

Common Core State Standards for Mathematics Flip Book

Kindergarten

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Compiled by Melisa J. Hancock, for questions or comments about the flipbooks please contact
Melisa (melisa@ksu.edu). Formatted by Melissa Fast (mfast@ksde.org)

Planning Advice--Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom.

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As the Kansas College and Career Ready Standards (KCCRS) are carefully examined, there is a realization that with time constraints of the classroom, not all of the standards can be done equally well and at the level to adequately address the standards. As a result, priorities need to be set for planning, instruction and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "While the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. ... It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)



The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In planning for instruction "grain size" is important. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Daro (Teaching Chapters, Not Lessons—Grain Size of Mathematics), strands are too vague and too large a grain size, while lessons are too small a grain size. About 8 to 12 units or chapters produce about the right "grain size". In the planning process staff should attend to the clusters, and think of the standards as the ingredients of cluster, while understanding that coherence exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions that argue 2 days instead of 3 days on a topic because it is a lower priority detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, lenses focused on lessons can also provide too narrow a view which compromises the coherence value of closely related standards.



The video clip [Teaching Chapters, Not Lessons—Grain Size of Mathematics](#) that follows presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with “grain size”, clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they demine distribution of time for both planning and instruction, helping to assure that students really understand before moving on. Each cluster has been given a priority level. As professional staffs begin planning, developing and writing units as Daro suggests, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level by Zimba. The three levels are referred to as:—**Focus**, **Additional** and **Sample**. Furthermore, Zimba suggests that about 70% of instruction should relate to the Focus clusters. In planning, the lower two priorities (Additional and Sample) can work together by supporting the Focus priorities. The advanced work in the high school standards is often found in “Additional and Sample clusters”. Students who intend to pursue STEM careers or Advance Placement courses should master the material marked with “+” within the standards. These standards fall outside of priority recommendations.

Appropriate Use:

Recommendations for using cluster level priorities

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through: sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possibility quality; the additional work of the grade should indeed support the Focus priorities and not detract from it.
- Set priorities for other implementation efforts taking the emphasis into account such as: staff development; new curriculum development; revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards rather than connecting the Additional and Sample clusters to the other work of the grade
- Sorting clusters from Focus to Additional to Sample and then teaching the clusters in order. To do so would remove the coherence of mathematical ideas and miss opportunities to enhance the focus work of the grade with additional clusters.
- Using the clusters' headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise and coherence of the standards (grain size).

Depth Opportunities

Each cluster, at a grade level, and, each domain at the high school, identifies five or fewer standards for in-depth instruction called Depth Opportunities (Zimba, 2011). Depth Opportunities (DO) is a qualitative recommendation about allocating time and effort within the highest priority clusters --the **Focus** level. Examining the Depth Opportunities by standard reflects that some are beginnings, some are critical moments or some are endings in the progressions. The DO's provide a prioritization for handling the uneven grain size of the content standards. Most of the DO's are not small content elements, but, rather focus on a big important idea that students need to develop.

DO's can be likened to the Priorities in that they are meant to have relevance for instruction, assessment and professional development. In planning instruction related to DO's, teachers need to intensify the mode of engagement by emphasizing: **tight focus, rigorous reasoning and discussion** and **extended class time devoted to practice and reflection** and have **high expectation for mastery**. (See Table 7, Depth of Knowledge (DOK) Appendix)

In this document, Depth Opportunities are highlighted **pink** in the Standards section. For example:

5.NBT.6 Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.

Depth Opportunities can provide guidance for examining materials for purchase, assist in professional dialogue of how best to develop the DO's in instruction and create opportunities for teachers to develop high quality methods of formative assessment.

Standards for Mathematical Practice in Kindergarten

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Kindergarten students complete.

Practice	Explanation and Example
1) Make Sense and Persevere in Solving Problems.	Mathematically proficient students in Kindergarten begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of the problem and look for ways to solve it. Students in Kindergarten use concrete objects or pictures to help them conceptualize and solve problems. Kindergarten students also are expected to persevere while solving tasks; that is, if students reach a point in which they are stuck, they don't "give up", they try another strategy. For example, young students might use concrete objects or pictures to show the actions of a problem or seeing a way to begin, they ask questions that will help them get started.
2) Reason abstractly and quantitatively.	Mathematically proficient students in Kindergarten begin to recognize that a number represents a specific quantity. Then, they connect the quantity to objects and written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. For example, Kindergartners use concrete objects to "act out" a context, they represent the problem with mathematical objects or symbols. This is also an example of modeling with mathematics (SMP #4).
3) Construct viable arguments and critique the reasoning of others.	<p>Mathematically proficient students in Kindergarten construct arguments using concrete objects, pictures, drawings, and actions. They begin to develop their mathematical communication skills as they participate in mathematical discussions. Questions like "How did you get that?" and "Why is that true?" encourage them to explain their thinking to others and respond to others' thinking.</p> <p>For example, in order to demonstrate what happens to the sum when the same amount is added to one addend and subtracted from another, students might represent a story about children moving between two classrooms: the number of children in each classroom is an addend; the total number of children in the two classrooms is the sum. When some students move from one classroom to the other, the number of students in each classroom changes by that amount—one addend decreases by some amount and the other addend increases by that same amount—but the total number of students does not change.</p>

<p>4) Model with mathematics.</p>	<p>Mathematically proficient students in Kindergarten can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an addition equation to describe a situation or using concrete objects to “show” the situation. Kindergartners experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need many opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. For example, they might arrange counters to solve problems such as this one: there are seven animals in the yard, some are dogs and some are cats, how many of each could there be? They are using the counters to model the mathematical elements of the contextual problem—that they can split a set of 7 into a set of 3 and a set of 4. When they learn how to write their actions with the counters in an equation, $4 + 3 = 7$, they are modeling the situation with numbers and symbols.</p>
<p>5) Use appropriate tools strategically.</p>	<p>Mathematically proficient students in Kindergarten begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergartners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side. The tools that Kindergartners might use include physical objects (cubes, geometric shapes, place value manipulatives, etc.) drawings or diagrams (number paths, tally marks, tape diagrams, arrays, tables, etc.) paper and pencil, rulers and other measuring tools, scissors, tracing paper, or other available technologies.</p>
<p>6) Attend to precision.</p>	<p>Mathematically proficient students in Kindergarten start by using everyday language to express their mathematical ideas and begin to develop their communication skills. They try to use clear and precise language in their discussions with others and in their own reasoning. For example, students can “show” and “explain” that the equivalence of 8 and $5 + 3$ can be written both as $5 + 3 = 8$ and $8 = 5 + 3$. They “show” this relationship using concrete objects. Similarly, the equivalence of $6 + 2$ and $5 + 3$ is expressed as $6 + 2 = 5 + 3$.</p>
<p>7) Look for and make use of structure.</p>	<p>Mathematically proficient students in Kindergarten look for patterns and structures in numbers, place value, properties of operations, etc. They USE structure to solve problems. Examples: The less you subtract, the greater the difference. Recognizing that adding 1 results in the next counting number, and recognizing the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that $3 + 2 = 5$ and $2 + 3 = 5$.</p>

8) Look for and express regularity in repeated reasoning.

Mathematically proficient students in Kindergarten notice repetitive actions in counting and computation, etc. and find shortcuts. For example, they may notice that the next number in a counting sequence is “one more”. When counting by tens, the next number in the sequence is “ten more” (or one more group of ten) or they notice that when tossing two-colored counters to find combinations of a given number, they always get what they call “opposites”—*when tossing 6 counters, they get 2 red, 4 yellow and 4 red, and 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow.* Or on a Ten Frame, with 8 counters, they notice there are 2 spaces, or with 4 counters on the Ten Frame, there are 6 spaces. As they look for and explain their reasoning they continually ask themselves, “Does this make sense”?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
<p>1. Make sense of problems and persevere in solving them.</p> <ul style="list-style-type: none"> • Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Can monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Can understand various approaches to solutions. • Continually ask themselves; “Does this make sense?” 	<ul style="list-style-type: none"> • How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. • What might you change? • Talk me through the steps you’ve used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize, represent, and show?
<p>2. Reason abstractly and quantitatively.</p> <ul style="list-style-type: none"> • Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them. 	<ul style="list-style-type: none"> • What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _____ related to _____? • What is the relationship between _____ and _____? • What does _____ mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use? • Could we have used another operation or property to solve this task? Why or why not?
<p>3. Construct viable arguments and critique the reasoning of others.</p> <ul style="list-style-type: none"> • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic. 	<ul style="list-style-type: none"> • What mathematical evidence would support your solution? How can we be sure that _____? / How could you prove that. _____? Will it still work if. _____? • What were you considering when. _____? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn’t it work? Would it ever work? Why or why not? • What is the same and what is different about. _____? • How could you demonstrate a counter-example?
<p>4. Model with mathematics.</p> <ul style="list-style-type: none"> • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, “How can I represent this mathematically?” 	<ul style="list-style-type: none"> • What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What’s an equation or expression that matches the diagram, number line, chart, table? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table? • What are some ways to visually represent? • What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
<p>5. Use appropriate tools strategically.</p> <ul style="list-style-type: none"> • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics. 	<ul style="list-style-type: none"> • What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use: a graph, number line, ruler, diagram, calculator, manipulative? • Why was it helpful to use. _____? • What can using a _____ show us, that _____ may not? • In what situations might it be more informative or helpful to use. _____?
<p>6. Attend to precision.</p> <ul style="list-style-type: none"> • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately. 	<ul style="list-style-type: none"> • What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • Is there a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties can you use to explain. _____? • How could you test your solution to see if it answers the problem?
<p>7. Look for and make use of structure.</p> <ul style="list-style-type: none"> • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects. 	<ul style="list-style-type: none"> • What observations do you make about. _____? • What do you notice when. _____? • What parts of the problem might you eliminate, simplify? • What patterns do you find in. _____? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to. _____? • In what ways does this problem connect to other mathematical concepts?
<p>8. Look for and express regularity in repeated reasoning.</p> <ul style="list-style-type: none"> • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results. 	<ul style="list-style-type: none"> • Will the same strategy work in other situations? • Is this always true, sometimes true or never true? • How would we prove that. _____? • What do you notice about. _____? • What is happening in this situation? • What would happen if. _____? • What is there a mathematical rule for. _____? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Critical Areas for Mathematics in Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the [Dynamic Learning Maps and Essential Elements](#) website.

Grade Kindergarten Content Standards Overview

By Clicking on the standard below you can go directly to the standard

Counting and Cardinality (CC)

- Know number names and the count sequence [CC.1](#) [CC.2](#)

[CC.3](#) [CC.4](#) [CC.5](#) [CC.6](#) [CC.7](#)

Operations and Algebraic Thinking (OA)

- Understand addition as putting together and adding to and understanding subtraction as taking apart and taking from. [OA.1](#) [OA.2](#)

[OA.3](#) [OA.4](#) [OA.5](#)

Number and Operations in Base Ten (NBT)

- Work with numbers 11-19 to gain foundations for place value. [NBT.1](#)

Measurement and Data (MD)

- Describe and compare measurable attributes. [MD.1](#) [MD.2](#)
- Classify objects and count the number of objects in each category.

[MD.3](#)

Geometry (GE)

- Identify and describe shapes. [G.1](#) [G.2](#) [G.3](#)
- Analyze, compare, create and compose shapes [G.4](#)

Major

Supporting

Additional
[Type here]

Depth Opportunities (DO)

Domain: Counting and Cardinality (CC)

Cluster: Know Number names and count sequence.

Standard: Grade K.CC.1

Count to 100 by ones and by tens. (CC)

Suggested Standards for Mathematical Practice (MP):

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Connections:

This is the beginning of the core standards.

