see? What works well? Share them with one another. Pass around the two-mirrored kaleidoscope for comparison. How many reflections does each yield?

Discussion: Bring students together without the kaleidoscopes. Have students try to explain how the kaleidoscope works: What is going on? How does your kaleidoscope differ from the two-mirrored kaleidoscope? Record responses on a white board or chart paper so everyone can see.

STEP 2: EXPLORING MIRRORS

Materials: 4 Plexiglass mirrors, each 5 inches x 7 inches

1. Stand two mirrors about 12 inches apart. Place an object between them. How many reflections do you see?

2. Tape two mirrors together – the tape should act like a hinge so that you can swivel the mirrors. Tape a protractor to the table so that the straight edge is perpendicular to the edge of the table. Place the mirrors so that the left mirror touches the straight edge of the protractor and adjust until the hinge lines up with the 0 mark on the protractor. Place a small object on the table at the open end of the mirrors.

Holding the left mirror in place, move the right mirror. How many images can you make? What is the fewest number of images you can make?

Find the angle of the mirrors for various mirror positions. Use the angle numbers that are on the inside of the protractor arc. At what angle do you get the largest number of images?

For older students:

3. Calculate the number of images seen at any angle using the following formula: 360 divided by ___ angle, then subtract 1. Example: For an angle of 40° , $360 \div 40 = 9 - 1 = 8$ images.
4. Predict the mirror angle that will show a certain number of images. Add 1 to the number of images and divide 360 by this number.
Example: For 6 images, $6 + 1 = 7$; $360 \div 7 = 51^\circ$

Do an experiment to check. Set the right hand mirror at 51 degrees. How many images do you see? Is the mathematical prediction correct?

5. Practice predicting the number of images seen at different mirror angles or the mirror angle needed to see a certain number of images.

Part 2 - Fun with Geometry Main Idea: Geometry explores the math of shapes, such as squares, triangles, pentagons, and others. If shapes are similar in form and size, geometry can be used to create repeating patterns known as tessellations. By understanding the concept of symmetry, students can arrange tessellations and create unique, interesting art.

INTRODUCTION

A tessellation is created when one or more geometric shapes are repeated over and over again to fill a surface without any gaps or overlaps. Tessellations are used to tile floors and ceilings, to create mosaics, and more.

Preparation: Either use our tessellation examples or bring in some examples of tessellations. These could include photographs of different floor tile patterns, ceiling tiles, a checker board, M.C. Escher's "Butterflies," etc. Cut out squares and triangles in two colors of paper so that each student can cover a sheet of paper with either triangles or squares. Practice making a tessellation by alternating colors and positions of triangles on a sheet of paper.

MATERIALS

- Paper
- Pencils
- Art supplies
- Rulers
- Protractors
- Scissors
- Glue
- Tessellation examples (pages AM 11 and AM 12)

ACTIVITY: FUN WITH GEOMETRY

STEP 1: TESSELLATION EXAMPLES

Show several examples of tessellations. Use the samples provided or ones you find in books at the library or online. Ask: "What do you notice about these works of art? What is special about this symmetrical pattern? (It can go on forever.) Where have you seen other types of symmetrical patterns?" (e.g., tiles or bricks on buildings and floors, public art). Tell students that these symmetrical patterns are called tessellations.

Hold up a triangle and ask: "What do you think it means that these triangles are symmetrical?" Try to get each student to express an idea. (Symmetry: one side exactly mirrors the other). Then demonstrate how to make a tessellation by making a triangle tessellation. Reiterate that the triangles are symmetrical and tessellations are symmetrical patterns.

STEP 2: TESSELLATIONS

Give each student a sheet of paper, a glue stick, and a handful of the squares or triangles in both colors.

Starting at the center of the paper, glue down a triangle or a square. Next, add another shape, but in the other color. Continue this process until the entire page is covered. Pieces may stick out beyond the edge of the paper.

Discussion: If you had a larger paper, could you continue this pattern? Could you cover the side of a building? What is special about this pattern? (It goes on forever.) This continuous repetition is a key feature of a tessellation. Could you arrange the shapes in a different way and still have a tessellation? Point out the photographs or other examples. How are they similar?

ACTIVITY: SYMMETRY TRANSFORMATION TYPES

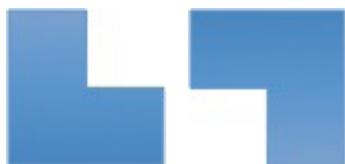
There are many ways that a single pattern can be repeated. If you look closely at many tessellations, you'll see figures using these transformation techniques: translation, reflection, and rotation, or a combination of any of these.



Translation (above) – This is a type of symmetry in which the shape is simply copied and repeated. This technique is called “slide” or “glide” or “translate.” One shape is translated over and over.



Reflection (above) – The shape is repeated, but is facing a different way.



Rotation (above) – The shape is repeated, but is turned around a point.

Combination: You can create a tessellation that combines all three of these patterns. You can create one row doing translation, then the next row doing reflection, or have a rotational tile that repeats along a row.

ACTIVITY: CREATING TESSELLATION ART

STEP 1: MAKE ART

Have students pick a simple shape or figure from at least one transformation type above. Place the shape or figure to tessellate on a sheet of paper and trace it. Slide the shape over, line it up with the first shape so there are no overlaps or gaps, and trace it again. Continue tracing the shape until the entire paper is covered. Use crayons or markers to color the shapes or figures. Can you color the page in a way that makes a pattern?

