Common Core Standards for Mathematical Practices (SMP) 1-8

Standards Mathematical Practices (SMP)	Making Sense of SMPs	Exemplification of SMP

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

MSA Support Resources:
6.NS.A.1a. Fraction Divided by Fraction
https://www.youtube.com/watch?v=jNWU0B6cALs

Mathematically proficient students (MPS) draw upon their previous knowledge, skills, problem-solving strategies, and heuristics to persevere in solving mathematical problems (real world or invented).

Making Sense of Mathematical Practices
MPS have the tenacity to persevere evolves as the students' ability to communicate their mathematical thinking and reasoning through multiple forms of representation (i.e., using equations, models, graphs, tables, verbal descriptions, manipulatives, and through writing) also improves and evolves.

How many ¼ teaspoon doses are in $\frac{7}{8}$ teaspoons of medicine?

1/8							
¼=2/8		1/4		1/4			

½ of ¼

$$\frac{\frac{7}{8}\times\frac{8}{2}}{\frac{2}{8}\times\frac{8}{2}}=\frac{7}{2}=3\frac{1}{2}$$

One-fourth is equivalent to 2/8, so 3(1/4) plus ½ of ¼ doses can be measured out of 7/8 medicine.

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

MSA Support Resources:
6.NS.A.1a. Fraction Divided by Fraction
https://www.youtube.com/watch?v=jNWU0B6cALs

6.NS.A.1.b
https://www.youtube.com/watch?v=zdRi1MfgWd8

MPS have the ability to make sense of and translate mathematical situations into symbols.
i.e., Concrete →Representational→Abstract.

Making Sense of Mathematical Practices

Question is presented as text.	MPS translate text into concepts, i.e., ideas and models.
MPS translate text into mathematical statement, i.e., symbols and numbers	MPS communicates mathematical thinking and reasoning via dialogue and writing.

How much chocolate will each person get if 3 people share 2 lbs. of chocolate equally?	2 lbs. 2/3 lb. 2/3 lb. 2/3 lb. $3\left(\frac{2}{3}\right) lbs. = \frac{6}{3} = 2$ lbs.
$2\text{lbs} \div 3\text{ple} = \frac{2lbs}{3ple}$	By imagining 2 lbs. as a unit and partitioning it into 3 equal parts, each person gets $\frac{2}{3}$ lb. of chocolate.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

MSA Support Resources:
7.RP.A.2.B
https://www.youtube.com/watch?v=6IGI0SHiufQ

MPS have the ability to understand the mathematical reasoning behind various problem-solving strategies and effectively connecting and communicating plausible arguments.

Making Sense of Mathematical Practices
The tub-filling problem (Right, Tables 1-3) illustrate three problem-solving strategies, all connecting and concluding that the filling rate of both cold and hot-water faucets is 1hr./3 tubs or 3 tubs/1 hr., Table 2 also includes finding the slope of a line from two points (1,3) and (2,6), and Table 3 even a finer unit fill-rate as 1 tub/20 min. — All problem-solving strategies effectively communicating mathematical thinking and reasoning.

If the cold-water faucet can fill the tub in a half-hour, and the hot-water faucet can fill it in an hour, how long will it take to fill the tub when they're running together?

Table 1

	1 hr. (60 min.)	
C-W	1 tub filled	1 tub filled
H-W	1 tub filled	

Table 2

	x	y	
1<	1	3	>3
	2	6	
	3	9	
	4	12	

$$y = \frac{3x}{1} + 0$$

In one hour, two cold water tubs and two half hot-water tubs can be filled for a ratio of 1:3 or 60 min./3 tubs. The constant of proportionality (unit rate) is 1:3.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

MSA Support Resources:
4.NF.B.4.B Fraction Model
https://www.youtube.com/watch?v=2HgONJ2FN3M

MPS have the ability to convert text and mathematical concepts into multiple forms of representations with the use of concrete and representational models, symbols, and technology tools to demonstrate and communicate their mathematical thinking and reasoning.

Making Sense of Mathematical Practices
MPS can translate and connect mathematical ideas, equations, and text into appropriate representations (i.e., models, diagrams, tables, graphs) to solve problems, analyze situations and draw conclusions assists and deepens mathematical thinking and reasoning.
(See SMP 3, above and problem and problem solutions in *Tables 1-3* to the right.)