Explanation and Examples:

The emphasis of this standard is on the counting sequence. When counting by ones, students need to understand that the next number in the sequence is one more. When counting by tens, the next number in the sequence is “ten more” (or one more group of ten).

Students are to rote count (verbal saying of numbers in sequence) by starting at one and count to 100. (They are only expected to master counting on the decade (0, 10, 20, 30, 40 ...). This objective does not require recognition of numerals. It is focused on the rote number sequence.

Provide settings that connect mathematical language and symbols to the everyday lives of kindergarteners. Support students’ ability to make meaning and mathematize (*the process of seeing and focusing on the mathematical aspects and ignoring the non-mathematical aspects.*

Mathematizing in Kindergarten: Solving problems, Communicating or showing their thinking, Connecting and Representing Ideas) the real world. Help them see patterns, make connections and provide repeated experiences that give students time and opportunities to develop understandings and increase fluency. Encourage students to explain their reasoning by asking probing questions such as “How do you know?” “How did you figure that out?”

Instruction on the counting sequence should be scaffold (e.g. 1-10, then 1-20, etc.)

Major

Supporting

Additional

Depth Opportunities (DO)

[Type here]

Accurate in counting depends on three things:

1. Knowing the patterns in the number-word list so that a correct number-word list can be said
2. Correctly assigning one number word to one object (one-to one-correspondence)
3. Keeping track of which objects have already been counted so that they are not counted more than once.

Keeping track—differentiating counted from uncounted entities—is more easily done by moving objects into a counted set. Doing so is not possible with things that cannot be moved, such as pictures in a book. Strategies for keeping track of messy, large sets continue to develop for many years.

Regularity and rhythm are important aspects of counting. Activities that increase these aspects can be helpful to children making lots of correspondence errors.

Errors in Counting

Four factors strongly affect accuracy in counting correspondence:

- Amount of counting experiences (more experience leads to fewer errors)
- Size of set (children become accurate on small sets first)
- Arrangements of objects (objects in rows make it easier to keep track of what has been counted and what has not)
- Effort

NCTM Focus In, Kindergarten

Instructional Strategies:

Counting should be reinforced throughout the day, not in isolation. (Meaningful Counting)

See Table 6, Developing Counting, Appendix

Examples:

- Count the number of chairs of the students who are absent
- Count the number of stairs, shoes, etc.
- Counting groups of ten such as “fingers in the classroom” (ten fingers per student).
- Count the number of students in a group.
- Count the number of specific object they have in their desk (e.g. crayons)

When counting orally, students should recognize the patterns that exist from 1 to 100. They should also recognize the patterns that exist when counting by 10s. Have students verbalize the patterns they see.

Help them see patterns, make connections and provide repeated experiences that give students time and opportunities to develop understandings and increase fluency.

Games that require students to add on to a previous count to reach a goal number encourage developing this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. These concepts emerge over time and cannot be forced.

Like counting to 100 by either ones or tens, writing numbers from 0 to 20 is a rote process. Initially, students mimic the actual formation of the written numerals while also assigning it a name.

Over time, children create the understanding that number symbols signify the meaning of counting. Practice count words and written numerals paired with pictures, representations of objects, and objects that represent quantities within the context of life experiences for kindergarteners.

For example, dot cards, dominoes and number cubes all create different mental images for relating quantity to number words and numerals.

One way students can learn the left to right orientation of numbers is to use a finger to write numbers in air (sky writing). Children will see mathematics as something that is alive and that they are involved.

As with many physical activities, counting will improve with practice and does not need to be perfect each time. It is much more important for all children to get frequent counting practice and watch and help one another, with occasional help and corrections from the teacher.

Tools /Resources

For detailed information, see: Learning Progressions Counting and Cardinality:
<http://ime.math.arizona.edu/progressions/>

See Appendix Table 6: “Development of Counting”

K.CC.A. Know number names and the count sequence.

- [K.CC Assessing Reading Numbers](#)
- [K.CC Teen Go Fish](#)
- [K.CC Find The Numbers 0-5 or 5-10](#)
- [K.CC Assessing Sequencing Numbers](#)
- [K.CC Five by Two](#)
- [K.CC More and Less Handfuls](#)

K.CC.A.1. Count to 100 by ones and by tens.

- [K.CC Choral Counting](#)
- [K.CC Counting Circles](#)
- [K.CC Assessing Counting Sequences Part I](#)
- [K.CC Counting by Tens](#)

See: “How Many Letters in Your Name” Students review numbers 1 to 10 by counting the number of letters in their names and their classmates' names. They also write and order numbers. The class compiles students' finished product in a class.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L869>

Let's Count to 10 is a unit in which students make groups of zero to 10 objects, connect number names to the groups, compose and decompose numbers, and use numerals to record the size of a group. Visual, auditory, and kinesthetic activities are included in each lesson. The unit is most appropriate for students learning at the kindergarten and grade one level.

<http://illuminations.nctm.org/LessonDetail.aspx?id=U147>

Common Misconceptions:

Some students might not see zero as a number. Ask students to write 0 and say *zero* to represent the number of items left when all items have been taken away. Avoid using the word *none* to represent this situation. Find instances for which the response would be zero in real-world settings to provide experiences with the concept of zero.

As long as children *understand* that correct counting requires one point and one word for each object and are trying to do that, parents and teachers do not need to correct errors all the time.

Major

Supporting

Additional
[Type here]

Depth Opportunities (DO)

Domain: Counting and Cardinality (CC)**Cluster: Know number names and the count sequence.****Standard: Grade K.CC.2**

Count forward beginning from a given number within known sequence (instead of having to begin at 1). (CC)

Suggested Standards for Mathematical Practice (MP):

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Connections:

This is the beginning of the core standards.

Explanation and Examples:

The emphasis of this standard is on the counting sequence to 100. **K.CC.2** includes numbers 0-100. This asks for students to begin rote counting forward counting in a sequence from a number other than one.. (E.g. Given the number 4, the student would count, “4, 5, 6”) This objective does not require recognition of numerals. It is focused on the rote number sequence.

Instructional Strategies:

Games that require students to add on to a previous count to reach a goal number encourage developing this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. **These concepts emerge over time and cannot be forced.**

Tools/Resources

K.CC.A.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

- [K.CC Assessing Counting Sequences Part II](#)
- [K.CC Number Line Up](#)
- [K.CC Start-Stop Counting](#)
- [K.CC Assessing Counting Sequences Part I](#)
- [K.CC Number After Bingo 1-15](#)
- [K.CC Pick a Number, Counting On](#)
- [K.CC “One More” Concentration](#)

Major**Supporting****Additional****Depth Opportunities (DO)**

[Type here]

This is a series of games and activities that focus on counting. It contains activity cards and many of the resources so that it can be used for instruction:

<http://www.kidscount1234.com/mathcentersandgames.html>

Common Misconceptions:

Counting on or counting from a given number conflicts with the learned strategy of counting from the beginning. In order to be successful in counting on, students must understand **cardinality** (*the number that ends the counting sequence represents how many objects are in the collection*).

Students often merge or separate two groups of objects and then re-count from the beginning to determine the final number of objects represented. For these students, counting is still a rote skill or the benefits of counting on have not been realized.

Games that require students to add on to a previous count to reach a goal number encourage developing this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. These concepts emerge over time and cannot be forced.

Major

Supporting

Additional

Depth Opportunities (DO)

[Type here]

Domain: Counting and Cardinality (CC)
Cluster: <i>Know number names and the count sequence.</i>
<p>Standard: Grade K.CC.3</p> <p>Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects). (CC)</p>
<p>Suggested Standards for Mathematical Practice (MP)</p> <p>MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects.</p> <p>This cluster is connected to the other clusters in the Counting and Cardinality Domain and to <i>Classify objects and count the number of objects in each category</i> in Kindergarten, and to <i>Add and subtract within 20</i> and <i>Extend the counting sequence</i> in Grade 1. K.CC.4; KNBT.1; K.MD.3</p>
<p>Explanation and Examples:</p> <p>This standard asks for students to represent a set of objects with a written numeral. The number of objects being recorded should not be greater than 20. Students can record the quantity of a set by selecting a number card/tile (numeral recognition) or writing the numeral. Students can also create a set of objects based on the numeral presented.</p> <p>Students should be given multiple opportunities to count objects and recognize that a number represents a specific quantity. Once this is established, students begin to read and write numerals (numerals are the symbols for the quantities). The emphasis should first be on quantity and then connecting quantities to the written symbols.</p> <ul style="list-style-type: none"> • A sample unit sequence might include: <ol style="list-style-type: none"> 1. Counting up to 20 objects in many settings and situations over several weeks. 2. Beginning to recognize, identify, and read the written numerals, and match the numerals to given sets of objects. 3. Writing the numerals to represent counted objects.

Major

Supporting

Additional

Depth Opportunities (DO)

[Type here]

- Since the teen numbers are not written as they are said, teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represents each teen number.
- For example, when focusing on the number “14,” students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten and four extra ones. Students should connect the representation to the symbol “14” and say, “*Ten and four*”.

Instructional Strategies:

One way students can learn the left to right orientation of numbers is to use a finger to write numbers in air (sky writing).

Children will see mathematics as something that is alive and that they are involved. Students should study and write numbers 0 to 20 in this order: numbers 1 to 9, the number 0, then numbers 10 to 20.

They need to know that 0 is the number items left after all items in a set are taken away. Do not accept “none” as the answer to “How many items are left?” for this situation.

Resources/Tools

K.CC.A.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

- [K.CC Rainbow Number Line](#)
- [K.CC Race to the Top](#)
- [K.CC Number TIC TAC TOE](#)
- [K.CC Assessing Writing Numbers](#)
- [K.CC,OA Dice Addition 1](#)
- [K.CC Bags of Stuff](#)

K.CC.B. Count to tell the number of objects.

- [K.CC Counting Cup](#)
- [K.CC More and Less Handfuls](#)
- [K.CC Counting Overview](#)
- [K.CC Number Rods](#)
- [K.CC Color Week](#)

See: "Counting Sheep", NCSM, [Great Tasks for Mathematics K-5](#), (2013).

NCTM, [Focus on Kindergarten](#).

See Also: K.CC.1

Common Misconceptions:

K.CC.3 addresses the writing of numbers and using the written numerals (0-20) to describe the amount of a set of objects. Recognize varied development of fine motor and visual development, a reversal of numerals will occur in a majority of the students. While reversals should be pointed out to students, the emphasis is on the use of numerals to represent quantities rather than the correct handwriting formation of the actual numeral itself.

Some students might not see zero as a number. Ask students to write 0 and say *zero* to represent the number of items left when all items have been taken away. Avoid using the word *none* to represent this situation.

Domain: Counting and Cardinality (CC)
Cluster: <u>Count and tell the number of objects.</u>
<p>Standard: Grade K.CC.4</p> <p>Understand the relationship between numbers and quantities; connect counting to cardinality.</p> <ol style="list-style-type: none"> When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. Understand that each successive number name refers to a quantity that is one larger. <p>(CC)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects. This cluster is connected to the other clusters in the Counting and Cardinality Domain and to <i>Classify objects and count the number of objects in each category</i> in Kindergarten, and to <i>Add and subtract within 20</i> in Grade 1.</p>
<p>Explanation and Examples:</p> <p>This standard asks students to count a set of objects and see sets and numerals in relationship to one another, rather than as isolated numbers or sets. These connections are higher-level skills that require students to analyze, to reason about, and to explain relationships between numbers and sets of objects. This standard should first be addressed using numbers 1-5 with teachers building to the numbers 1-10 later in the year. The expectation is that students are comfortable with these skills with the numbers 1-10 by the end of Kindergarten.</p>

Major**Supporting****Additional****Depth Opportunities (DO)**

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K.CC.4a reflects the ideas that students implement correct counting procedures by pointing to one object at a time (one-to-one correspondence) using one counting word for each object (one-to-one touching/synchrony), while keeping track of objects that have and have not been counted.. This is the foundation of counting.