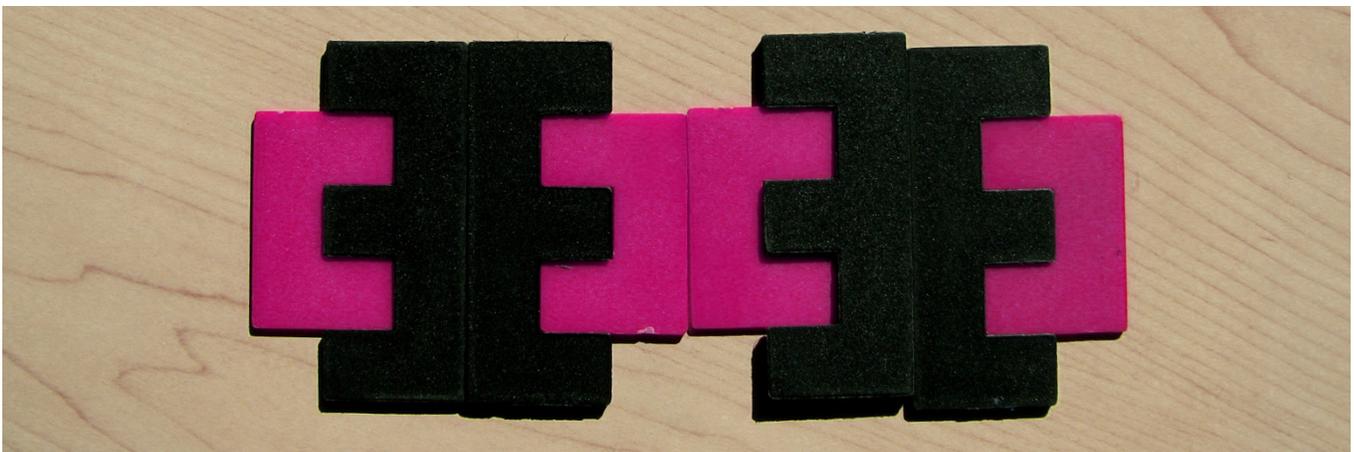
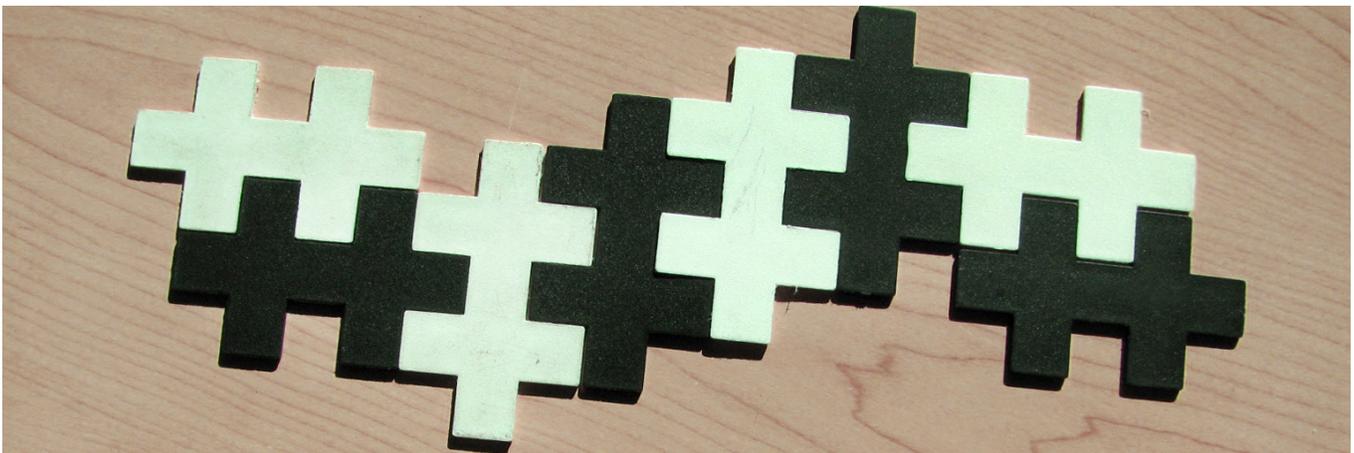
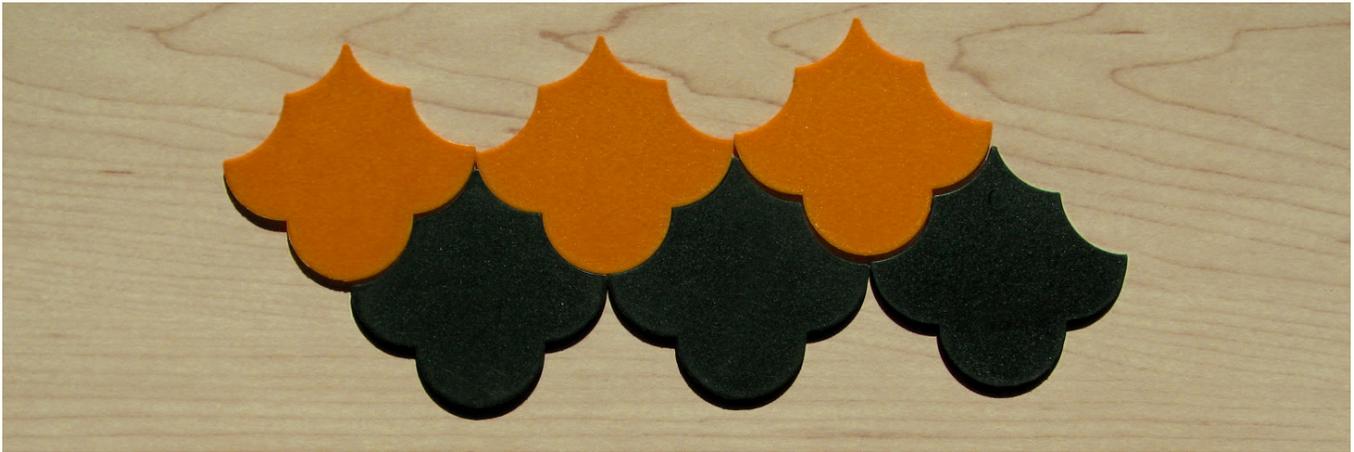
STEP 2: SHARE THE ART

Post the student art on the wall and gather students for a discussion.

Discussion: Why did you do what you did? How do the pictures you made compare to what you see in a kaleidoscope?

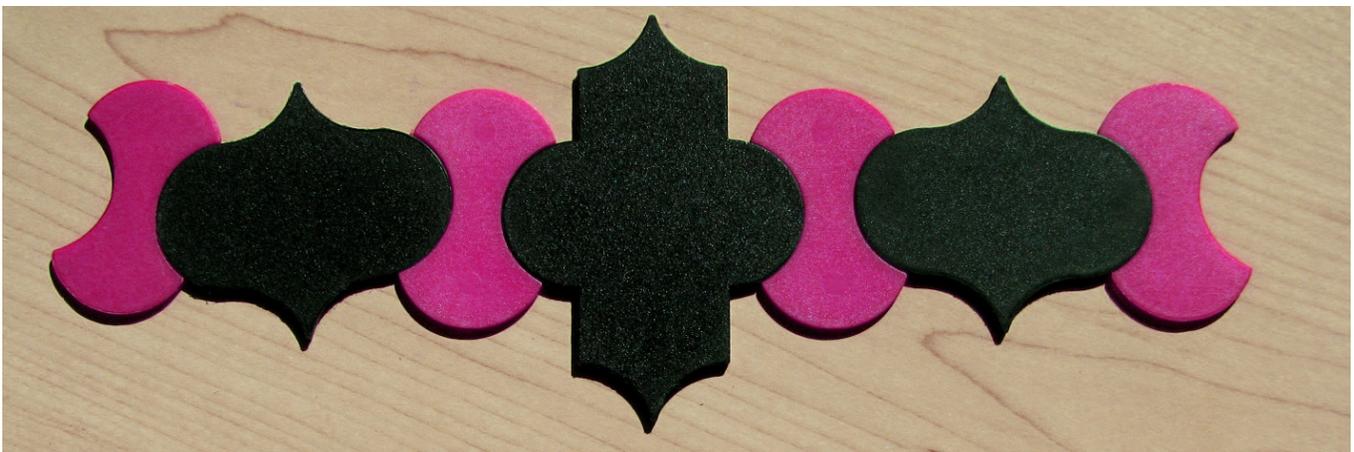
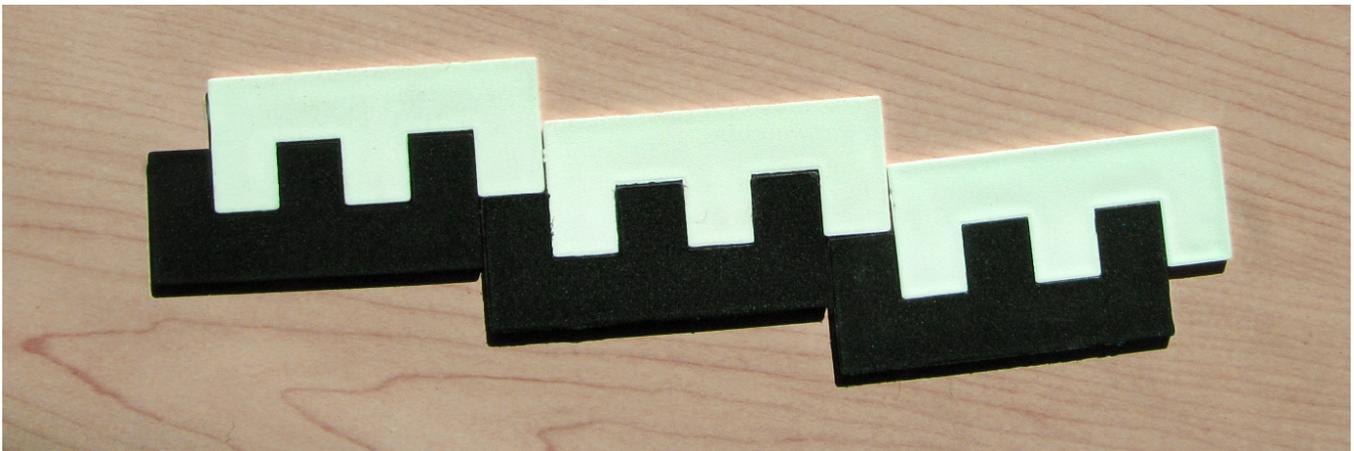


Tessellation Examples





Tessellation Examples



Part 3 - Fractal Art Main Idea: *Fractals* are never-ending, self-similar patterns; they repeat themselves on different scales. **Geometric fractals** are made by following a simple process over and over again to generate the pattern. Using measurement and precision, students will generate their own *Sierpinski triangles* that are interesting art pieces alone, but can also be combined to make larger fractal art in groups of 3, 9, and 27.

