What to Look for in Students' Writing in Mathematics

(6th version of 5th grade problem)

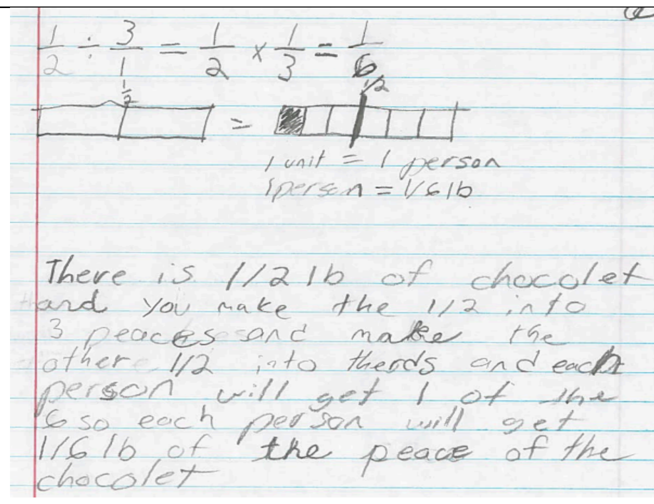
What cognitive tools do we use to solve problems and communicate one's mathematical reasoning?

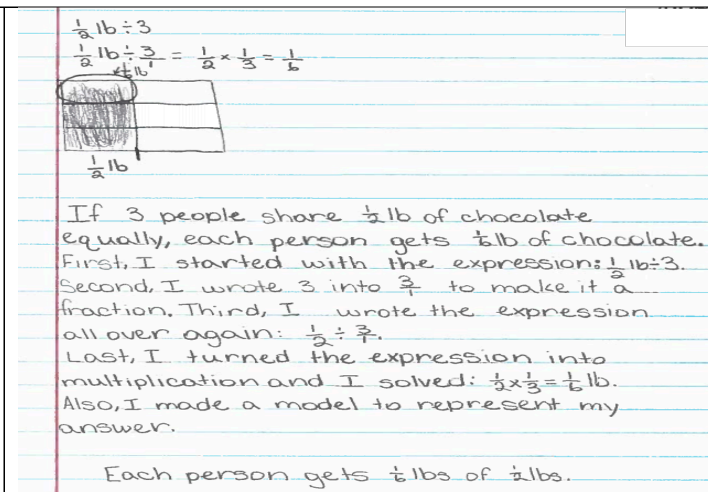
Writing in the mathematics is no different than any other objective writing. "Writing is thinking on paper, or talking to someone on paper" (William, 1976, p. vii). Writing in mathematics allows the teacher to peek into the writer's mind as the writer attempts to communicate her mathematical reasoning. In fact, the perceptive teacher can witness all Common Core Standards for Mathematics, Math Standards for Practices (MSP), one through eight, in the students' writing.
In addition to seeking evidence of the MSP, the teacher is looking for evidence of certain thinking tools:

- Images (e.g., models, drawings, pictures, schematics, etc.)
- Concepts (i.e., mathematical ideas that govern the math problem)
- Facts (i.e., declarative knowledge within the context of the specific problem)
- Language (i.e., formal mathematical vocabulary and text in context), and
- Procedures (i.e., strategies, algorithms, heuristics, etc.)

Problem: How Much Chocolate?

How much chocolate will each person get if 3 people share ½ lb. of chocolate equally? Use numbers, words, and images (e.g., models, tables, schematics, etc.) to communicate your mathematical thinking.





Student's writing verbatim

Student A	Student B
"There is ½ lb of chocolet and you make the ½ into 3 peaces and make the other ½ into therds and each person will get 1/6 lb of the peace of the chocolet"	"If 3 people share ½ lb of chocolate equally, each person gets $\frac{1}{6}$ lb of chocolate. First, I started with the expression: ½ lb÷3. Second, I wrote 3 into $\frac{3}{1}$ to make it a fraction. Third, I wrote the expression all over again: $\frac{1}{2} \div \frac{3}{1} = \frac{1}{6}$. Last, I turned he expression into multiplication and I solved: $\frac{1}{2}$ x $\frac{1}{3} = \frac{1}{6}$ lb. Also, I made a model to represent my answer. Each person gets $\frac{1}{6}$ lbs of $\frac{1}{2}$ lbs."