K.CC.4b calls for students to answer the question “How many are there?” by counting objects in a set and understanding that the last number stated when counting a set (...8, 9, **10**) represents the total amount of objects: “There are **10** bears in this pile.” (*cardinality*).

It also requires students to understand that the same set counted three different times will end up being the same amount each time. The idea is to develop a purpose for counting as keeping track of objects is developed. Therefore, a student who moves each object as it is counted recognizes that there is a need to keep track in order to figure out the amount of objects present.

Conservation of number, (regardless of the arrangement of objects, the quantity remains the same); conservation of number is a developmental milestone which some Kindergarten children will not have achieved. The goal of this objective is for students to be able to count a set of objects; regardless of the formation those objects are placed.

K.CC.4c represents the concept of “one more” while counting a set of objects. Students are to make the connection that if a set of objects was increased by one more object then the number name for that set is to be increased by one as well.

Students are asked to understand this concept with and without objects. For example, after counting a set of 8 objects, students should be able to answer the question, “How many would there be if we added one more object?”; and answer a similar question when not using objects, by asking hypothetically, “What if we have 5 cubes and added one more. How many cubes would there be then?”

This concept should be first taught with numbers 1-5 before building to numbers 1-10. Students should be expected to be comfortable with this skill with numbers to 10 by the end of Kindergarten.

Instructional Strategies:

One of the first major concepts in a student’s mathematical development is cardinality. Cardinality, knowing that the number word said tells the quantity you have and that the number you end on when counting represents the entire amount counted. The big idea is that number means amount and, no matter how you arrange and rearrange the items, the amount is the same. Until this concept is developed, counting is merely a routine procedure done when a number is needed. To determine if students have the cardinality rule, listen to their responses when you discuss counting tasks with them. For example, ask, “How many are here?” The student counts correctly and says that there are seven. Then ask, “Are there seven?” Students may count or hesitate if they have not developed cardinality. Students with cardinality may emphasize the last count or explain that there are seven because they counted them. These students can now use counting to find a matching set. Students develop the understanding of counting and cardinality from experience. Almost any activity or game that engages children in counting and comparing quantities, such as board games, will encourage the development of cardinality. Frequent opportunities to use and discuss counting as a means of solving problems relevant to kindergarteners is more beneficial than repeating the same routine day after day. For example, ask students questions that can be answered by counting up to 20 items before they change and as they change locations throughout the school building.

As students develop meaning for numerals, they also compare numerals to the quantities they represent. The models that can represent numbers, such as dot cards and dominoes, become tools for such comparisons. Students can concretely, pictorially or mentally look for similarities and differences in the representations of numbers. They begin to “see” the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities that are one larger. In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will build students’ ability to break numbers into parts. Provide a variety of experiences in which students connect count words or number words to the numerals that represent the quantities. Students will arrive at an understanding of a number when they acquire cardinality and can connect a number with the numerals and the number word for the quantity they all represent.

Resources/Tools:

K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality.

- [K.CC Goody Bags](#)
- [K.CC Counting Mat](#)
- [K.CC The Napping House](#)

<http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathBelinda>

See Also: K.CC.1

Common Misconceptions:

When counting objects student may think the last object counted represents the quantity last “said” When seven chips are lined up and student counts (one-to-one) until s/he gets to seven, when asked to “show” the quantity 7, the student picks up the last chip.

Students may think that the count word used to tag an item is permanently connected to that item. So when the item is used again for counting and should be tagged with a different count word, the student uses the original count word. For example, a student counts four geometric figures: triangle, square, circle and rectangle with the count words: one, two, three, four. If these items are rearranged as rectangle, triangle, circle and square and counted, the student says these count words: four, one, three, two.

Students count objects without seeing sets and numerals in relationship to one another. They see a set of objects as isolated numbers or sets,

Domain: Counting and Cardinality (CC)
Cluster: <u>Counts and tells the number of objects.</u>
<p>Standard: Grade K.CC.5</p> <p>Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. (CC)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.</p>
<p>Connections:</p> <p>See K.CC.4</p>
<p>Explanation and Examples:</p> <p>This standard addresses various counting strategies. From the research in early childhood mathematics, students go through a progression of four general ways to count (Kathy Richardson).</p> <p>These counting strategies progress from least difficult to most difficult:</p> <ol style="list-style-type: none"> 1) students move objects and count them as they move them, 2) students line up the objects and count them, 3) students have a scattered arrangement and they touch each object as they count 4) students have a scattered arrangement and count them by visually scanning without touching them. <p>Since the scattered arrangements are the most challenging for students, K.CC.5 calls for students to only count 10 objects in a scattered arrangement, and count up to 20 objects in a line, rectangular array, or circle. Out of these 3 representations, a line is the easiest type of arrangement to count.</p>

K.CC.4-6

Students should develop counting strategies to help them organize the counting process to avoid re-counting or skipping objects.

Note it is not appropriate to use number lines for Pre-K through first grade. See additional information in Grade 1. One of the first major concepts in a student's mathematical development is cardinality. Cardinality, knowing that the number word said tells the quantity and that the number said last represents the entire amount counted.

The big idea is that number means amount and, no matter how you arrange and rearrange the items, the amount is the same. Until this concept is developed, counting is merely a routine procedure done when a number is needed.

To determine if students have the cardinality rule, listen to their responses when you discuss counting tasks with them. For example, ask, "How many are here?". The student counts correctly and says that there are seven. Then ask, "Are there seven?". Students may count or hesitate if they have not developed cardinality. Students with cardinality may emphasize the last count or explain that there are seven because they counted them.

Students can concretely, pictorially or mentally look for similarities and differences in the representations of numbers. They begin to "see" the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities that are one larger.

Instructional Strategies:

Students should develop counting strategies to help them organize the counting process to avoid re-counting or skipping objects.

Suggestions::

- If items are placed in a circle, the student may mark or identify the starting object.
- If items are in a scattered configuration, the student may move the objects into an organized pattern.
- Some students may choose to use grouping strategies such as placing objects in twos, fives, or tens (note: this is not a kindergarten expectation).
- Counting up to 20 objects should be reinforced when collecting data to create charts and graphs.
(A student may use a clicker (electronic response system) to communicate his/her count to the teacher).

Students develop the understanding of counting and cardinality from experience. Almost any activity or game that engages children in counting and comparing quantities, such as board games, will encourage the development of cardinality.

Students should have frequent opportunities to use and discuss counting as a means of solving problems relevant to kindergarteners. This is more beneficial than repeating the same routine day after day. For example, ask students questions that can be answered by counting up to 20 items before they change and as they change locations throughout the school building.

As students develop meaning for numerals, they also compare numerals to the quantities they represent. The models, such as dot cards and dominoes that represent numbers become tools for such comparisons.

In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will build students' ability to break numbers into parts.

Provide a variety of experiences in which students connect count words or number words to the numerals that represent the quantities.

Resources/Tools

See Also: K.CC.1

K.CC.B.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

- [K.CC Finding Equal Groups](#)

K.CC.C. Compare numbers.

- [K.CC Biggest Number Wins](#)
- [K.CC More and Less Handfuls](#)

Common Misconceptions:

Some students might think that the count word used to tag an item is permanently connected to that item. So when the item is used again for counting and should be tagged with a different count word, the student uses the original count word. For example, a student counts four geometric figures: triangle, square, circle and rectangle with the count words: one, two, three, four. If these items are rearranged as rectangle, triangle, circle and square and counted, the student says these count words: four, one, three, two.

Domain: Counting and Cardinality (CC)

Cluster: Compare numbers.

Standard: Grade K. CC.6

Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects) **(CC)**

This is a Depth Opportunity for instruction.

Suggested Standards for Mathematical Practice (MP):

MP.2 Reason abstractly and quantitatively.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Connections:

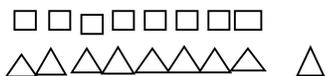
See K.CC. 1-5

Explanation and Examples:

This standard expects mastery of up to ten objects. Students can use matching strategies (Student 1), counting strategies or equal shares (Student 3) to determine whether one group **is greater than, less than, or equal to** the number of objects in another group (Student 2).

Student 1

I lined up one square and one triangle. Since there is one extra triangle, there are more triangles than squares.



Student 2

I counted the squares and I got 8. Then, I counted the triangles and got 9. Since 9 is greater than 8, there are more triangles than squares.

Student 3

I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares.

As children develop meaning for numerals, they also compare these numerals to the quantities represented and their number words.

Major

Supporting

Additional

[Type here]

Depth Opportunities (DO)

Modeling numbers with manipulatives such as dot cards and five- and ten-frames are tools for such comparisons. Children can look for similarities and differences in these different representations of numbers.

They begin to “see” the relationship of one more, one less, two more and two less, leading to the concept that successive numbers name quantities where one is larger.

In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10.

Children demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a “proof” by demonstrating a one-to-one match, by counting again or other similar means (concretely or pictorially) that makes sense.

An ultimate level of understanding is reached when children can compare two numbers from 1 to 10 represented as written numerals without counting.

Students should develop a strong sense of the relationship between quantities and numerals before they begin comparing numbers.

Students state whether the number of objects in a set is more, less, or equal to a set that has 0, 5, or 10 objects.

Instructional Strategies:

Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. (Five-Frame and Ten Frame) This flexibility with numbers will greatly impact children’s ability to break numbers into parts.

Students need to explain their reasoning when they determine whether a number is greater than, less than, or equal to another number. Teachers need to ask probing questions such as “How do you know?” to elicit their thinking. For students, these comparisons increase in difficulty, from greater than to less than to equal. It is easier for students to identify differences than to find similarities.

Strategies:

- Matching: Students use one-to-one correspondence, repeatedly matching one object from one set with one object from the other set to determine which set has more objects.
- Counting: Students count the objects in each set, and then identify which set has more, less, or an equal number of objects.
- Observation: Students may use observation to compare two quantities (e.g., by looking at two sets of objects, they may be able to tell which set has more or less without counting).

- Observations in comparing two quantities can be accomplished through daily routines of collecting and organizing data in displays. Students create object graphs and pictographs using data relevant to their lives (e.g., favorite ice cream, eye color, pets, etc.). Graphs may be constructed by groups of students as well as by individual students.
- Benchmark Numbers: This would be the appropriate time to introduce the use of 0, 5 and 10 as benchmark numbers to help students further develop their sense of quantity as well as their ability to compare numbers.

See also K.CC.4

Resources/Tools:

K.CC.C.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.*

- [K.CC Which number is greater? Which number is less? How do you know?](#)

Also See: “It’s All in the Bag” Georgia Department of Education. Students will work in partner groups to compare two sets of colored blocks. Discussions should include terms more than, less than, and equal. Students will use counting strategies for sets that have been put together, removed, or are compared.

<http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathBag>

Common Misconceptions:

One misconception is the relationship between sets and the numbers. Children who are not *conservers* of number will experience difficulties. Also, It is easier for students to identify differences than to find similarities.