INTRODUCTION

Fractals are commonly found in nature, geometry, and algebra. Two common fractal patterns found in nature are branches and spirals. Today, students will generate a geometric fractal called a **Sierpinski triangle (examples are easily found online)**. It is made of **equilateral triangles** (triangles in which all three sides are equal). The midpoints are connected to make a new, smaller triangle inside of the larger one and that new triangle is then removed from that generation, and the process continues in the remaining triangles. The removed triangle will always be pointing down while the rest will be pointing up. The number of triangles pointing up will increase by a factor of three in each new generation.

Preparation: Make copies of the triangle handout on page AM 15. Practice making a Sierpinski triangle with four generations. If possible, find and share fractal examples.

MATERIALS

- Triangle template (see page AM 15)
- Sierpinski carpet and Koch curve samples (see page AM 16)
- #2 pencils and erasers
- Crayons, colored pencils, or markers
- Rulers (1 per student)
- Scissors

ACTIVITY: EXPLORING FRACTALS

STEP 1: WHAT DO FRACTALS LOOK LIKE?

Discussion: Gather images of fractals from nature and math. Excellent math examples include the Mandelbrot set and Koch curve. These can be found in books at the library or online. Show several examples of fractals from nature. Ask students to describe what they see. How are these fractals? What shape is repeating? Show examples of fractals from mathematics. How are these fractals? What shape is repeating?

Fractals are **self-similar**. Does anyone know what that means? It means the parts are the same as the whole. As you zoom in on a fractal and look at a smaller piece of it, it will have the same shape and pattern as the whole and as every other part. (Demonstrate using Koch curve or Sierpinski carpet).

ACTIVITY: SIERPINSKI TRIANGLES**STEP 1: FIND THE MIDPOINTS**

Give each student a pencil, a ruler, and a triangle template. Using the ruler, students should measure each side of their triangle and put a dot or a small line at the midpoint of each side. Once all of the midpoints are marked, students then use a ruler to draw a line connecting the midpoints to form a new triangle in the middle of the big one.

STEP 2: COLOR IN THE TRIANGLE

Using colored pencils, crayons, or markers, have students color in the upside-down triangle they just made. This is the triangle that is being removed and is no longer included in the number of triangles or area calculations.

Now, there are three triangles remaining. Find the midpoints and draw in the downward-facing triangles for each.

STEP 3: REPEAT FOR FOUR GENERATIONS

Color in the three new downward-facing triangles. Encourage students to be creative and use different materials, colors, and techniques to fill in each generation of triangles. Repeat the process two more times until students have completed four generations of their fractals. After the last generation, they can color in the remaining upward-facing triangles too, so their whole triangle is colored in.

Discussion: How many triangles are in the second generation of the Sierpinski triangle? Is it possible to calculate how many triangles are in each stage rather than counting?

STEP 4: COMBINE TO MAKE A BIGGER FRACTAL

Have each student cut out his or her fractal. Ask students if they can make bigger triangle fractals by combining with their peers. How many students need to join their triangles to make another Sierpinski triangle? What's the biggest triangle the class can make?

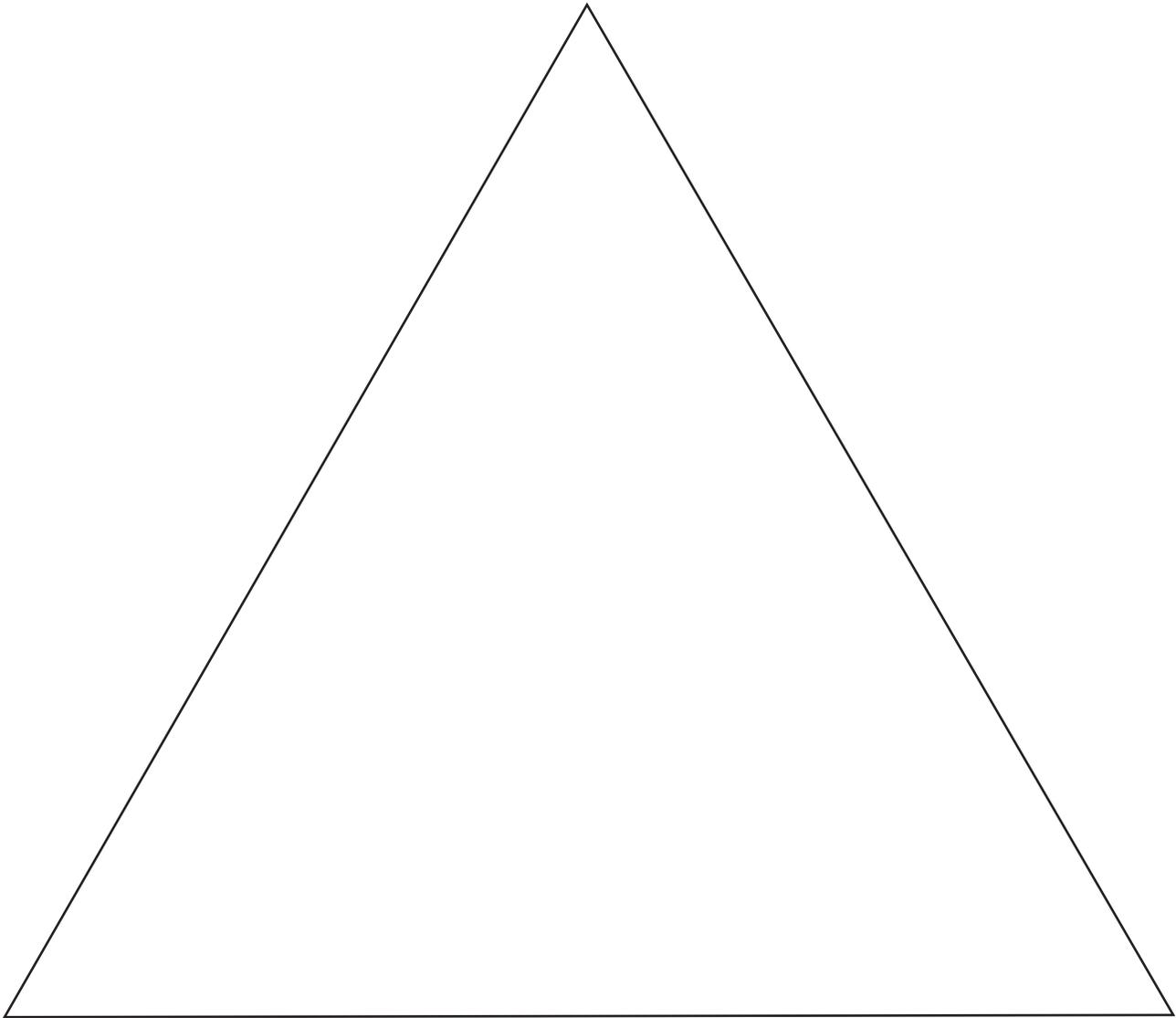
Have students break into groups of 3 and join their triangles to make new, bigger fractals. If there are enough students, join 3 of the group's triangles to form an even bigger triangle of 9 student triangles. How many student triangles would it take to make an even bigger version than the 9-triangle fractal?