Students' Thinking Tools: ICFLP

Image: Student uses two bar models to show ½ lb. of chocolate partitioned into thirds.
Concept: Student illustrates partitioning of ½ lb. into 3 pieces, which equates to partitioning the whole into sixths, so one unit equals 1/6 and each person gets one of the 6 units.
Facts: Student correctly declares one-half pound of chocolate is partitioned into 3 pieces and each person gets 1/6 pound
Language: Student exhibits cursory use of mathematics language with phrases like: "½ lb of chocolet'"; "…make the ½ into 3 peaces"; and make the other ½ into therds and each person will get 1/6 lb…
Procedure: Student understands procedure used when he writes: "There is ½ lb of chocolet and you make the ½ into 3 peaces and make the other ½ into therds…"

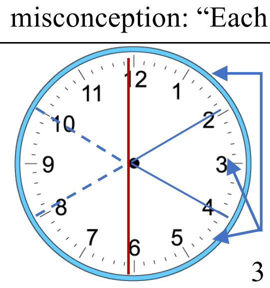
Comparing Students' mathematical writings. Student B's use of syntax, grammar, punctuation, and spelling are certainly better applied. Student B, however, is recounting the application of the "invert and multiply algorithm," and her making a model to represent her answer and not her mathematical reasoning *per se*. While Student A struggles with syntax, grammar, punctuation, and spelling, her conceptual and procedural understanding of the problem are clearly stated, "There is ½ lb of chocolet and you make the ½ into 3 peaces and make the other ½ into therds…" Although Student B clearly states the solution in the first sentence "If 3 people share ½ lb of chocolate equally, each person gets $\frac{1}{6}$ lb of chocolate." The last sentence illustrates his confusion of $\frac{1}{6}$ within context, "Each person gets $\frac{1}{6}$ lbs of 1/2 lbs."

Implications

The opportunities for advancing both student's mathematical thinking, reasoning, and discourse through writing are considerable. Although the students are at different stages in their abilities to communicate their mathematical thinking, both students demonstrate extensive prior knowledge, therefore, entry points for instruction. Student A, for example, has access to bar modeling and the concept of unitizing as well as the understanding that if a fraction of the whole is partitioned the entire whole is equally partitioned, i.e., "There is ½ lb of chocolet and you make the ½ into 3 peaces and make the other ½ into therds…" Here is where the teacher's intervention is most effective: mathematical vocabulary development, which will provide the student with diversity in word choice.

Student B, on the other hand, has a clear understanding of procedural knowledge and the ability to communicate it through enumeration, a sentence structure device, that makes her thinking and reasoning easier to follow (i.e., first, second, last). This, then, provides a window into Student A's mathematical focus, in this case, procedure. Here is the teacher's entry point into advancing the student's mathematical thinking and discourse. The Student's clear communication and use of the procedural strategy for solving the problem, and communicating his mathematical thinking and reasoning makes the concepts governing the problem more accessible to the student, and easier for the teacher to assist the student is see his misconception: "Each person gets $\frac{1}{6}$ lbs of $\frac{1}{2}$ lbs."

Similarities of thinking tools (ICFLP) used by teacher to communicate mathematical reasoning.



3 of $\frac{1}{6}$

One-half pound divided into three equal portions is equals to three 1/3 units of ½ lb., which is equivalent to three one-sixth units of ½ lb., so each person gets $\frac{1}{6}$ lb. of chocolate.

Solution A: $\frac{1}{2}\ lb \div 3 = \frac{1}{3} x \frac{1}{2} lb. = \frac{1}{6} lb.$

Solution B: $\frac{\frac{1}{2}lb.}{3} = \frac{.5lb.}{3}\ x\ \frac{2}{2} = \frac{1}{6} lb.$