Major

Supporting

Additional
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Depth Opportunities (DO)

Domain: Counting and Cardinality (CC)
Cluster: <u>Compare numbers.</u>
Standard: Grade K.CC.7
Compare two numbers between 1 and 10 presented as written numerals. (CC)
Suggested Standards for Mathematical Practice (MP):
<p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>
Connections:
<p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects. More information about this critical area of focus can be found by clicking here. This cluster is also connected to <i>Work with numbers 11-19 to gain foundations for place value</i> in Kindergarten, and to all clusters in the Operations and Algebraic Thinking Domain in Grade 1.</p>
Explanation and Examples:
<p>This standard asks students to apply their understanding of numerals 1-10 to compare one from another. Thus, looking at the numerals 8 and 10, a student must be able to recognize that the numeral 10 represents a larger quantity than the numeral 8. Students should begin this standard by having ample experiences with sets of objects (K.CC.3 and K.CC.6) before completing this standard with just numerals. Based on early childhood research, students should not be expected to be comfortable with this skill until the <u>end of Kindergarten</u>.</p> <p>As children develop meaning for numerals, they also compare these numerals to the quantities represented and their number words.</p> <p>Children demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a “proof” by demonstrating a one to-one match, by counting again or other similar means (concretely or pictorially) that makes sense. An ultimate level of understanding is reached when children can compare two numbers from 1 to 10 represented as written numerals without counting.</p>

Major

Supporting

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Depth Opportunities (DO)

Instructional Strategies:

Modeling numbers with manipulatives such as dot cards and five- and ten-frames are tools for such comparisons. Children can look for similarities and differences in these different representations of numbers. They begin to “see” the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities where one is larger.

In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10.

Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will impact children’s ability to break numbers into parts.

Resources/Tools

See Also: K.CC.1

K.CC.C.7. Compare two numbers between 1 and 10 presented as written numerals.

- [K.CC Guess the Marbles in the Bag](#)

Common Misconceptions:

See K.CC.6

Domain: Operations and Algebraic Thinking (OA)
Cluster: Understand addition as putting together and adding to, and subtraction as taking apart and taking from.
<p>Standard: Grade K.OA.1</p> <p>Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problems. This applies wherever drawings are mentioned in the Standards.) (OA)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects.</p>
<p>Explanation and Examples:</p> <p>All standards in the Operations and Algebraic Thinking cluster should only include numbers through 10. Students will model simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$.</p> <p>Kindergarten students should see addition and subtraction equations. Student writing of equations in kindergarten <u>is encouraged, but it is not required</u>.</p> <p>Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.</p> <p>This standard asks subtraction) by representing addition and subtraction situations in various ways. This objective is primarily focused on understanding the concept of addition and subtraction, rather than merely reading and solving addition and subtraction number sentences (equations).</p>

Major**Supporting****Additional****Depth Opportunities (DO)**

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Instructional Strategies: K.OA.1-5

Using addition and subtraction in a word problem context allows students to develop their understanding of what it means to add and subtract.

Students should use objects, fingers, mental images, drawing, sounds, acting out situations and verbal explanations in order to develop the concepts of addition and subtraction. Then, they should be introduced to writing expressions and equations using appropriate terminology and symbols which include $+$, $-$, and $=$.

- Addition terminology: *add, join, put together, plus, combine, total*
- Subtraction terminology: *minus, take away, separate, difference, compare*

Have students decompose numbers less than or equal to 5 during a variety of experiences to promote their fluency with sums and differences less than or equal to 5 that result from using the numbers 0 to 5. For example, ask students to use different models to decompose 5 and record their work with drawings or equations.

Next, have students decompose 6, 7, 8, 9, and 10 in a similar fashion. As students begin to understand the role and meaning of arithmetic operations in number systems, they will gain computational fluency, and using efficient and accurate methods for computing.

The teacher can use back-mapping and scaffolding to teach students who show a need for more help with counting. For instance, ask students to build a tower of 5 using 2 green and 3 blue linking cubes while you discuss composing and decomposing 5. Have them identify and compare other ways to make a tower of 5. Repeat the activity for towers of 7 and 9. Help students use counting as they explore ways to compose 7 and 9.

If students progress from working with manipulatives to writing numerical expressions and equations, and they skip using pictorial thinking—students will then be more likely to use finger counting and rote memorization for work with addition and subtraction.

Counting forward builds to the concept of addition while counting back leads to the concept of subtraction. However, counting is an inefficient strategy. Teachers need to provide instructional experiences so that students progress from the concrete level, to the pictorial level, then to the abstract level when learning mathematical concepts. **(Concrete, Pictorial, Abstract CPA)**

Just knowing the basic facts is not enough. We need to help students develop the ability to quickly and accurately understand the **relationships** between numbers. They need to make sense of numbers as they find and make strategies for joining and separating quantities. (Table 1 in Appendix)

Tools/Resources

See Table 1 in Appendix

For detailed information, see: Learning Progressions Operations and Algebraic Thinking
http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_0a_k5_2011_05_30_2.pdf

K.OA.A.1. Represent addition and subtraction with objects, fingers, mental images, drawings*, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

- [K.OA Dice Addition 2](#)
- [K.OA Ten Frame Addition](#)

See Also: “It’s All In the Bag” Students will work in partner groups to compare two sets of colored blocks. Discussions should include terms more than, less than, and equal. Students will use counting strategies for sets that have been put together, removed, or are compared.

<http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathBag>

Common Misconceptions:

Students may over-generalize the vocabulary in word problems and think that certain words indicate solution strategies that must be used to find an answer. They might think that the word *more* always means to add and the words *take away* or *left* always means to subtract.

When students use the words *take away* to refer to subtraction and its symbol, teachers need to repeat students’ ideas using the words *minus* or *subtract*. For example, students use addition to solve this Take from/Start Unknown problem: Melisa took the 8 stickers she no longer wanted and gave them to Anna. Now Melisa has 11 stickers *left*. How many stickers did Melisa have to begin with?

Note on vocabulary: The term “total” should be used instead of the term “sum”. “Sum” sounds the same as “some”, but has the opposite meaning. “Some” is used to describe problem situations with one or both addends unknown, so it is better in the earlier grades to use “total” rather than “sum”.

Formal vocabulary for subtraction (“minuend” and “subtrahend”) is not needed in Kindergarten. (“total” and “addend” are sufficient for classroom discussion).

Students should be encouraged to use create drawings /pictorial representations of the problems and/or situation (See Table 1 in Appendix)

Domain: Operations and Algebraic Thinking (OA)
Cluster: <i>Understand addition as putting together and adding to, and subtraction as taking apart and taking from.</i>
Standard: Grade K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. (OA)
Suggested Standards for Mathematical Practice (MP): MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of other. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.
Connections: See Grade K.OA.1
Explanation and Examples: All standards in the Operations and Algebraic Thinking cluster should only include numbers through 10. Students will model simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. Kindergarten students should see addition and subtraction equations. Student writing of equations in kindergarten <u>is encouraged, but it is not required</u> . Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. This standard asks students to solve problems presented in a story format (context) with a specific emphasis on using objects or drawings to determine the solution. This builds upon the students' understanding of addition and subtraction from K.OA.1, to solve problems. Once again, numbers should not exceed 10. Teachers should focus on three types of problems during instruction. There are three types of addition and subtraction problems are:

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Depth Opportunities (DO)

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Result Unknown, Change Unknown, and Start Unknown. These types of problems become increasingly difficult for students. Research has found that Result Unknown problems are easier than Change and Start Unknown problems. Kindergarten students should have experiences with all three types of problems. The level of difficulty can be decreased by using smaller numbers (up to 5) or increased by using larger numbers (up to 10). (See Table 1, Appendix)

Create situations in which students experience the following addition and subtraction problem types (see Table 1, Appendix).

- Add To word problems, such as, “Mia had 3 apples. Her friend gave her 2 more. How many does she have now?”
 - A student’s “think aloud” of this problem might be, “I know that Mia has some apples and she’s getting some more. So she’s going to end up with more apples than she started with.”
- Take From problems such as:
 - José had 8 markers and he gave 2 away. How many does he have now? When modeled, a student would begin with 8 objects and remove two to get the result.
- Put Together/Take Apart problems with Total Unknown gives students opportunities to work with addition in another context such as:
 - There are 2 red apples on the counter and 3 green apples on the counter. How many apples are on the counter?
- Solving Put Together/Take Apart problems with Both Addends Unknown provides students with experiences with finding all the decompositions of a number and investigating the patterns involved.
 - There are 10 apples on the counter. Some are red and some are green. How many apples could be green? How many apples could be red?

Using a word problem context allows students to develop their understanding about what it means to add and subtract. (*Addition is putting together and adding to. Subtraction is taking apart and taking from*). Instruction that helps Kindergarteners to develop the concept of addition/subtraction is modeling the actions in word problem using objects, fingers, mental images, drawings, sounds, acting out situations, and/or verbal explanations.

Students may use different representations based on their experiences, preferences, etc. They may connect their conceptual representations of the situation using symbols, expressions, and/or equations. Students should experience the addition and subtraction problem types found in Table 1, Appendix).

Instructional Strategies:

See K.OA.1

Tools/Resources

See: http://commoncoretools.me/wp-content/uploads/2011/04/ccss_progression_nbt_2011_04_073_corrected2.pdf for additional detailed information.

K.OA.A.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

- [K.OA What's Missing?](#)
- [K.CC,OA Dice Addition 1](#)
- [K.OA Dice Addition 2](#)
- [K.OA Ten Flashing Fireflies](#)

Common Misconceptions:

See K. OA.1

Domain: Operations and Algebraic Thinking (OA)
Cluster: <i>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</i>
<p>Standard: Grade K.OA 3</p> <p>Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). (OA)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects.</p>
<p>Explanation and Examples:</p> <p>All standards in the Operations and Algebraic Thinking cluster should only include numbers through 10. Students will model simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$.</p> <p>Kindergarten students should see addition and subtraction equations. Student writing of equations in kindergarten <u>is encouraged, but it is not required</u>. Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.</p> <p>This standard asks students to understand that a set of (5) object can be broken into two sets (3 and 2) and still be the same total amount (5). The focus is on number pairs which add to a specified total, 1-10.</p>

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In addition, this standard asks students to understand that a set of objects (5) can be broken in multiple ways (3 and 2; 4 and 1). Thus, when breaking apart a set (decomposing), students develop the understanding that a smaller set of objects exists within that larger set (inclusion). This should be developed in context before moving into how to represent decomposition with symbols (+, -, =).

After the students have had numerous experiences with decomposing sets of objects and recording with pictures and numbers, the teacher eventually makes connections between the drawings and symbols: $5=4+1$, $5=3+2$, $5=2+3$, and $5=1+4$

Number sentence only comes after pictures or work with manipulatives, and students should never give the number sentence without a mathematical representation.

Example:

“Bobby Bear is missing 5 buttons on his jacket. How many ways can you use blue and red buttons to finish his jacket? Draw a picture of all your ideas.

Students could draw pictures of:

- 4 blue and 1 red button
- 3 blue and 2 red buttons
- 2 blue and 3 red buttons
- 1 blue and 4 red buttons

Students may use objects such as cubes, two-color counters, square tiles, etc. to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, etc.

After the students have had numerous experiences with decomposing sets of objects and recording with pictures and numbers, the teacher eventually makes connections between the drawings and symbols: $5=4+1$, $5=3+2$, $5=2+3$, and $5=1+4$

The number sentence only comes after pictures or work with manipulatives, and students should never give the number sentence without a mathematical representation.

Students may use objects such as cubes, two-color counters, square tiles, etc. to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, etc.