Hang the group fractals (of 3, 9, or 27 student triangles) on the wall and have students look at them from a distance.

Discussion: What does it look like? Is it art? Is it math? Ask students to verbalize how math can be used to make interesting art.



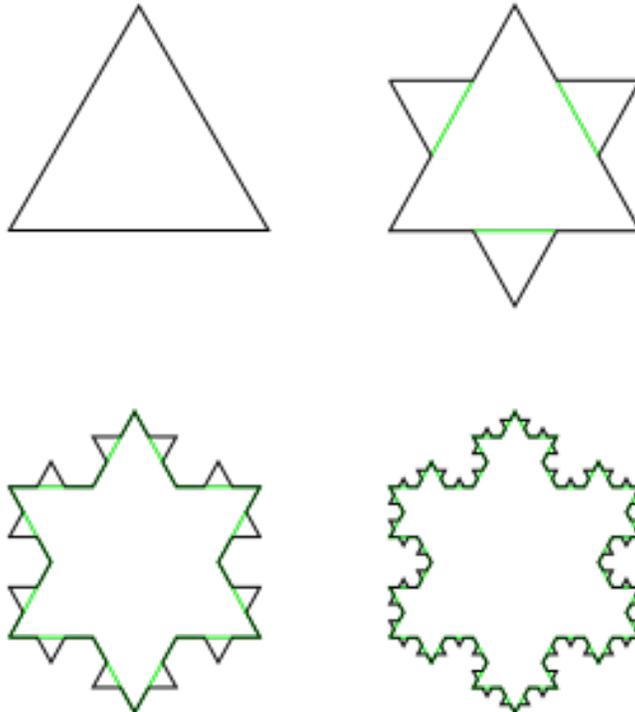
Triangle Template



Sierpinski carpet and Koch curve samples



Sierpinski Carpet, which is made of many Sierpinski Triangles



Koch Snowflake which includes many Koch curves forming all the edges

I am a creative designer at Sandia National Laboratories, which means I help scientists communicate their work by designing posters, brochures, reports, and presentations. I like my job because I get to work with people from all over Sandia, learn about their work, and then help them share it with other people. In school, I knew I wanted to use my creativity to help people, but I knew I'd need math skills, too. All the math classes I took—even though they were hard—helped me learn to think logically and taught me many skills that inform my design work. I use geometry and proportions every day to come up with interesting new designs in different sizes that attract people's attention. Drawing pictures really helped me in my math classes, because I'm a visual learner, so picturing the math helped me remember it. I didn't always like math in school, but I'm glad I stuck with it, because I use it every day!



STACEY LONG
Creative Designer

I am a civil engineer at Sandia National Laboratories, working in renewable energy. I help American Indian tribes and Alaskan Native villages learn about and take advantage of renewable energy technologies, like solar panels. When I was in elementary school, I liked math and science and always wanted to figure out how different gadgets worked. Now, I get to educate others about our natural resources and how renewable energy technologies work. Math isn't always easy, but it is very useful and is part of what helped me become an engineer. Engineering is a great way to help people! I had to practice my math and engineering skills a lot to get to the point that I can help to educate others. Keep practicing your math skills, and you can find an exciting and fulfilling career!



SANDRA BEGAY
Civil Engineer

Changing the Equation: After-School Math Curriculum

THE BUILT ENVIRONMENT MATH

Based on *After-School Math PLUS*
from the Educational Equity Center at
Academy for Educational Development



Funding provided by



Sandia National Laboratories

KIT: BUILT ENVIRONMENT

This After-School Math Kit includes engaging activities that are fun for both students and after-school staff. Students learn math while working in cooperative, supportive groups facilitated by staff members. Even better, after-school staff don't need to be math experts!

Through these activities, students and staff hone math skills, gain confidence in math, and increase their enjoyment of math.

THIS KIT INCLUDES

■ **Instructions to facilitate 4 activities.** These activities are simplified summaries and extensions of the “After-School Math PLUS: Built Environment” curriculum created by the Educational Equity Center at Academy for Educational Development (AED).

The 4 activities are:

- Part 1. According to Height
- Part 2. Planning the Ideal Community
- Part 3. Mapping Your Community

■ **Scientist Spotlight** Ask your students “who is a scientist?” and you'll typically get answers that include white lab coats, microscopes, and bubbling beakers. All of these images reflect some aspect of science and STEM (Science, Technology, Engineering, Math), but they don't provide a full picture. We include stories of two STEM professionals that work at Sandia National Laboratories. Read these with your students, and together list some of the activities, skills, and experiences from the stories. What surprises your students? What was unexpected? Does this change how they think of scientists and engineers?

TIPS FOR LEADING ACTIVITIES

- Give students opportunities to share their ideas with you and with each other.
- Make sure to introduce each activity. Learning happens best when learners know what's coming up and why it matters to them.
- Ask open-ended questions, rather than those that have a “yes” or “no” answer.
- Ask questions that inspire the learner to thoughtfully analyze a situation and consider consequences, such as, “What do you think will happen if you do this?”
- Give the learner time to answer the question. Ask the question, then wait. A while. Trust us: thoughtful answers take time.
- When a learner tells you what they think, respond by repeating and paraphrasing what they have said without criticism.
- Don't give too much praise or reject ideas. Telling a learner they are right or wrong can discourage them from generating additional ideas or pursuing deeper exploration.

Part 1 - According to Height Main Idea: Did you know that you are about as tall as your outstretched arms? Students will explore scale, ratio, and proportion, then use graph paper and measuring tapes to create a scale picture of themselves.

INTRODUCTION

Scale is the ratio between the actual size of an object and a representation of it. A **ratio** compares one part of an object to a different part – how does the size of your head compare to the length of your arm? **Proportion** compares one part to the whole – your head is about $1/8$ of your height. These are important concepts for creating accurate representations. Scale and proportion are important to artists, engineers, and many other professionals.

MATERIALS

- Graph paper (2 sheets per student)
- Colored pencils (several per group)
- Measuring tape (1 per group)
- *Body Proportion Activity Sheet* (1 per student, found on pages BE 5 - BE 6)

ACTIVITY: ACCORDING TO HEIGHT

STEP 1: DRAW YOURSELF (15 MINUTES)

Have students draw a horizontal line across their paper, four boxes up from the bottom of their graph paper. Then, starting at the line, ask them to draw a picture of themselves from head to feet, with the bottoms of their feet touching the line. A stick figure is fine, but have them include the shoulders, elbows, hands, pelvis, knees, and feet.

Collect the drawings and hang them on the wall, arranged by height using the line at the bottom of the pages to align the pictures. Then, ask students to physically line up according to height. Are they in the same order as the drawings?

Discussion: Why or why not? Ask students for ideas about how they could draw pictures of themselves that would accurately represent their height. Explain that these would be “drawn to scale.”

STEP 2: MEASURE YOURSELF (30 MINUTES)

Have students work in small groups of three. Group members will take turns measuring each other and recording their results on the *Body Proportion Activity Sheet*. Then, group members will work together to answer the questions.

STEP 3: DRAW YOURSELF TO SCALE (15 MINUTES)

Using the new information on the *Body Proportion Activity Sheet*, have students use their measurements to draw a picture of themselves to scale on the graph paper. As a group, agree on the scale to use. For example, each square on the graph paper can represent 10 cm.

Discussion: Which body proportion is the most surprising? Have students describe in their own words what “scale” and “proportion” mean and what purpose they served in their drawings. Was it easier or harder to create their drawings the second time? Why?