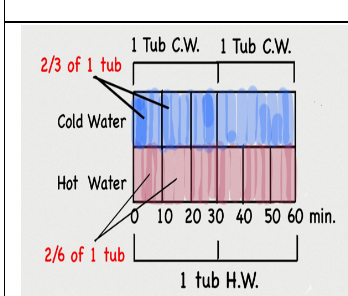


Figure 1. Illustrates filling of tub simultaneously using both cold and hot water faucets per minute.

Both cold and hot water faucets together fill three tubs in one hour, a 3:1 ratio – 1 tub/20 min. unit rate – because in 20 min., 2/3 of a cold-water tub is filled and 2/6 of a hot-water tub is filled, and $\frac{2}{3}$ tub + $\frac{1}{3}$ tub =1 tub.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

MSA Support Resources:
8.EE.B.6. Slope and Similar Triangle
https://www.youtube.com/watch?v=qorPoQQzEiY

Making Sense of Mathematical Practices
MPS are able to envision previous knowledge in the form of diagrams, numbers, words, formulas, and symbols, blend these, and map their relationships to incoming information and create new knowledge and draw conclusions.

Find two fractions between ¼ and ½.

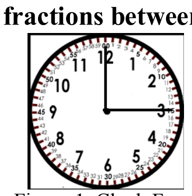


Figure 1. Clock Face

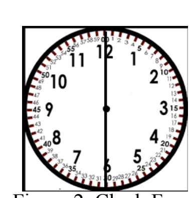


Figure 2. Clock Face

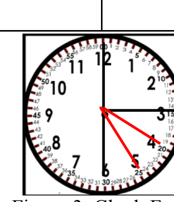


Figure 3. Clock Face

I can imagine two clock faces, one partitioned into one-fourth and a second partitioned into one-halves. Then, I imagine blending these two clock faces and can readily see 4/12 and 5/12 or their equivalents, 1/3 and 25/60.

Making Sense of Mathematical Practices
MPS have the ability to identify, find, and use appropriate concrete, representational, abstract, and technological tools to visualize mathematics problems, find problem solutions, and effectively represent and communicate their mathematical thinking and reasoning.

A person walks ½ miles in each ¼ hour, how many miles does he walk in 1 hour, in 1 and ½ hour, and in 2 hours?

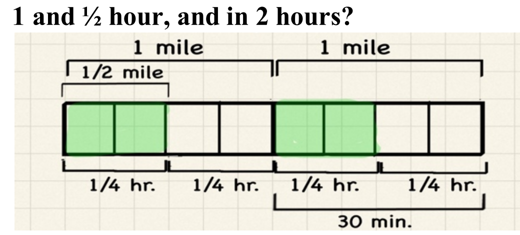


Figure 4, 4a. All images on this table were created on *NoteShelf*, an iPad App; the *Value Tables* and *Graphs* were created with the iPad App *Quick Graph*; all images are iPad screenshots and edited with iPad photo editing tools, then emailed and pasted on this page as an example of students' use of technological tools to produce and communicate mathematical ideas. (Also see all MPS 1-8)

Figure 4. The bar model shows that in ¼ hr. (15 min.) a person walks ½, and there are 4(1/4) hrs. in an hour, so 4 quarter hrs. x ½ mile = 4/2 hr. miles, which is equivalent to 2 hr. miles.

Value Table Done

X	$y=\frac{.5\cdot x}{.25}+0$	$y=\frac{2\cdot x}{1}+0$
1	2	2
1.25	2.5	2.5
1.5	3	3
1.75	3.5	3.5
2	4	4
2.25	4.5	4.5
2.5	5	5
2.75	5.5	5.5
3	6	6

Figure 4a. Value table illustrating y=mx+b for walking ½ mi. per ¼ hr.