Students may also use drawings to show different number pairs for a given number. For example, students may draw 5 objects, showing how to decompose in several ways.

$$\begin{array}{l} x \ x \ x \ x \ x \quad 5 \text{ objects} \\ \boxed{x \ x} \ \boxed{x \ x \ x} \quad 5 = 2 + 3 \\ \boxed{x \ x \ x \ x} \ \boxed{x} \quad 5 = 4 + 1 \end{array}$$

Instructional Strategies:

See K.A0.1

Sample unit sequence:

- A contextual problem (word problem) is presented to the students such as, “Melisa goes to Debbie’s house. Debbie tells her she may have 5 pieces of fruit to take home. There are lots of apples and bananas. How many of each can she take?”
- Students find related number pairs using objects (such as cubes or two-color counters), drawings, and/or equations. Students may use different representations based on their experiences, preferences, etc.
- Students may write equations that equal 5 such as:

$$* 5=4+1$$

$$* 3+2=5$$

$$* 2+3=4+1$$

$$* 5+ 0 = 5$$

This is a good opportunity for students to systematically list all the possible number pairs for a given number. For example, all the number pairs for 5 could be listed as 0+5, 1+4, 2+3, 3+2, 4+1, and 5+0. Students should describe the pattern that they see in the addends, e.g., each number is one less or one than the previous addend. (Continue to make sure students include the number plus zero as a possible solution).

Tools/Resources:

K.OA.A.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5=2+3$ and $5=4+1$).

- [K.OA Make 9](#)
- [K.OA Bobbie Bear's Buttons](#)
- [K.OA Christina's Candies](#)
- [K.OA Pick Two](#)
- [K.OA Shake and Spill](#)
- [K.OA My Book of Five](#)

Common Misconceptions:

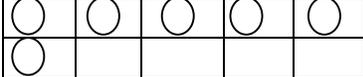
See K.OA.1

Domain: Operations and Algebraic Thinking (OA)
Cluster: <u>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</u>
Standard: Grade K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (OA)
Suggested Standards for Mathematical Practice (MP): MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Connections: See K. OA.1
Explanations and Examples: This standard builds upon the understanding that a number can be decomposed into parts (K.OA.3). The number pairs that total ten are foundational for students' ability to work fluently within numbers and operations. Different models, such as ten-frames, cubes, two-color counters, etc., assist students in visualizing these number pairs for ten. Once students have had experiences breaking apart ten into various combinations, this asks students to find a missing part of 10.

Major**Supporting****Additional**
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Example 1:

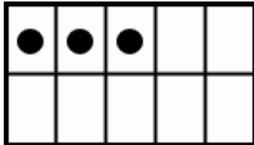
"A full case of juice boxes has 10 boxes. There are only 6 boxes in this case. How many juice boxes are missing?"

<u>Student 1</u>	<u>Student 2</u>	<u>Student 3</u>
<p>Using a Ten Frame "I used 6 counters for the 6 boxes of juice still in the case. There are 4 blank spaces so 4 boxes have been removed. This makes sense since 6 and 4 more equal 10".</p> 	<p>Think addition. "I counted out 10 cubes because I knew there needed to be ten. I pushed these 6 over here because they were in the container. These are left over. So there's 4 missing"</p>	<p>Basic Fact. "I know that it's 4 because 6 and 4 is the same amount as 10".</p>

Example 2:

Students place three objects on a ten frame and then determine how many more are needed to "make a ten."

Students may use electronic versions of ten frames to develop this skill.

**Example 3:**

The student snaps ten cubes together to make a "train."

- Student breaks the "train" into two parts. S/he counts how many are in each part and record the associated equation ($10 = \underline{\quad} + \underline{\quad}$).
- Student breaks the "train into two parts. S/he counts how many are in one part and determines how many are in the other part without directly counting that part. Then s/he records the associated equation (if the counted part has 4 cubes, the equation would be $10 = 4 + \underline{\quad}$).
- Student covers up part of the train, without counting the covered part. S/he counts the cubes that are showing and determines how many are covered up. Then s/he records the associated equation (if the counted part has 7 cubes, the equation would be $10 = 7 + \underline{\quad}$).

Example 4:

The student tosses ten two-color counters on the table and records how many of each color are facing up.

Instructional Strategies:

See K.OA.1

Tools/Resources:

See engageNY Modules:

<https://www.engageny.org/resource/kindergarten-mathematics>

See: http://commoncoretools.me/wp-content/uploads/2011/04/ccss_progression_nbt_2011_04_073_corrected2.pdf for additional detailed information.

Common Misconceptions:

See K.OA.1

Domain: Operations and Algebraic Thinking (OA)
Cluster: <i>Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.</i>
Standard: Grade K.OA.5 Fluently add and subtract within 5. (OA)
Suggested Standards for Mathematical Practice (MP): MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Connections: See K.OA.1
Explanation and Examples: All standards in the Operations and Algebraic Thinking cluster should only include numbers through 10. Students will model simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. Kindergarten students should see addition and subtraction equations. Student writing of equations in kindergarten is <u>encouraged</u> , but it is <u>not required</u> . Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. This standard uses the word fluently, which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property and/or those shown below). Fluency is developed by working with many different kinds of objects over an extended amount of time. This objective does not require students to instantly know the answer. <i>Traditional flash cards or timed tests have not been proven as effective instructional strategies for developing fluency.</i>

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Depth Opportunities (DO)

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This standard focuses on students being able to add and subtract numbers within 5. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategies students may use to attain fluency include:

- Counting on (e.g., for $3+2$, students will state, “3,” and then count on two more, “4, 5,” and state the solution is “5”)
- Counting back (e.g., for $4-3$, students will state, “4,” and then count back three, “3, 2, 1” and state the solution is “1”)
- Counting up to subtract (e.g., for $5-3$, students will say, “3,” and then count up until they get to 5, keeping track of how many they counted up, stating that the solution is “2”)
- Using doubles (e.g., for $2+3$, students may say, “I know that $2+2$ is 4, and 1 more is 5”)
- Using commutative property (e.g., students may say, “I know that $2+1=3$, so $1+2=3$ ”)
- Using fact families (e.g., students may say, “I know that $2+3=5$, so $5-3=2$ ”)

Students may use electronic versions of five frames to develop fluency of these facts.

Instructional Strategies:

See K.OA.1

Tools/Resources:

K.OA.A.5. Fluently add and subtract within 5.

- [K.OA Many Ways to Do Addition 1](#)
- [K.OA My Book of Five](#)

Common Misconceptions:

See K.OA.1

Domain: Number and Operations in Base Ten (NBT)
Cluster: <i>Work with the numbers 11 to 19 to gain a foundation for place value.</i>
<p>Standard: Grade K.NBT.1</p> <p>Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (MP.4 <i>model</i>) (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (NBT)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects, K.CC.3; and language arts standards K.RI.3; K.W.2</p>
<p>Explanation and Examples:</p> <p>This is the first time that students move beyond the number 10 with representations, such as objects (manipulatives) or drawings.</p> <p>The spirit of this standard is that students separate out a set of 11-19 objects into a group of ten objects with leftovers. This ability is a pre-cursor to later grades when they need to understand a more complex concept that a group of 10 objects is also one ten (<i>unitizing</i>). Ample experiences with ten frames will help solidify this concept.</p> <p>Research states that students are not ready to unitize until the end of first grade. Therefore, this work in Kindergarten lays the foundation of composing tens and recognizing leftovers.</p>

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Supporting

Additional

Depth Opportunities (DO)

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Example:

Teacher: “Please count out 15 chips.”

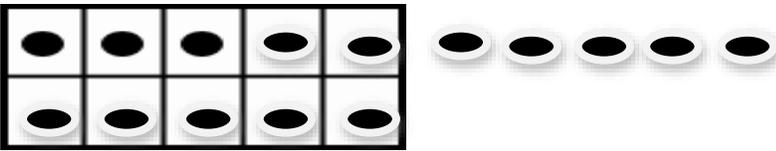
Student: Student counts 15 counters (chips or cubes).

Teacher: “Do you think there is enough to make a group of ten chips? Do you think there might be some chips leftover?”

Student: Student answers.

Teacher: “Use your counters to find out.”

Student: Student can either fill a ten frame or make a stick of ten connecting cubes. They answer, “There is enough to make a group of ten”



Teacher: How many leftovers do you have?

Student: Students say, “I have 5 left over.”

Teacher: How could we use words and/or numbers to show this?

Student: Students might say “Ten and five is the same amount as 15”, “ $15 = 10 + 5$ ”

Special attention needs to be paid to this set of numbers as they do not follow a consistent pattern in the verbal counting sequence.

- Eleven and twelve are special number words.
- “Teen” means one “ten” plus ones.
- The verbal counting sequence for teen numbers is backwards – we say the ones digit before the tens digit. For example “27” reads tens to ones (twenty-seven), but 17 reads ones to tens (seven-teen).
- In order for students to interpret the meaning of written teen numbers, they should read the number as well as describe the quantity. For example, for 15, the students should read “fifteen” and state that it is one group of ten *and* five ones and record that $15 = 10 + 5$.

Teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represent each teen number. For example, when focusing on the number “14,” students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten ones and four additional ones.

Students should connect the representation to the symbol “14.” Students should recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated.

Instructional Strategies:

Kindergarteners need to understand the idea of a ten so they can develop the strategy of adding onto 10 to add within 20 in Grade 1. Students need to construct their own base-ten ideas about quantities and their symbols by connecting to counting by ones.

They should use a variety of manipulatives to model and connect equivalent representations for the numbers 11 to 19. For instance, to represent 13, students can count by ones and show 13 beans. They can anchor to five and show one group of 5 beans and 8 beans or anchor to ten and show one group of 10 beans and 3 beans.

Students need to eventually see a ten as different from 10 ones.

After the students are familiar with counting up to 19 objects by ones, have them explore different ways to group the objects that will make counting easier. Have them estimate before they count and group.

Discuss their groupings and lead students to conclude that grouping by ten is desirable. *10 ones make 1 ten* makes students wonder how something that means a lot of things can be one thing.

Students need to first use materials that can be grouped to represent numbers 11 and 19 because a group of ten such as a bundle of 10 straws or a cup of 10 beans makes more sense than *a ten* in pre-grouped materials. They need to see that there are 10 single objects represented on the item for ten in pre-grouped materials, such as the rod in base-ten blocks.

Students need to learn to attach words to materials and groups and understand what they represent. Eventually, they need to see the rod as *a ten* that they did not group themselves.

Students should impose their base-ten concepts on a model made from “groupable” and “pre-groupable” materials. Students can transition from “groupable” to “pre-groupable” materials by leaving a group of ten intact to be reused as a pre-grouped item.

When using pre-grouped materials, students should reflect on the ten-to-one relationships in the materials, such as the “tenness” of the rod in base-ten blocks. After many experiences with pre-grouped materials, students can use dots and a stick (one tally mark) to record singles and a ten. Kindergartners should use proportional base-ten models, where a group often is physically 10 times larger than the model for one.

Non-proportional models such as an abacus and money should not be used at this grade level. (Exception: penny can be used to for counting, grouping, etc. to represent one, the nickel can be

used when working with 5-frames as a representation of five, and the dime as a representation of the 10 frame).

Encourage students to use base-ten language to describe quantities between 11 and 19. At the beginning, students do not need to use *ones* for the singles. Some of the base-ten language that is acceptable for describing quantities such as 18 includes *one ten and eight, a bundle and eight, a rod and 8 singles* and *ten and eight more*. Write the horizontal equation $18 = 10 + 8$ and connect it to base-ten language.

Encourage, but do not require, students to write equations to represent quantities. Students have difficulty with *ten* as a singular word that means 10 things. For many students, the understanding that a group of 10 things can be replaced by a single object and they both represent 10 is confusing.

