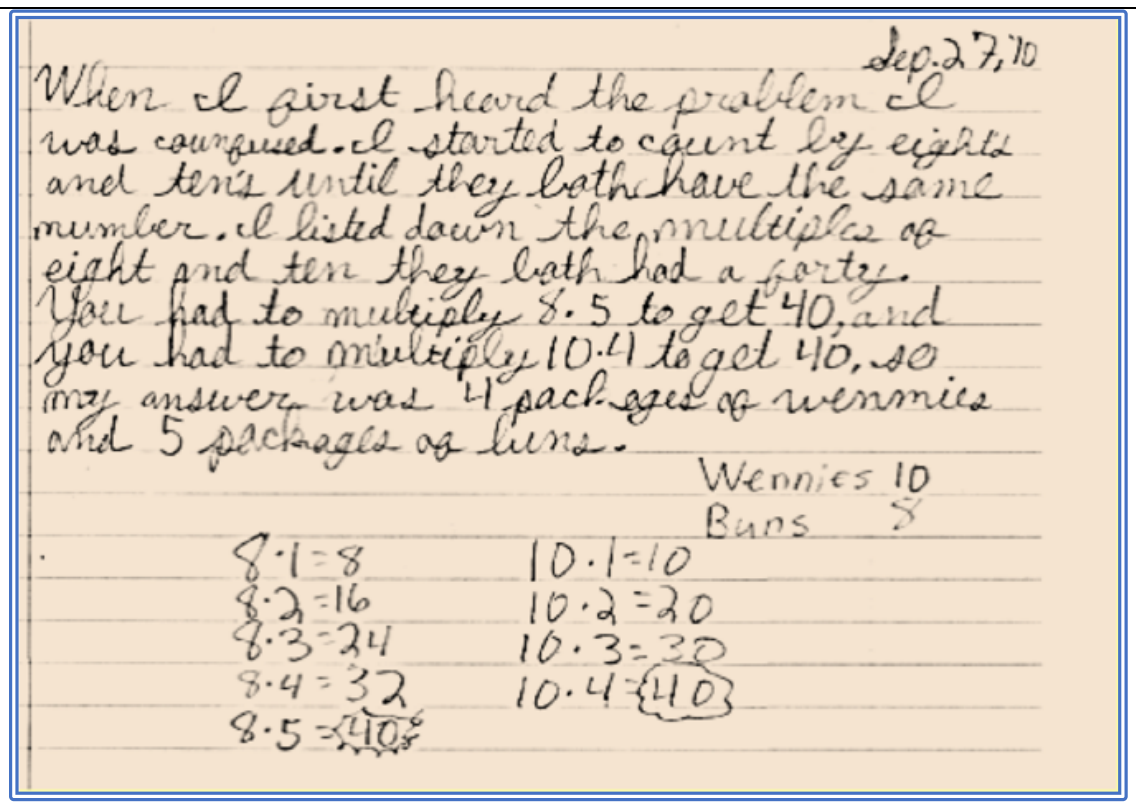
What to Look for in Students' Writing in Mathematics

What cognitive tools do we use to solve problems and communicate one's mathematical reasoning?	Writing in the mathematics is no different than any other objective writing. "Writing is thinking on paper, or talking to someone on paper" (William, 1976, p. vii). Writing in mathematics allows the teacher to peek into the writer's mind as the writer attempts to communicates her mathematical reasoning. In fact, the perceptive teacher can witness all Common Core Standards for Mathematics, Math Standards for Practices (MSP), one through eight, in the students' writing. In addition to seeking evidence of the MSP, the teacher is looking for evidence of certain thinking tools: • Images (e.g., models, drawings, pictures, schematics, etc.) • Concepts (i.e., mathematical ideas that govern the math problem) • Facts (i.e., declarative knowledge) • Language (i.e., formal mathematical vocabulary and text in context), and • Procedures (i.e., strategies, algorithms, heuristics, etc.)
Problem: Hotdogs vs. Buns	Hotdogs come in packages of 10. Hot dog buns come in packages of 8. What is the least number of packages of hot dogs and packages of buns should I buy so that I have the same number of servings of each?
5th grader's work	 Sep. 27, 10 When I first heard the problem I was confused. I started to count by eights and ten's until they both have the same number. I listed down the multiples of eight and ten they both had a forty. You had to multiply 8·5 to get 40, and you had to multiply 10·4 to get 40, so my answer was 4 packages of wennies and 5 packages of buns. Wennies 10 Buns 8 8·1=8 10·1=10 8·2=16 10·2=20 8·3=24 10·3=30 8·4=32 10·4=40 8·5=40
Student's writing transcribed	When I first heard the problem I was confused. I started to count by eights and ten's until they both have the same number. I listed down the multiples of eight and ten they both had a forty. You had to multiply 8*5 to get 40, and you had to multiply 10*4 to get 40, so my answer was 4 packages of wennies and 5 packages of buns. Wennies 10 Buns 8 8*1=8 10*1=10 8*2=16 10*2=20 8*3=24 10*3=30 8*4=32 10*4=40 8*5=40
Finding Thinking Tools: ICFLP	**Image:** Student provides a quasi-T-chart she used to determine Least Common Multiple problem (LCM). **Concept:** Student determines mathematical concept that governs problem; although does not identify as LCM. **Facts:** Student declares facts of problem: 10 "wennies" and 8 buns as well as gives the facts that 8*5=40 and 10*4=40 as solution to LCM problem. **Language:** Student uses mathematical language with phrases like: Started to count by eights and ten; multiples of eight and ten; multiply 8*5. **Procedure:** Student communicates procedure (i.e., heuristic, method, strategy) of determining solution.
Similarities of thinking tools (ICFLP) used by teacher to communicate mathematical reasoning.	"This is a LCM problem. I have to find the LCM of both 10 wieners and 8 buns. Multiples (10): 10, 20, 30 40 Multiples (8): 8, 16, 24, 32, 40 So, 4 packages of 10 wieners equal 40, and 5 packages of 8 buns equals 40."
Self-awareness & social-emotional implications	The first sentence is not a throwaway line. "When I first heard the problem I was confused," is a statement of the student's state of mind and hint of the student's underlying emotions, both of which can be intrinsically motivating to the student and a jump-off point for the teacher's inquiries (i.e., "What do you want to find?" "How is this procedure connected to your mathematical model?" "Can you think of a similar problem?") to guide and provoke the student's mental operations. "Emotion forms the rudder that steers learners' thinking, in effect helping them to call up information and memories that are relevant to topic or problem at hand" (Immordino-Yang, 2016, p.86).

Immordino-Yang, M.H. (2016). Emotions, learning, and the brain: Exploring the educational implications of affective neuroscience. New York, N.Y. W.W. Norton & Co.