Connect: Where else is scale used? Ideas might include maps and models (cars, planes, trains). Discuss how scale is used in each idea.

Many artists use standard human figure proportions, included in this activity. For example, the height of a person is the same as the measurement of their outstretched arms (a 1:1 ratio). And the length of a person’s head is typically about one-eighth ($1/8$) of their overall height (a 1:8 ratio). Did you know that the width of your shoulders is about a quarter ($1/4$) of your height?



Body Proportion Activity Sheet

	Your Measurement (in inches)	Human Figure Proportions for artists
Total height (measure from the bottom of your feet to the top of your head)		8 heads
Head height (bottom of your chin to the top of your head)		1/8 of total height
Hip height (measure from your feet to your hip)		1/2 of total height
Arm Span (with arms raised parallel to the ground at shoulder height out to your sides, measure left hand fingertip to right hand fingertip)		Same as total height
Elbow to fingertip		1/4 of total height
Elbow to armpit		1/8 of total height
Wrist to fingertip		1/10 of total height
Width of shoulders (measure between the points of your shoulders)		1/4 of total height
From the bottom of the foot to the bottom of the knee		1/4 of total height

QUESTIONS:

1. Write down the ratio of your head height to your total height. How does your own ratio compare with that used by many artists?
2. What is the ratio of your total height to your hip height? How does it compare with the proportions for artists?
3. What is the ratio of your total height to your arm span? Is it 1:1? If not, which one is larger?
4. Find three other ratios and see how they compare with the proportions for artists.

Part 2 - Planning the Ideal Community **Main Idea:** What would your ideal community look like? Students will work in small groups to design a community. Each group will be given a budget and will choose components that they want included in their community (such as parks, transportation systems, services, and recreation).

INTRODUCTION

This “simulation” style activity challenges students to apply what they know about community to create a model community for a fictitious group of 20 households. The imposed budget and cost list will force groups to make choices. While there will be no absolute “right” or “wrong” choices, your role will be to challenge students to justify their choices and to ensure students are being thoughtful. Giving specific praise—“I really like how you planned this to minimize the walking distance between the houses here and the park”—as well as specific challenges—“If you locate the landfill here, don’t you think the smell will impact the houses right next to it?”—will work wonders in motivating teams to make careful decisions.

MATERIALS

- Graph paper (6 sheets per group)
- Colored pencils (several per group)
- Ruler (1 per group)
- *Community Structures and Features Cost Sheet* (1 per group, found on page BE 11)

ACTIVITY: PLANNING THE IDEAL COMMUNITY

STEP 1 (15 MINUTES)

Discussion: Challenge the students to write what community means to them in one or two sentences. Ask students to volunteer to share their definitions. Responses might include a group of people who all have something in common or ideas around people coming together to help each other and others in need. Use chart paper to categorize responses. Category headings might include: Place, Interdependency, Belonging, Common Interests, Culture, Family, Support, Common Good, etc.

STEP 2 (15 MINUTES)

Divide students into small groups of three or four. Have each group develop a list of features and structures they associate with their community.

Discussion: Help them with their list by prompting them: What brings people together? How are community members supported? What do families need? How do communities provide support, including food, water, shelter, and medical care? How do people move around?

Connections: Have each group share their list. Compare these lists with the categories from the original discussion. What is missing? What can you add to your community to promote a sense of belonging, support, culture, safety, comfort? Tell the students that they will now be given the role of “Community Planning Team” tasked with creating a community for 20 families.

STEP 3 (30 MINUTES)

Each small Community Planning Team will be given a budget of \$10,000 that they can spend on a list of potential structures and features as listed in the handout: *Community Structures and Features Cost List*. Each team should spend their budget to obtain the features they think should be included in their “ideal community.”

Discussion: Challenge each group to compare their final list with the group’s definition of community. What is missing? Have each team share their list and justify the choices they made. What features will provide a feeling of belonging? What features will help promote culture and common interests? Are there enough trees and other green spaces included? Why are these important? What are their favorite aspects of community, and why?

Part 3 - Mapping Your Community **Main Idea:** What would your ideal community look like? Each team will use measurement, scale, and proportion to create a blueprint drawing of their ideal community.

INTRODUCTION

This “simulation” style activity challenges students to apply what they know about community to create a model community for a fictitious group of 20 households. The group will have to agree upon the scale that they will use for their model, and think about how to draw the community structures and features to match that scale.

MATERIALS

- Graph paper (6 sheets per group)
- Colored pencils (several per group)
- Ruler (1 per group)
- *Community Structures and Features Cost Sheet* (1 per group, found on page BE 11)
- *Proposed Size and Scale for Community Structures Sheet* (1 per group, found on page BE 12)

ACTIVITY: MAPPING YOUR COMMUNITY

STEP 1: (10 MINUTES)

Have each Community Planning Team review their *Community Structures and Features Cost Sheet*. They should have spent \$10,000 to provide structures, features, and services to their community of 20 houses. Have them give their community a name.

Next, ask each group to think about the arrangement of the features in their new community. Should the school be located next to the landfill? Should the hospital be located next to the outdoor theater where loud music is played?

STEP 2: (20 MINUTES)

Have each team take a piece of graph paper and cut out shapes to represent each feature and structure that they purchased for their community. Each shape should be made to scale. Give the students the *Proposed Size and Scale for Community Structures Sheet*. They can use the ideas listed on the sheet, or choose to make their own scale. Emphasize that the structures should be in proportion to each other.

Next, each team should tape together four sheets of graph paper. This will be the space in which their community will be located. Then, each team should arrange their features on their community space.

Discussion: Have each team justify their choices by asking questions as they try different ideas. Challenge the teams to think about the relationship among the features. Is it easy to get from one place to another? How would a child get from their house to the playground? To school? How easy is it for the police department or fire department to respond? If there are no roads, how will someone carry their groceries and other shopping bags from the store to their house?

STEP 3: (20 MINUTES)

Now that each team has an idea of how they would like their community to be arranged, they should transfer their ideas onto their community space (the four pieces of graph paper taped together) using colored pencils and the ruler. Have the students use words and symbols to label all structures and other features. They should also indicate the scale that they used; for example, 1 square = 10 feet.

Reflection: Ask each team to share their blueprint. Ask the other teams to share two things they like about the plan. Follow with one or two questions about why that team chose to do something in a specific way. Ask the presenting team: Are you satisfied with the plan? What trade-offs did you have to make? Would you make different choices if you had more money? What if you had less money?



COMMUNITY STRUCTURES AND FEATURES COST SHEET

CATEGORY	FEATURE DESCRIPTION	COST	Included in your community? List cost here
Environment	Trees	\$10 each	
	Pond	\$250	
	Desert ecosystem - desert scrub, grasses, and animals	\$5 per 10 square feet	
	Grassy Area	\$10 per 10 square feet	
Services	Hospital	\$3,000	
	Doctor's Office	\$500	
	Water Tower	\$250	
	Fire Department	\$500	
	Garage collection and sewage treatment	\$350	
	Landfill	\$250	
	Grocery Store	\$500	
	School	\$500	
	Police Station	\$500	
	Community Center / Cultural Center	\$500	
	Shopping Mall	\$2,500	
	Small Store	\$350 each	
Transportation	Airport	\$5,000	
	Roads for Cars	\$500 per mile	
	Train Tracks	\$2,000 per mile	
	Bicycle Lanes	\$250 per mile	
	Sidewalks / Walking Paths	\$150 per mile	
Recreation	Playground - small (slides, teeter totter, merry-go-round, monkey bars, and tether ball)	\$300	
	Playground - Medium (everything listed above + climbing structures, basketball court, and features for toddlers)	\$600	
	Playground - Large (everything listed above + zip lines, large tree house, musical area)	\$1,000	
	Mountain Bike Park	\$250	
	Skateboard Park	\$250	
	Amusement Park (like Cliff's)	\$4,000	
	Swimming Pool	\$500	
	Water Park	\$1,000	
	Outdoor Theater for Concerts, Plays	\$2,500	
			Total Cost:



PROPOSED SIZE AND SCALE FOR COMMUNITY STRUCTURES SHEET

FEATURE	SUGGESTED SCALE	OR CREATE YOUR OWN SCALE
Each square on the graph paper	10 ft by 10 ft	
Each house (20 houses total)	10 ft by 10 ft	
Pond	20 ft by 30 ft	
Desert ecosystem - desert scrub, grasses, and animals	per 10 square feet	
Grassy area	per 10 square feet	
Hospital	100 ft by 100 ft	
Doctor's office	20 ft by 20 ft	
Water tower	10 ft by 10 ft	
Fire department	20 ft by 20 ft	
Garbage collection and sewage treatment	50 ft by 50 ft	
Landfill	100 ft by 100 ft	
Grocery store	100 ft by 100 ft	
School	100 ft by 100 ft	
Police Station	20 ft by 20 ft	
Community center / Cultural center	20 ft by 20 ft	
Shopping mall	100 ft by 100 ft	
Small store	10 ft by 10 ft	
Airport	100 ft by 100 ft + runways	
Playground - small (slides, teeter-totter, merry-go-round, monkey bars, and tether ball)	20 ft by 20 ft	
Playground - medium	30 ft by 30 ft	
Playground - large	50 ft by 50 ft	
Mountain bike park	Your choice	
Skateboard park	20 ft by 20 ft	
Amusement park (like Cliff's)	100 ft by 100 ft	
Swimming pool	50 ft by 50 ft	
Water park	50 ft by 50 ft	
Outdoor theater for converts, plays	50 ft by 50 ft	

I review the work conducted at Sandia National Laboratories for potential emissions that pollute the air and contribute to climate change and ozone depletion. I like my job, because I'm helping to keep people and the environment safe now and into the future! It's important that big companies like Sandia Labs follow the rules to protect the environment, just like everyone else. In school, I took math classes every year and built up a solid foundation that helped me in my engineering and science classes once I got to college. I still use math all the time, especially showing my work and converting between different units—like from inches to centimeters. Math skills have helped me learn to think analytically and ask questions, two skills that have helped me in all the different parts of my life. Remember that different people learn in different ways, so if you don't understand what your math teacher says, ask questions and keep trying—it will be worth it!



ELIZABETH QUINLEY
Air Quality Compliance Engineer

I am a geophysicist at Sandia National Laboratories who plans experiments using maps and information about an area's geology (rock formations). The data from these experiments helps me determine what is underground without having to dig a big, expensive hole. People need to know what is underground so they can build buildings, run plumbing and utility lines, and find oil, gas, and other resources. My job takes me all over the world; I've even done experiments in Antarctica! I use math in my experiments to predict wave speeds, amounts of energy, and directions to make sure my experiments run smoothly and the data is accurate. It took me a long time to learn all of the math I needed and to get good at it, but practice helped. Everything can be hard when you are first learning, but if you ask for help when you need it and keep practicing, it will get easier!



NEDRA BONAL
Geophysicist

Changing the Equation: After-School Math Curriculum

MUSIC MATH

Based on *After-School Math PLUS*
from the Educational Equity Center at
Academy for Educational Development



Funding provided by



Sandia National Laboratories

KIT: MUSIC MATH

This After-School Math Kit includes engaging activities that are fun for both students and after-school staff. Students learn math while working in cooperative, supportive groups facilitated by staff members. Even better, after-school staff don't need to be math experts!

Through these activities, students and staff hone math skills, gain confidence in math, and increase their enjoyment of math.

THIS KIT INCLUDES

■ **Instructions to facilitate 4 activities.** These activities are simplified summaries and extensions of the “After-School Math PLUS: Music Math” curriculum created by the Educational Equity Center at Academy for Educational Development (AED).

The 4 activities are:

- Part 1. Creating Rhythm
- Part 2. Making Rhythm Patterns
- Part 3. Composing Music
- Part 4. Discovering Music Fractions.

■ **Scientist Spotlight** Ask your students “who is a scientist?” and you'll typically get answers that include white lab coats, microscopes, and bubbling beakers. All of these images reflect some aspect of science and STEM (Science, Technology, Engineering, Math), but they don't provide a full picture. We include stories of two STEM professionals that work at Sandia National Laboratories. Read these with your students, and together list some of the activities, skills, and experiences from the stories. What surprises your students? What was unexpected? Does this change how they think of scientists and engineers?

TIPS FOR LEADING ACTIVITIES

- Give students opportunities to share their ideas with you and with each other.
- Make sure to introduce each activity. Learning happens best when learners know what's coming up and why it matters to them.
- Ask open-ended questions, rather than those that have a “yes” or “no” answer.
- Ask questions that inspire the learner to thoughtfully analyze a situation and consider consequences, such as, “What do you think will happen if you do this?”
- Give the learner time to answer the question. Ask the question, then wait. A while. Trust us: thoughtful answers take time.
- When a learner tells you what they think, respond by repeating and paraphrasing what they have said without criticism.
- Don't give too much praise or reject ideas. Telling a learner they are right or wrong can discourage them from generating additional ideas or pursuing deeper exploration.



Part 1 - Creating Rhythm Main Idea: Many children enjoy music, but few are aware of how much math is involved. Some pieces of music are popular because of their mathematical structure. In this activity, students will start to explore the ways music and math are related by identifying and replicating rhythms.

INTRODUCTION

This activity involves gathering student ideas about music and the relationship between math and music. These concepts are complex. If students are struggling, add some ideas of your own and give students time to think about and add to what has been said.

Rhythm is a strong, regular, repeated pattern of movement or sound, while the **beat** is the underlying tempo or pace of the music. If you tap your foot along with music, you are tapping out the beat. If you are humming or singing along with the music, you are copying the rhythm.

MATERIALS

- Chart paper
- Markers
- Paper and pencils
- Music (CD, online streaming, iPod, or MP3 player; speakers)

ACTIVITY: CREATING RHYTHM

STEP 1: WHAT IS MUSIC? (15 MINUTES)

Introduce the activity by asking students what they know about music. Ask questions such as: What kind of music do you listen to? Where do you listen? Who plays a musical instrument? What is it about the music that you like?

Have students make a bar graph of “Our Favorite Kind of Music” (country, rock, hip-hop, etc.). For extra practice, make a bar graph for another question. What are different ways to visualize this data? For example, many students will have heard of a pie chart. You could also have students stand up or move to different parts of the room to indicate which music they like to listen to (“move to this corner if you like 90’s music; move to this corner if you like classic rock”; etc.).

Discussion: In pairs, have students think about and discuss the following question: “Do you think there are ways that music and math are related?” Record student ideas on chart paper. If needed, prompt additional ideas by adding concepts like musical beat, rhythm, counts, patterns, measures, whole and half notes, etc. to the chart paper. Tell students that they will be exploring music during the next few programs and discovering ways that music and math are connected. Hang up the chart and invite students to add to the list when they discover new links between music and math.

STEP 2: LISTEN TO MUSIC (15 MINUTES)

Play a short piece of music for the students once or twice – e.g., Dave Brubeck’s “Take 5” (<https://www.youtube.com/watch?v=vmDDOFXSgAs>).

Discussion: What instruments do you think you hear in the music? Have the students try to imitate one of the instruments or sounds they hear. Does the instrument or sound repeat more than once? Is there a beat or rhythm to the music? Have students try to articulate the difference between beat and rhythm. If possible, watch <https://www.youtube.com/watch?v=2SqOWtZXDeg>, a video of students exploring the difference between beat and rhythm.