Tools/Resources

For detailed information, see Learning Progressions Number and Operations in Base Ten: http://commoncoretools.me/wp-content/uploads/2011/04/ccss_progression_nbt_2011_04_073_corrected2.pdf

K.NBT.A.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

- [K.NBT What Makes a Teen Number?](#)

Common Misconceptions:

Students have difficulty with *ten* as a singular word that means 10 things. For many students, the understanding that a group of 10 things can be replaced by a single object and they both represent 10 is confusing. Help students develop the sense of *ten* by first using groupable materials then replacing the group with an object or representing 10, such as a rod or 10 Frame.

Watch for and address the issue of attaching words to materials and groups without knowing what they represent. If this misconception is not addressed early on it can cause additional issues when working with numbers 11-19 and beyond.

At this stage you may encounter some students who when working with “grouped” materials will continue to count each object in the “ten group”. The students who do this are developmentally at the beginning of the idea of ten as a group.

Domain: Measurement and Data (MD)
Cluster: <i>Describe and compare measurable attributes.</i>
Standard: Grade K.MD.1 Describe measurable attributes of objects, such as length or weight . Describe several measurable attributes of a single object. (MD)
Suggested Standards for Mathematical Practice (MP): MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.
Connections: This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects.
Explanation and Examples: This standard calls for students to describe measurable attributes of objects, such as length and weight . In order to describe attributes such as length and weight; students must have many opportunities to informally explore these attributes. <ul style="list-style-type: none"> • Students should state comparisons of objects verbally and then focus on specific attributes when making verbal comparisons for K.MD.2. • They may identify measurable attributes such as length, width, height, and weight. For example, when describing a soda can, a student may talk about how tall, how wide, how heavy, or how much liquid can fit inside. These are all measurable attributes. Non-measurable attributes include: words on the object, colors, pictures, etc. This standard focuses on using descriptive words and does not mean that students should sort objects based on attributes.

Instructional Strategies:

It is critical for students to be able to identify and describe measurable attributes of objects. An object has different attributes that can be measured, like the height and weight of a can of food.

Students should be given many opportunities to compare directly where the attribute becomes the focus. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: Which box will hold the most? Which box will hold least? Will they hold the same amount? “How could you find out?” Students can decide to fill one box with dried beans then pour the beans into the other box to determine the answers to these questions.

Have students work in pairs to compare their arm spans. As they stand back-to-back with outstretched arms, compare the lengths of their spans, then determine who has the shortest arm span. Ask students to explain their reasoning.

Then ask students to suggest other measurable attributes of their bodies that they could directly compare, such as their height or the length of their feet.

Tools/ Resources

For detailed information, see Learning Progression

http://commoncoretools.files.wordpress.com/2012/07/ccss_progression_gm_k5_2012_07_21.pdf

K.MD.A. Describe and compare measurable attributes.

[K.MD Longer and Heavier? Shorter and Heavier?](#)

K.MD.A.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

[K.MD Which is heavier?](#)

See “Chrysanthemum is My Name”, Georgia Department of Education. In this task, the students will compare the length of their names with the length of their classmate’s names using cube towers. They will share and discuss the differences and similarities in the lengths using math vocabulary such as longer, shorter, more and less.

<http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathBelinda>

Common Misconceptions:

Some students may experience difficulty in being able to classify the same object into different categories. They see it as belonging to only one category.

Domain: Measurement and Data (MD)
Cluster: <i>Describe and compare measurable attributes.</i>
<p>Standard: Grade K.MD.2</p> <p>Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. <i>For example, directly compare the heights of two children and describe one child as taller/shorter.</i> (MD)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.6 Attend to precision. MP.7 Look for and make use of structure.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects. This cluster is connected to <i>Measure lengths indirectly and by iterating length units</i> in Grade 1.</p>
<p>Explanation and Examples:</p> <p>This standard asks for direct comparisons of objects. Direct comparisons are made when objects are put next to each other, such as two children, two books, two pencils. For example, a student may line up two blocks and say, “This block is a lot longer than this one.” Students are not comparing objects that cannot be moved and lined up next to each other. The objects do not have to be the same. A book might be compared to a pencil or a pencil to a crayon.</p> <p>When making direct comparisons for length, students must attend to the “starting point” of each object and recognize that objects should be matched up at the end of objects to get accurate measurements. For example, the ends need to be lined up at the same point, or students need to compensate when the starting points are not lined up.</p> <p>Conservation of length includes understanding that if an object is moved, its length does not change; an important concept when comparing the lengths of two objects). Since this understanding requires conservation of length, a developmental milestone for young children, children need multiple experiences to move beyond the idea that . . .</p>

Major

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[Type here]

Depth Opportunities (DO)

*“Sometimes this block is **longer than** this one and sometimes it’s **shorter** (depending on how I lay them side by side) and that’s okay.” “This block is always longer than this block (with each end lined up appropriately).”*

Before conservation of length: *The blue block is longer or shorter than the plain block when they are lined up like this. But when I move the blocks around, sometimes the plain block is longer than the blue block.*



After conservation of length: *I have to line up the blocks to measure them.*

Language plays an important role in this standard as students describe the similarities and differences of measurable attributes of objects (e.g., shorter than, taller than, lighter than, the same as, etc.).

Instructional Strategies:

Students should have many opportunities to compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third objects).

A student can be given an object as part of a scavenger hunt in the classroom and be asked to find one or two objects that are the same length as; two that are longer and two that are shorter.

Tools/Resources

K.MD.A.2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

- [K.MD Size Shuffle](#)
- [K.MD Which weighs more? Which weighs less?](#)
- [K.MD Which is heavier?](#)
- [K.MD Longer and Shorter](#)
- [K.MD Which is Longer?](#)
- [K.MD Which is Heavier?](#)

Common Misconceptions:

Many students have difficulty understanding that when an object is moved away from the object they are comparing it with, the length does not change. With multiple opportunities, students learn that they have to line up the items they are comparing and/or measuring. (*Conservation of Length: includes understanding that if an object is moved, its length does not change; an important concept when comparing the lengths of two objects*).

Domain: Measurement and Data (MD)
Cluster: Classify objects and count the number of objects in each category.
<p>Standard: Grade K.MD.3</p> <p>Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. <i>(Limit category counts to be less than or equal to 10).</i> (MD)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #1, Representing and comparing whole numbers, initially with sets of objects.</p> <p>This cluster is connected to <i>Know number names and the count sequence</i> and <i>Count to tell the number of objects</i> in Kindergarten, and to <i>Represent and interpret data</i> in Grade 1.</p>
<p>Explanation and Examples:</p> <p>This standard asks students to identify similarities and differences between objects (e.g., size, color, shape) and use the identified attributes to sort a collection of objects.</p> <p>Once the objects are sorted, the student counts the amount in each set. Once each set is counted, then the student is asked to sort (or group) each of the sets by the amount in each set.</p> <p>For example, when given a collection of buttons, the student separates the buttons into different piles based on color (all the blue buttons are in one pile, all the orange buttons are in a different pile, etc.). Then the student counts the number of buttons in each pile: blue (5), green (4), orange (3), purple (4). Finally, the student organizes the groups by the quantity in each group (Orange buttons (3), Green buttons (4), Purple buttons with the green buttons because purple also had (4), Blue buttons last (5).</p> <p>Other possible objects to sort include: shells, shapes, beans, small toys, coins, rocks, etc. After sorting and counting, it is important for students to:</p> <ul style="list-style-type: none"> • <i>explain how they sorted the objects;</i> • <i>label each set with a category;</i> • <i>answer a variety of counting questions that ask, “How many ...”;</i> and • <i>compare sorted groups using words such as, “most”, “least”, “alike” and “different”.</i> <p>This objective helps to build a foundation for data collection in future grades. In later grade, students will transfer these skills to creating and analyzing various graphical representations.</p>

Major**Supporting****Additional**
[Type here]**Depth Opportunities (DO)**

Instructional Strategies:

Provide categories for students to use to sort a collection of objects. Each category can relate to only one attribute, like *Red* and *Not Red* or *Hexagon* and *Not Hexagon*, and contain up to 10 objects. Students count how many objects are in each category and then order the categories by the number of objects they contain.

Ask questions to initiate discussion about the attributes of shapes. Then have students sort a collection of two-dimensional and three-dimensional shapes by their attributes. Provide categories like *Circles* and *Not Circles* or *Flat* and *Not Flat*.

Have students count the objects in each category and order the categories by the number of objects they contain.

Have students infer the classification of objects by guessing the rule for a sort. First, the teacher uses one attribute to sort objects into two loops or regions without labels. Then the students determine how the objects were sorted, suggest labels for the two categories and explain their reasoning.

Tools/Resources

Visit : “Take it to the store”

<http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathStore> for some ideas that could be used.

K.MD.B.3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.*

- [K.MD Sort and Count I](#)
- [K.MD Sort and Count II](#)
- [K.MD Goodie Bags](#)

Domain: Geometry (G)
Cluster: <i>Identify and describe shapes (squares, circles, triangles, rectangles, hexagon, cubes, cones, cylinders and spheres).</i>
Standard: Grade K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as <i>above, below, beside, in front of, behind, and next to</i> . (GE)
Suggested Standards for Mathematical Practice (MP): MP.6 Attend to precision. MP.7 Look for and make use of structure.
Connections: This cluster is connected to the Kindergarten Critical Area of Focus #2, Describing shapes and space . This cluster is connected to <i>Analyze, compare, create and compose shapes</i> in Kindergarten, and to <i>Reason with shapes and their attributes</i> in Grade 1.
Explanation and Examples: This standard expects students to use positional words such as: <i>above, below, beside, in front of, behind, and next to</i> in describing objects in the environment. Kindergarten students need to focus first on location and position of two-and-three-dimensional objects in their classroom prior to describing location and position of two-and-three-dimensional representations on paper. Examples of environments in which students would be encouraged to identify shapes would include nature, buildings, and the classroom using positional words in their descriptions. Teachers should work with children and pose four mathematical questions: Which way? How far? Where? And what objects? To answer these questions, children develop a variety of important skills contributing to their spatial thinking.

Major

Supporting

Additional
[Type here]

Depth Opportunities (DO)

Examples:

- Teacher holds up an object such as an ice cream cone, a number cube, ball, etc. and asks students to identify the shape. Teacher holds up a can of soup and asks, "What shape is this can?" Students respond "cylinder!"
- Teacher places an object next to, behind, above, below, beside, or in front of another object and asks positional questions. Where is the water bottle? (water bottle is placed behind a book) Students say "The water bottle is behind the book."

Students should have multiple opportunities to identify shapes; these may be displayed as photographs, or pictures using the document camera or interactive whiteboard.

Instructional Strategies:

Develop spatial sense by connecting geometric shapes to students' everyday lives. Initiate natural conversations about shapes in the environment. Have students identify and name two- and three-dimensional shapes in and outside of the classroom and describe their relative position.

Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see, e.g. *This rectangle (a poster) is over the sphere (globe)*. Teachers can use a digital camera to record these relationships.

Hide shapes around the room. Have students say where they found the shape using positional words, e.g. *I found a triangle UNDER the chair.*

Have students create drawings involving shapes and positional words: *Draw a window ON the door* or *Draw an apple UNDER a tree*. Some students may be able to follow two- or three-step instructions to create their drawings.

Use a shape in different orientations and sizes along with non-examples of the shape so students can learn to focus on defining attributes of the shape.

Manipulatives used for shape identification actually have three dimensions. However, Kindergartners need to think of these shapes as two-dimensional or "flat" and typical three-dimensional shapes as "solid."

Students will identify two-dimensional shapes that form surfaces on three-dimensional objects. Students need to focus on noticing two and three dimensions, not on the words *two-dimensional* and *three-dimensional*.

You may create an activity where an object is identified and whispered into the teacher's ear. Students then ask question of the first student---"is it in front of", etc.