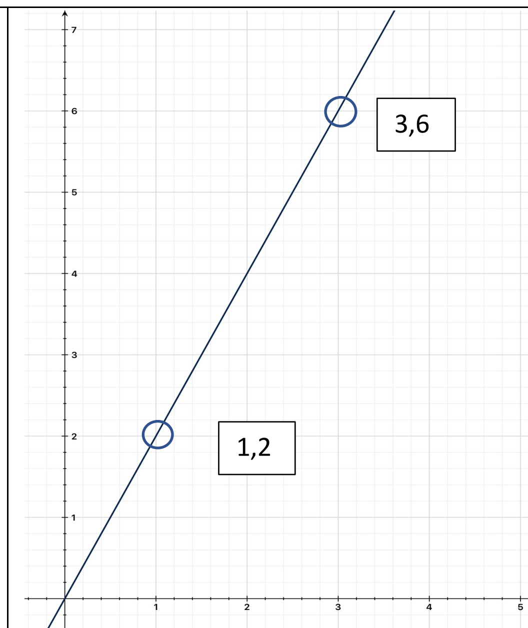


Figure 4b. Slope of graph showing points (1,2) and (3,6), and illustrating slope $y_2 - y_1/x_2 - x_1 = y$.

CCSS.MATH.PRACTICE.MP6
Attend to precision.

MSA Support Resources:
6.NS.A.1.b
https://www.youtube.com/watch?v=zdRi1MfgWd8

MPS are able to compute with precision as well as communicate their mathematical thinking and reasoning precisely to others using various mathematical tools concrete models, drawing, and diagrams, mathematical concepts, facts, mathematical language, and procedures.

Making Sense of Mathematical Practices
At the appropriate grade level, the MPS is able to identify, use, connect, and communicate the relationships among the five representations of a problem, i.e., algebraic equations, arithmetic expressions, tables and graphs, concrete or pictorial representations, and verbal description and written communication. (See SMP.4, 5, 7)

One-half of a group are children. One-third of the adults are men. There are 36 women. How many people are in the group?

3/6+1/6+2/6=6/6			Adults		
——— ½ Children ———			Men	—— 36 Women ——	
18	18	18	18	18	18

By converting $\frac{1}{2}$ and $\frac{1}{3}$ to its equivalents, sixths, I find that two-sixths is equal to 36 women, so $\frac{1}{6}$ is equal to 18 men, which is the unit of measure; therefore, 6 groups of 18 is equal to 108 people.

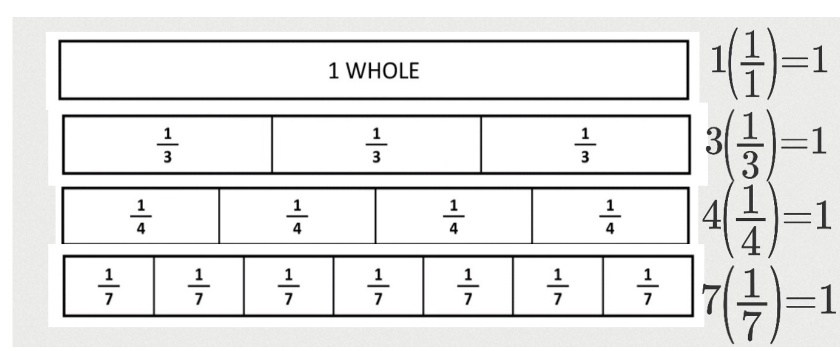
CCSS.MATH.PRACTICE.MP7
Look for and make use of structure.

MSA Support Resources:
5.NF.B.7.C
https://www.youtube.com/watch?v=vksnUoqCi38

MPS look for and use mathematical structures and patterns to solve problems and show relationships between and among mathematical concepts, operations, and procedures. MPS can see algebraic expressions as concepts, processes, and objects simultaneously.