Definitions: The **beat** is the unchanging tempo or pulse of the tune; it is what you tap your foot to. The **rhythm** is the length and accent given to a series of notes in a piece. For example, when you sing “Hickory Dickory Dock,” the rhythm is the same as the words with a sound for each syllable. The **pitch** describes whether the sound of the words (notes) go up or down (high or low). Rhythm and pitch together create a melody - the rhythm determines the length of the notes and the pitch determines whether the notes are high or low.

Read more: Difference Between Beat and Rhythm: www.differencebetween.net/miscellaneous/difference-between-beat-and-rhythm/#ixzz4gt3rQgVs

STEP 3: EXPLORE RHYTHM (10 MINUTES)

Once more, play the piece of music students listened to earlier. Ask students to listen to the whole song once, then clap the rhythm together (not the beat, but the pattern of the music). What are the challenges to finding and keeping the rhythm?

STEP 4: MOVE TO MUSIC (20 MINUTES)

Ask students if they dance to music. When and where do you dance? Do you know dances like the Electric Slide, the Dab, or the Chicken Dance? If so, ask them to teach everyone. After the dances, discuss the patterns of steps with students.

Discussion: Can anyone think of ways we used math when we listened to music and danced? Students may come up with words like counting or pattern. Add them to the chart, if necessary.

Part 2 - Making Rhythm Patterns Main Idea: Rhythm patterns are essential and fundamental in music. They are one example of math in the design of music. Students will create rhythm patterns and make visual representations of them.

INTRODUCTION

In music, the rhythm is the strong, regular, repeated pattern of sound, while the beat is the underlying tempo or pace of the music. If you tap your foot along with music, you are tapping out the beat. If you are humming or singing along with the music, you are copying the rhythm. The creation of music involves math in the rhythm, beat, and notes. In this activity, students will design a musical pattern, then depict their pattern visually so others can play it.

MATERIALS

- *Rhythm Pattern Squares Example* (found on page MM 13)
- Markers or crayons
- Squares of paper or cardboard (8-inch squares work well; smaller squares also work fine)
- Glue sticks
- Large pieces of paper to hold four of the squares

ACTIVITY: MAKING RHYTHM PATTERNS

STEP 1: WHAT IS RHYTHM? (20 MINUTES)

In the previous activity, students listened to music and clapped the rhythm together. Ask the students to define rhythm in their own words. Challenge their sense of rhythm by having a “Clap Off.” Have everyone sit in a circle. Clap out a simple rhythm and pass it on to the next student in the circle. This student should repeat the rhythm and pass it to the next student. Keep going until the rhythm has passed all the way around the circle. If a student doesn’t repeat the rhythm correctly they are eliminated. Continue playing. After each round, increase the difficulty of the rhythm until there is only one student left.

STEP 2: CREATING RHYTHM PATTERNS (20 MINUTES)

Divide students into groups of four; they are now in their “music groups.” Give each group four 8-inch squares of colored paper, markers or crayons, a glue stick, and a large piece of paper to accommodate the four squares. The four squares grouped together onto the paper represent a four-count measure.

Ask each group to create a four-count rhythm pattern using claps, stomps, or snaps. For example, one pattern could be 2 claps and 2 stomps. Another four-count pattern could be 1 stomp, 2 finger snaps, and 1 clap. Different colors could be used to represent stomps, snaps, or claps.

Ask each group to visually depict each of their counts on one of the squares of paper. They can use words, pictures, or symbols, but someone else must be able to understand and repeat the pattern. See *Rhythm Pattern Squares Example* on page MM 13.

Have the students glue down their squares in the order of their pattern on the large piece of paper.

STEP 3: SHARE AND DISCUSS (20 MINUTES)

Ask each group to perform its pattern, show their visual depiction, and then trade with another group who should try to perform the pattern.

After everyone has had a chance to perform, talk about the various rhythm patterns. How were they the same? How were they different? Were there differences in the tempo (speed of the rhythms)? Did the sounds seem like music?

Was it hard to read someone else's representation? Ask the students what they would need to do so that others could play their rhythm patterns the way the authors intended. Point out that it would be easier to "read" the music if everyone used the same symbols (like notes on a sheet of music). As a class, decide on symbols and/or colors to represent each action. For example, a foot for stomping and a hand for clapping. What symbol should be used to represent finger snaps?

Hand out more 8-inch squares of paper to each group so that they can rewrite their four-count patterns using the agreed-upon symbols or colors and glue them onto another large piece of paper.

Ask students to write the names of the people in their group on their rhythm pattern paper, to trade patterns with another group, and to play each other's music.

Part 3 - Composing Music Main Idea: Making pleasant-sounding music involves creating patterns and adding variations to those patterns. Students will arrange a series of sounds to create a piece of music.

INTRODUCTION

Students will identify parts of a pattern, integrate parts of one pattern with parts of another pattern, represent newly formed patterns, and develop sequences.

Rhythm is a pattern created by long and short sounds within a measure. To prepare for these activities, create two measures of four-count rhythm patterns using four 8-inch squares placed on each one of two larger sheets of paper (each set of 4 squares = one measure). Make one measure using the symbols for four claps and one using the symbols for four stomps and mount the two measures on chart paper.

MATERIALS

- Four-count rhythm patterns previously developed
- Markers, pencils, or crayons
- 8-inch squares of paper or cardboard
- Glue sticks or tape
- Scissors
- Chart paper
- Construction paper
- Post-it notes in different colors (optional)

ACTIVITY: COMPOSING MUSIC

STEP 1: LARGE GROUP COMPOSITION (20 MINUTES)

Remind students of the rhythm patterns they created last time. Explain that they are now going to arrange sounds to create a new piece of music.

1. Show students the four-count measures that you prepared for this activity. Ask students to clap the first measure together. Then repeat the first measure a second time and explain that now it is an eight-count piece of music. Then have the students perform the two measures together (4 claps, 4 stomps). What does it sound like? Guide students from one symbol to the next in a steady beat (e.g., point, tap your foot).

2. Have the students create two new measures using two elements from the first measure and two elements from the second. For example, if the first measure was four claps and the second measure was four stomps, the new measures will have two claps and two stomps each. Make visual depictions of these new measures using the class symbols.

You can cut the original measures into pairs and tape them to a new piece of paper to demonstrate.

3. Have students repeat this new eight-count piece several times. Ask them to describe this new composition. How does this piece sound?

4. Challenge: If we want to create another piece of music with just these symbols, what might we do now? If students need help getting started you could point out that in the first composition, we had four sounds that were repeated. In the second, we took two of the sounds and combined them with two of the sounds from another piece of music. Do you see a pattern? What do you think we will do next?

Once students have recognized the pattern of four, then two, ask them what would follow if the pattern continued: 4 divided in half is 2; 2 divided in half is 1.

5. Have students cut the previous pattern into single sounds and place them on a sheet so that one sound is followed by a different sound. There will be eight sounds of alternating claps and stomps. Have students construct two new measures of four sounds each. (Different color post-it notes can be used here instead of the cut-paper squares for speed or for students with motor skills challenges.)

6. Repeat this new eight-count piece several times. Ask students to describe this new composition. How does this piece sound?

PART 2: A COMMUNITY COMPOSITION (40 MINUTES)

Have students form into their four-person music groups and give each group its original four-count rhythm pattern (which they can now call a measure). Ask each group to use this measure to create an eight-count piece of music (two measures). Then, as in the large group, ask each music group to come up with three variations on their eight-count pattern.

Have each group perform its eight-count composition for the class. Ask students to describe their compositions.

Have each group write four measures of “song” (four beats per measure) and perform them. Then, have the groups perform their compositions simultaneously. Ask the students to describe what it was like to have the musical groups come together like this.