Resources/Tools

For detailed information see Geometry Learning Progression:

http://commoncoretools.files.wordpress.com/2012/06/ccss_progression_g_k6_2012_06_27.pdf

K.G.A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

- [K.G Shape Sequence Search](#)
- [K.G Shape Hunt Part 1](#)
- [K.G Shape Hunt Part 2](#)

Common Misconceptions:

Students many times use incorrect terminology when describing shapes. For example students may say a *cube* is a *square* or that a *sphere* is a *circle*. The use of the two-dimensional shape that appears to be part of a three-dimensional shape to name the three-dimensional shape is a common misconception. Work with students to help them understand that the two-dimensional shape is a part of the object but it has a different name.

Domain: Geometry (G)
Cluster: <i>Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cylinders, and spheres).</i>
Standard: Grade K.G.2 Correctly name shapes regardless of their orientations or overall size. (GE)
Suggested Standards for Mathematical Practice (MP): MP.6 Attend to precision. MP.7 Look for and make use of structure.
Connections: This cluster is connected to the Kindergarten Critical Area of Focus #2, Describing shapes and space . This cluster is connected to <i>Analyze, compare, create and compose shapes</i> in Kindergarten, and to <i>Reason with shapes and their attributes</i> in Grade 1.
Explanation and Examples: Children also need to see examples of shapes beyond circles, squares, rectangles, and triangles. Without these, children develop limited notions. Kindergartners should also learn to recognize these shapes whether they are in “standard position” or rotated so that their bases are not horizontal. Kindergartners can begin to develop explicit and sophisticated levels of thinking and communication. They can learn to describe, and even define, shapes in terms of their <i>parts</i> or <i>attributes</i> (properties). For example, they can build accurate representations of shapes from physical models of line segments, such as sticks. As they discuss what they have built, attributes of the shapes will arise naturally. Example: Student: “I built a rectangle” Teacher: “How do you know it is a rectangle?” Student: “ Because the two opposite sides are the same length and all the angles are the same—they are right angles like in a square.”

The experience of discussing attributes of rectangles (or any shape they build) helps children begin to understand the *geometric structure* of all rectangles at an explicit level of thinking.

Having students build shapes and feel shapes hidden in a bag or box and describe what they feel or built helps them explore the properties/attributes of shapes.

Such activities help children learn to identify and describe shapes by the number of their sides or corners. Such descriptions build geometric concepts but also reasoning skills and language.

They encourage children to view shapes analytically. Children begin to describe some shapes in terms of their properties, such as saying that squares have four side of *equal length*. They informally describe properties of blocks in functional contexts, such as that some blocks roll and those that do not roll.

Instructional Strategies:

Allow children to build shapes using straws/toothpicks and clay or soft items such as candy or marshmallows.

Another valuable activity is the tactile-kinesthetic exploration of shapes—feeling shapes hidden in a box. Kindergartners can name the shape they are feeling rather than just match shapes. After this, they can extend the activity further as they **describe** the shape without using its name, so that their friends can name the shape. In this way, children learn the properties of the shape, moving from intuitive to explicit, verbalized knowledge. All these variations can be repeated with less familiar shapes.

Common Misconceptions:

Many children come to believe incorrectly that shapes such as a trapezoid “is not a shape” because it is not a shape for which they know a name for the shape.

Domain: Geometry (G)
Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, spheres).
Standard: Grade K.G.3 Identify shapes as two-dimensional (lying in a plane, “flat”) or three dimensional (“solid”). (GE)
Suggested Standards for Mathematical Practice (MP): MP.6 Attend to precision. MP.7 Look for and make use of structure.
Connections: This cluster is connected to the Kindergarten Critical Area of Focus #2, Describing shapes and space . This cluster is connected to <i>Analyze, compare, create and compose shapes</i> in Kindergarten, and to <i>Reason with shapes and their attributes</i> in Grade 1.
Explanation and Examples: Students are asked to identify two-dimensional (flat objects) and three-dimensional (solid objects). This standard can be done by having students sort 2-dimensional and 3-dimensional objects, or by having students describe the appearance or thickness of shapes. A final type of relationship between shapes that is very important is the difference between two-dimensional (flat) and three-dimensional shapes. Student should be able to differentiate between two dimensional and three dimensional shapes. <ul style="list-style-type: none"> • Student names a picture of a shape as two dimensional because it is flat and can be measured in only two ways (length and width). • Student names an object as three dimensional because it is not flat (it is a solid object/shape) and can be measured in three different ways (length, width, height/depth). Faces of three dimensional shapes can be identified as specific two-dimensional shapes.
Instructional Strategies: See K.G.2
Common Misconceptions: See K.G.2

Domain: Geometry (G)
Cluster: Analyze, compare, create, and compose shapes.
<p>Standard: Grade K.G.4</p> <p>Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length). (GE)</p>
<p>Suggested Standards for Mathematical Practice (MP):</p> <p>MP.4 Model with mathematics. MP.6 Attend to precision. MP.7 Look for and make use of structure.</p>
<p>Connections:</p> <p>This cluster is connected to the Kindergarten Critical Area of Focus #2, Describing shapes and space.</p> <p>This cluster is connected to <i>Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)</i> in Kindergarten, and to <i>Reason with shapes and their attributes</i> in Grade 1.</p>
<p>Explanation and Examples:</p> <p>This standard asks the student to note similarities and differences between and among 2-D and 3-D shapes using informal language. These experiences help young students begin to understand how 3-dimensional shapes are composed of 2-dimensional shapes (e.g., The base and the top of a cylinder is a circle; a circle is formed when tracing a sphere).</p> <p>Students analyze and compare two- and three-dimensional shapes by observations. Their visual thinking enables them to determine if things are alike or different based on the appearance of the shape.</p> <p>Students sort objects based on appearance. Even in early explorations of geometric properties, they are introduced to how categories of shapes are subsumed (<i>contained</i>) within other categories. For instance, they will recognize that a square is a special type of rectangle.</p> <p>Students should be exposed to triangles, rectangles, and hexagons whose sides are not all congruent.</p>

They first begin to describe these shapes using everyday language and then refine their vocabulary to include sides and vertices/corners.

These geometric competencies are at the foundation of geometry, but also arithmetic (e.g., composing and decomposing numbers and arrays in multiplication), measurement, and higher-order geometric work. Creating and then iterating units and higher-order units in the context of construction patterns, measuring, and computing, are established bases for mathematical understanding and analysis.

Instructional Strategies:

Opportunities to work with pictorial representations, concrete objects, as well as technology helps student develop their understanding and descriptive vocabulary for both two- and three-dimensional shapes.

It is important to allow students to explore and build geometric understanding themselves. One important step to take is to switch from making assertions and generalizations to framing ideas as questions. Rather than saying, “Every time you put two triangles together, you get a square”—a mathematically incorrect statement. Ask the following:

“How many different ways can you put these two triangles together to make a new shape?”
 “What shapes will you get?”

This allows children to see that even with two right triangles made from a square, they can put these together to make a triangle or a parallelogram.

Kindergartners can develop the ability to intentionally and systematically combine shapes to make new shapes and complete puzzles. They do so with increasing anticipation, on the basis of the shapes’ attributes, and thus, children developmental imagery of the component shapes. They move from using shapes separately to putting them together to make pictures.

A significant advance is that they can combine shapes with different properties, extending the pattern block shapes (whose angles are multiples of 30 degrees) to such shapes as tangrams (with angles that are multiples of 45 degrees).

In addition they can explore sets of various shapes that include angles that are multiples of 15 degrees, as well as sections of circles.

Combining these shape sets should be done after children have worked with the pattern-block shapes separately from the square/rectangle/right triangle shapes based on 90 degrees and 45 degrees because many compositions are possible when the angles are consistent.

Use shapes collected from students to begin the investigation into basic properties and characteristics of two- and three-dimensional shapes. Have students analyze and compare each shape with other objects in the classroom and describe the similarities and differences between the shapes.

Ask students to describe the shapes while the teacher records key descriptive words in common student language. Students may use the word *flat* to describe two-dimensional shapes and the word *solid* to describe three-dimensional shapes.

Use the sides, faces and vertices of shapes to practice counting and reinforce the concept of one-to-one correspondence.

The teacher and students orally describe and name the shapes found on a Shape Hunt. Students draw a shape and build it using materials regularly kept in the classroom such as construction paper, clay, wooden sticks or straws.

Students can use a variety of manipulatives and real-world objects to build larger shapes with these and other smaller shapes: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.

Kindergarteners can manipulate cardboard shapes, paper plates, pattern blocks, tiles, canned food, and other common items.

Have students compose (build) a larger shape using only smaller shapes that have the same size and shape. The sides of the smaller shapes should touch and there should be no gaps or overlaps within the larger shape. For example, use one-inch squares to build a larger square with no gaps or overlaps.

Have students also use different shapes to form a larger shape where the sides of the smaller shapes are touching and there are no gaps or overlaps. Ask students to describe the larger shape and the shapes that formed it.

-Focus in Kindergarten, NCTM 2011

Tools/Resources

K.G.B.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).

- [K.G Alike or Different Game](#)

Common Misconceptions:

One of the most common misconceptions in geometry is the belief that orientation is tied to shape. A student may see the first of the figures below as a triangle, but claim to not know the name of the second.

Students need to have many experiences with shapes in different orientations. For example, in the *Just Two Triangles* activity, ask students to form larger triangles with the two triangles in different orientations.

Another misconception is confusing the name of a two-dimensional shape with a related three-dimensional shape or the shape of its face. For example, students might call a *cube* a *square* because the student sees the face of the cube.

It is important when students are exploring 2-dimensional shapes (flat) that the shapes they are working with are on paper or other “FLAT” material.

Domain: Geometry (G)
Cluster: <i>Analyze and compare shapes.</i>
Standard: Grade K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. (G)
Suggested Standards for Mathematical Practice (MP): MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.7 Look for and make use of structure.
Connections: This cluster is connected to the Kindergarten Critical Area of Focus #2, Describing shapes and space . This cluster is connected to <i>Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)</i> in Kindergarten, and to <i>Reason with shapes and their attributes</i> in Grade 1.
Explanation and Examples: Students are asked to apply their understanding of geometric attributes of shapes in order to create given shapes. For example, a student may roll a clump of play-doh into a sphere or use their finger to draw a triangle in the sand table, recalling various attributes in order to create that particular shape. Because two-dimensional shapes are flat and three-dimensional shapes are solid, students should draw two-dimensional shapes and build three-dimensional shapes. Shapes may be built using materials such as clay, toothpicks, marshmallows, gumdrops, straws, pipe cleaners, etc.
Instructional Strategies: See K. G.4
Common Misconceptions: See K. G.4

Domain: Geometry (G)
Cluster: <i>Analyze and compare shapes.</i>
Standard: Grade K.G.6 Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?” (G)
Suggested Standards for Mathematical Practice (MP): MP.1 Make sense of problems and persevere in solving them. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.7 Look for and make use of structure.
Connections: See K. G.1
Explanation and Examples: This standard moves beyond identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. This concept begins to develop as students first move, rotate, flip, and arrange puzzle pieces. Next, students use their experiences with puzzles to move given shapes to make a design (e.g., “Use the 7 tangrams pieces to make a fox.”). Finally, using these previous foundational experiences, students manipulate simple shapes to make a new shape.
Instructional Strategies: Students use pattern blocks, tiles, or paper shapes and technology to make new two- and three-dimensional shapes. Their investigations allow them to determine what kinds of shapes they can join to create new shapes. They answer questions such as “What shapes can you use to make a square, rectangle, circle, triangle? ...etc.” This is an opportunity to use blocks from a play center to create shapes composed of a series of blocks. Laying several rectangular prisms can make other identifiable shapes. Students may use a document camera to display shapes they have composed from other shapes. They may also use an interactive whiteboard to copy shapes and compose new shapes. They should describe and name the new shape.

Major

Supporting

Additional
[Type here]

Depth Opportunities (DO)

Common Misconceptions:

See K. G.4

Major**Supporting****Additional**
[Type here]**Depth Opportunities (DO)**

APPENDIX

TABLE 1. Common Addition and Subtraction Situations⁶

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown ¹
Put Together / Take Apart²	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ or $5 = 5 + 0$ $5 = 1 + 4$ or $5 = 4 + 1$ $5 = 2 + 3$ or $5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare³	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ or $? + 3 = 5$

¹These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

²Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

³For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

⁶Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common Multiplication and Division Situations⁷

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ And $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example:</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example:</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example:</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays,⁴ Area⁵	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example:</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example:</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example:</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example:</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example:</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example:</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

⁴The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

⁵Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

⁷The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The Properties of Operations

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every (a) there exists $(-a)$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Here a , b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system

TABLE 4. The Properties of Equality*

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$ then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$ then $a + c = b + c$
Subtraction property of equality	If $a = b$ then $a - c = b - c$
Multiplication property of equality	If $a = b$ then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$ then $a \div c = b \div c$
Substitution property of equality	If $a = b$ then b may be substituted for a in any expression containing a .

Here a , b and c stand for arbitrary numbers in the rational, real, or complex number systems.

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$
If $a > b$ then $b < a$
If $a > b$ then $-a < -b$
If $a > b$ then $a \pm c > b \pm c$
If $a > b$ and $c > 0$ then $a \times c > b \times c$
If $a > b$ and $c < 0$ then $a \times c < b \times c$
If $a > b$ and $c > 0$ then $a \div c > b \div c$
If $a > b$ and $c < 0$ then $a \div c < b \div c$

Here a , b and c stand for arbitrary numbers in the rational or real number systems.

TABLE 6. Development of Counting in K-2 Children

Levels	$8 + 6 = 14$	$14 - 8 = 6$
Level 1: Count all	<p>Count All</p> <p>a</p> <p>1 2 3 4 5 6 7 8</p> <p>○ ○ ○ ○ ○ ○ ○ ○</p> <p>b</p> <p>1 2 3 4 5 6</p> <p>○ ○ ○ ○ ○ ○</p> <p>c</p> <p>1 2 3 4 5 6 7 8 9 10 11 12 13 14</p>	<p>Take Away</p> <p>a</p> <p>1 2 3 4 5 6 7 8 9 10 11 12 13 14</p> <p>○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○</p> <p>b</p> <p>1 2 3 4 5 6 7 8 1 2 3 4 5 6</p> <p>○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○</p> <p>c</p>
Level 2: Count on	<p>Count On</p> <p>8</p> <p>○ ○ ○ ○ ○ ○ ○ ○</p> <p>→ 8</p> <p>9 10 11 12 13 14</p>	<p>To solve $14 - 8$ I count on $8 + ? = 14$</p> <p>10 11 12 13 14</p> <p>I took away 8</p> <p>8 to 14 is 6 so $14 - 8 = 6$</p>
Level 3: Recompose	<p>Recompose: Make a Ten</p> <p>Make a ten (general): one addend breaks apart to make 10 with the other addend</p> <p>○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○</p> <p>10 + 4</p> <p>Make a ten (from 5's within each addend)</p> <p>○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○</p> <p>10 + 4</p>	<p>$14 - 8$: I make a ten for $8 + ? = 14$</p> <p>○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○</p> <p>8 + 2 + 4</p> <p>6</p> <p>8 + 6 = 14</p>
Doubles $\pm n$	<p>$6 + 8$</p> <p>$= 6 + 6 + 2$</p> <p>$= 12 + 2 = 14$</p>	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Beginning--A child can count very small collections (1-4) collection of items and understands that the last word tells "how many" even. Beyond on small numbers the number words may be said without the lack of numerical understanding. This is often referred to as rote counting.

Level 1—The child uses one to one correspondence to find the number of objects in two sets. Even if the quantity is known for the first set, the child will start with the first set and continue counting on into the second set. The child begins the count with one. This also connects to Piaget's Hierarchical Inclusion – that in order to have 7 – you have to have 6, 5, 4, etc.

Level 2 – At this level the student begins the counting, starting with the known quantity of the first set and "counts on" from that point in the sequence to establish how many. This method is used to find the total in two sets. This counting is not rote. This level of counting requires several connections between cardinality and counting meanings of the number words.

Table 7. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	<ul style="list-style-type: none"> Recall conversions, terms, facts 			
Understand	<ul style="list-style-type: none"> Evaluate an expression Locate points on a grid or number on number line Solve a one-step problem Represent math relationships in words, pictures, or symbols 	<ul style="list-style-type: none"> Specify, explain relationships Make basic inferences or logical predictions from data/observations Use models/diagrams to explain concepts Make and explain estimates 	<ul style="list-style-type: none"> Use concepts to solve non-routine problems Use supporting evidence to justify conjectures, generalize, or connect ideas Explain reasoning when more than one response is possible Explain phenomena in terms of concepts 	<ul style="list-style-type: none"> Relate mathematical concepts to other content areas, other domains Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	<ul style="list-style-type: none"> Follow simple procedures Calculate, measure, apply a rule (e.g., rounding) Apply algorithm or formula Solve linear equations Make conversions 	<ul style="list-style-type: none"> Select a procedure and perform it Solve routine problem applying multiple concepts or decision points Retrieve information to solve a problem Translate between representations 	<ul style="list-style-type: none"> Design investigation for a specific purpose or research question Use reasoning, planning, and supporting evidence Translate between problem & symbolic notation when not a direct translation 	<ul style="list-style-type: none"> Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	<ul style="list-style-type: none"> Retrieve information from a table or graph to answer a question Identify a pattern/trend 	<ul style="list-style-type: none"> Categorize data, figures Organize, order data Select appropriate graph and organize & display data Interpret data from a simple graph Extend a pattern 	<ul style="list-style-type: none"> Compare information within or across data sets or texts Analyze and draw conclusions from data, citing evidence Generalize a pattern Interpret data from complex graph 	<ul style="list-style-type: none"> Analyze multiple sources of evidence or data sets
Evaluate			<ul style="list-style-type: none"> Cite evidence and develop a logical argument Compare/contrast solution methods Verify reasonableness 	<ul style="list-style-type: none"> Apply understanding in a novel way, provide argument or justification for the new application

Create	<ul style="list-style-type: none">Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	<ul style="list-style-type: none">Generate conjectures or hypotheses based on observations or prior knowledge and experience	<ul style="list-style-type: none">Develop an alternative solutionSynthesize information within one data set	<ul style="list-style-type: none">Synthesize information across multiple sources or data setsDesign a model to inform and solve a practical or abstract situation
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References, Resources, and Links

1. Achieve the Core. <http://www.achievethecore.org/>.
2. Bolam, R., McMahon, A., Stoll, L., Thomas, S., & Wallace, M. (2005). Creating and sustaining professional learning communities. *Research Report Number 637*. General Teaching Council for England. London, England: Department for Education and Skills.
3. Croft, A., Coggshall, J. G., Dolan, M., Powers, E., with Killion, J. (2010). *Job-embedded professional development: What it is, who is responsible, and how to get it done well*. National Comprehensive Center for Teacher Quality. Retrieved March 11, 2013, from <http://www.tgsource.org/>
4. Darling-Hammond, L., Wei, R.C., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: a status report on teacher development in the United States and abroad*. Oxford, OH: National Staff Development Council and the School Redesign Network at Stanford University.
5. Garet, M. S., Porter, A. C., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? *American Educational Research Journal*, 38(4), 915–945. Retrieved March 11, 2013, from www.aztla.asu.edu/ProfDev1.pdf
6. Guskey, T. (2000). *Evaluating professional development*. Thousand Oaks, CA: Corwin Press.
7. Illustrative Mathematics. <http://www.illustrativemathematics.org/>.
8. Inside Mathematics. <http://www.insidemathematics.org/>.
9. Kanold, T. & Larson, M. (2012). *Common Core Mathematics in a PLC at Work, Leaders Guide*.
10. Kansas Mathematics Standards (CCSS). (2012). <http://www.ksde.org/Default.aspx?tabid=4754>
11. Killion, J. (2002). *Assessing impact: Evaluating staff development*. Oxford, OH: National Staff Development Council.
12. Learning Forward. (2011). *Standards for Professional Learning*.
13. Learn NC. <http://www.learnnc.org/lp/editions/ccss2010-mathematics>.
14. Learn Zillion. <http://learnzillion.com/>.
15. Mathematics Assessment Project. <http://map.mathshell.org/materials/index.php>.
16. McCallum, W., Daro, P., & Zimba, J. *Progressions Documents for the Common Core Math Standards*. Retrieved March 11, 2013, from <http://ime.math.arizona.edu/progressions/#about> .
17. Mid-continent Research for Education and Learning. (2000). *Principles in action: Stories of award-winning professional development* [video]. Aurora, CO: Author.
18. National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics*.
19. National Council of Teachers of Mathematics. (2000). *Principles & Standards for School Mathematics*.
20. National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence*.
21. National Council of Teachers of Mathematics. (2009) *Focus in High School Mathematics: Reasoning and Sense Making*.
22. National Council of Teachers of Mathematics. *Core Math Tools*. <http://www.nctm.org/resources/content.aspx?id=32702>.
23. National Council of Teachers of Mathematics. *Illuminations*. <http://illuminations.nctm.org/>.

24. National Mathematics Advisory Panel. (2008). *The Final Report of the National Mathematics Advisory Panel*. U.S. Department of Education.
25. National Staff Development Council. (2001). *Standards for staff development*. Oxford, OH: Author. Retrieved March 11, 2013, from <http://www.nsd.org/standards/index.cfm>
26. Porter, A., Garet, M., Desimone, L., Yoon, K., & Birman, B. (2000). *Does professional development change teaching practice? Results from a three-year study*. Washington, DC: U.S. Department of Education. Retrieved March 11, 2013, from <http://www.ed.gov/rschstat/eval/teaching/epdp/report.pdf>
27. South Dakota Department of Education. *Curation Review Rubric for Mathematics Learning Resources*. Retrieved March 11, 2013, from <http://www.livebinders.com/play/play?id=367846>
28. Steiner, L. (2004). *Designing effective professional development experiences: What do we know?* Naperville, IL: Learning Point Associates.
29. Tools for the Common Core Standards. <http://commoncoretools.me/>.
30. Wisconsin Department of Public Instruction. (2010) *Characteristics of High-Quality and Evidence-Based Professional Learning*. Retrieved March 11, 2013 from, http://www.betterhighschools.org/MidwestSIG/documents/Rasmussen_Handout1.pdf
31. Yoon, K. S., Duncan T., Lee, S. W.-Y., Scarloss, B., & Shapley, K. L. (2007). *Reviewing the evidence on how teacher professional development affects student achievement*. (Issues & Answers Report, REL 2007–No. 033). Washington, DC: National Center for Education Evaluation and Regional Assistance, Regional Education Laboratory Southwest. Retrieved March 11, 2013, from <http://ies.ed.gov/>
32. Publishers Criteria: www.corestandards.org
33. Focus by Grade Level, Content Emphases by Jason Zimba: <http://achievethecore.org/page/774/focus-by-grade-level>
34. Georgie Frameworks: <https://www.georgiastandards.org/Standards/Pages/BrowseStandards/MathStandards9-12.aspx>
35. engageNY Modules: <http://www.engageny.org/mathematics>