Part 4 - Discovering Music Fractions **Main Idea:** Varying the number of beats in a four-count pattern can make a more interesting musical piece. Students will create a more complicated piece of music and represent it so others can play it.

INTRODUCTION

Students will further develop an understanding of parts of a whole and explore fractions through music. They will use fractional notation and practice adding fractions to create a whole. They will also become familiar with using combinations and permutations of a small set of pieces to make a variety of music.

In preparation, string a clothesline or twine across the room along with plenty of clothespins or paper clips. Also, print out copies of the *Note Values Chart*, found on page MM 13.

MATERIALS

- 8-inch squares of paper in various colors
- Scissors
- Glue or glue sticks
- Clothesline or twine
- Clothespins or paper clips
- Yarn
- *Note Values Chart* (found on page MM 13)

ACTIVITY: DISCOVERING MUSIC FRACTIONS

STEP 1: INTRODUCTION (5 MINUTES)

In this activity students will look at another way to create compositions based on the length of time allowed for each sound. Tell students that they are going to find a way to indicate how long they want each sound to last.

In this activity, the 8-inch squares are not just worth one beat; they are worth four beats—an entire measure! We will see how we can divide that measure into different beats to create different music patterns.

Remind students that they have been creating four-count measures and that each count was represented by a sound that was of equal duration. Ask them to think about music they like to listen to and point out how some sounds are longer and some shorter.

STEP 2: DEMONSTRATION (15 MINUTES)

Take an 8-inch square paper and explain that it represents a four-count measure. Fold the paper in half once and then in half again, so that it's divided into four smaller squares. Cut the small squares apart and hang them on a piece of twine using clothespins or paper clips. Hang a piece of yarn on both sides of the four pieces to show the beginning and end of the measure. Each square represents one of the sounds in the four-count measure.

Discussion: If we know that the four-count measure (the 8-inch square) is a whole, what part of the whole does each of these small squares represent? How can we write that? Help students discover they can write a mathematical equation: $1 = 1/4 + 1/4 + 1/4 + 1/4$.

How could you indicate that you want a sound (e.g., a clap) to be two quick sounds—that is, a half of a beat (two quick claps)? They could divide one of the squares into 2 pieces. If we divide one of these squares into two pieces, what will each of these pieces represent? One way to think about this is to figure out how many of these small rectangles it will take to cover the original 8-inch square. It will take 8, so each piece is $1/8$. For older students, explain $1/4 \div 2 = 1/8$.

Ask students to fold and cut a piece that represents $1/8$ into two pieces. What part of the whole does this represent now? $1/8 \div 2 = 1/16$.

Students can use these smaller pieces ($1/4$, $1/8$, & $1/16$) to show sound length. Suggest a color code like red = clap, blue = stomp, etc. to indicate which sound is being used. As a class, students should decide on a color code. Post it in a prominent place so that students can refer to it as they work in their small groups.

STEP 3: DISCOVERING MUSICAL FRACTIONS (20 MINUTES)

Hand out copies of *Note Values Chart*. Refer to the fraction graphic on the chart and discuss how the lengths of the notes are like fractions. For example, two half notes are equal to (last as long as) a whole note; four eighth notes are equal to (the same length as) one half note; and so on.

You could say that a $1/4$ note is a regular clap; a $1/8$ note indicates a shorter time for a clap: a quick clap. These might be hard to imagine, but practice will help.

STEP 4: SMALL-GROUP ACTIVITY (20 MINUTES)

In your music groups, try to find new ways to construct your four-count measure. Remember that the total for each of your measures will be equal to the large 8-inch square. Check that the various pieces you put together all fit together in the large square.

Have each group create a measure that uses a variety of different notes (both sounds & lengths). Once each group decides what sounds they want to use in their measure, they will need paper in the colors that correspond to those sounds.

Have each group experiment with a variety of notes for one sound. What does a $\frac{1}{4}$ note clap sound like? What does a $\frac{1}{8}$ note clap sound like? A $\frac{1}{16}$ note stomp? Have them make combinations to create a measure. Start out simply, maybe with just $\frac{1}{4}$ and $\frac{1}{8}$ notes.

Discussion: What combinations did you use for your measure? What else could you have done? Is there one combination you like better than the others? Why?

Remember, the total needs to add up to a whole measure. You might want to ask them to write a fraction sentence that proves that each measure is whole. For example, $4 \text{ eighths} + 1 \text{ half} = 1 \text{ whole}$.

Have each music group create and agree on two measures they would like to present to the class. Give them time to practice.

STEP 5: LARGE-GROUP ACTIVITY (20 MINUTES)

Ask each group to perform their two measures.

Discussion: How did you develop your measures? Why did you use those notes? Have each group hang their measures on the clothesline or twine so everyone can see.

CHANGING THE EQUATION: MUSIC MATH EXTENSION



The Atlanta Symphony has created activity sheets so that students can make their own musical instruments:

<https://www.atlantasymphony.org/aso/asoassets/downloadcenter/Symphony%20Street%20Activity%20Sheets.pdf>



Rhythm Pattern Squares Example



CHANGING THE EQUATION: MUSIC MATH NOTES VALUE CHART



explora!

Ideas You Can Touch
Ideas que puedes tocar

Whole Note

4 beats for each note



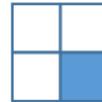
Half Note

2 beats for each note



Quarter Note

1 beat for each note



Eighth Note

1/2 beat for each note



Sixteenth

1/4 beat for each note

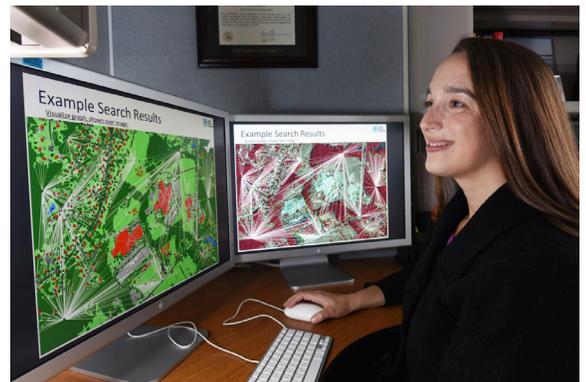


I am a mathematician at Sandia National Laboratories, where I calculate the probability of accidents happening and the consequences they would have for different energy sources, like nuclear power plants. I like my job because I get to use math, I help keep people safe, and I support the development of safe energy sources for the future. Everyone uses energy every day, so it's important that we have reliable and sustainable energy sources that are also safe for the environment. In school, I always liked math because it was challenging and useful, but some of my classes were really hard. I learned to ask for help and kept practicing until I mastered the concepts I needed. Now that I'm a mathematician, I don't just solve math problems all day. I get to use math to help scientists and engineers express and solve their problems with math by writing computer programs. When math gets hard, remember to be patient with yourself and don't be afraid to ask questions and ask others for help! Both the math and the communication skills you learn will serve you well.



AUBREY ECKERT-GALLUP
Mathematician

I manage a team of scientists and engineers at Sandia National Laboratories. Our job is to use math in clever ways to solve really hard problems. I love my job because it is challenging and there are so many unanswered questions we still need to figure out. I work with large sets of data and a team of people to figure out the patterns in the data to understand what the data tells us. Our work keeps the country safe! When I was in school, I took a lot of math, but it was really hard for me. I struggled for a while before I found someone I wanted to ask for help. I got better with the help of others while working with a study group. Those experiences taught me teamwork and perseverance, two skills I rely on now. Science, engineering, and math are great fields to go into, because there are always new, interesting things being discovered! Practice, practice, practice to keep your math muscle exercised, and you can do any job in the world.



KRISTINA CZUCHLEWSKI
Computer Scientist