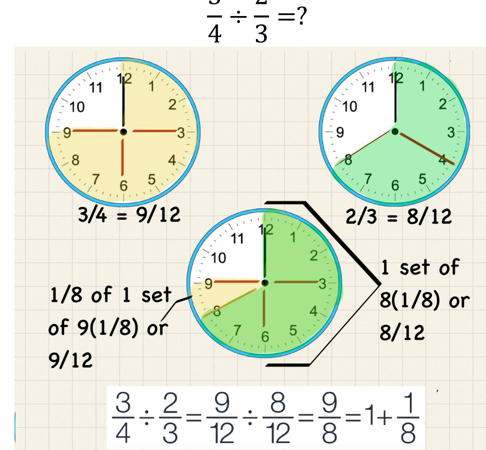
Making Sense of Mathematical Practices
The MPS are able to flexibly discern subtle mathematical structures and patterns in a math problem as processes, concepts, or objects. For example, $\frac{4}{4} = 1$ can be thought of as a product (quotient) of a process; it can be conceived as a concept that can be mentally manipulated, 4 parts of ¼ = 1 whole; or as an object, 3 pieces out of a whole pie cut into 4 equal pieces is ¾ of the whole pie. It is an explanation, a mental model that is carried around in the MPS' mind.
In addition, the MPS can flexibly conceive the underlying structure of a proportion as a procedural equation solving for x (i.e., $\frac{4}{4} = \frac{x}{7} = \frac{7\cdot 4}{4} = \frac{x\cdot 7}{7} = 7 = x$) as well as see relationally across the equal sign like so, $\frac{4}{4} = \frac{x}{7}$, the denominator to numerator relationship of the first ratio is one-to-one, so the second ratio, on the right side of the equal sign, must also have a one-to-one relationship, if, indeed, the two ratios are proportional, i.e., $\frac{4}{4} = \frac{7}{7}$. Also, looking across the equal sign, the right side denominator, 7, is $1\frac{3}{4}$ time greater than 4, the denominator of the left side ratio, so the numerator of the same side must also be $1\frac{3}{4}$ times greater than the numerator of the ratio on the left-hand side, that is, $4\ x\ 1\frac{3}{4} = 7$, so $\frac{4}{4} = 7/7$.
(See also MP.8)

Use numbers, words, and pictures to communicate your mathematical thinking, and explain why $\frac{4}{4}, \frac{7}{7}, and\ \frac{3}{3}$ are equivalent fractions?



My fraction model represents fractions of different sizes that makeup the same whole, that is, 3 parts of $\frac{1}{3}$ equal 1 whole, 4 parts of ¼ equal 1whole, and 7 parts of $\frac{1}{7}$ equal 1 whole — all wholes are of equal length and width.

$\frac{3}{4} \div \frac{2}{3} = ?$



I see the denominators of 3/4 and 2/3 as factors of 12; therefore, I blended the clock faces of 3/4 and 2/3 into one clock face that shows 1 set of 8(1/8) plus 1(1/8) remaining in 9(1/8) or 9/12÷8/12=1+1/8..

CCSS.MATH.PRACTICE.MP8
Look for and express regularity in repeated reasoning.

MSA Support Resources:
5.0A.B.3 Generate Patterns
https://www.youtube.com/watch?v=dfBuO2yox7w

3.0A.D.9 Identifying Patterns
https://www.youtube.com/watch?v=YaPDWQcoXwI

MPS notice repeated calculation and look for patterns, general methods, and shortcuts. They are able to discover and explain underlying relations while uncovering a model and/or unifying equations or function.

Making Sense of Mathematical Practices
MPS in middle school are able to identify a growing pattern, determine the constants, construct a value table to find constant rate of change and create a graph using order pairs to plot a line on a coordinate plane. In addition, the MPS is able to communicate her mathematical thinking and reasoning using

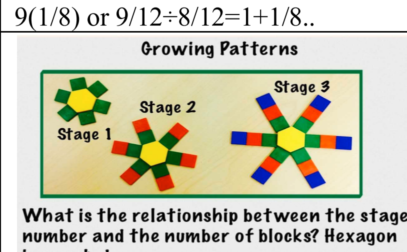


Figure 1. Growing pattern, 3 stages

<table>
<tr><th colspan="4">Common Core Standards for Mathematical Practices (SMP) 1-8</th></tr>
<tr>
<td rowspan="2">4.0A.C.5 Analyze Patterns
https://www.youtube.com/watch?v=YC3pmL_nTps</td>
<td rowspan="2">mathematical structure, mathematical tools, i.e., models, graphs, diagrams, mathematical concepts, facts, mathematical language, and procedures. (SMP.3,4,5,6,7)

Making Sense of Mathematical Practices
MPS are able to mentally compress (short-cut) mathematical processes and concepts, in effect, the MPS is able to visualize processes and concepts and their products as objects that can be mentally manipulated. (See Figures 1. and 2. to the right as compared to description of mental compression outline below.)

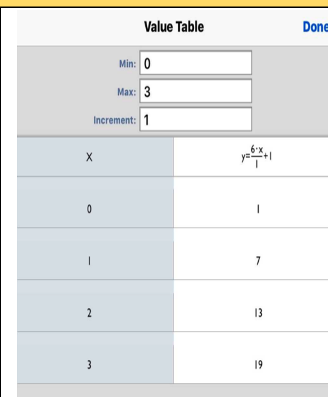
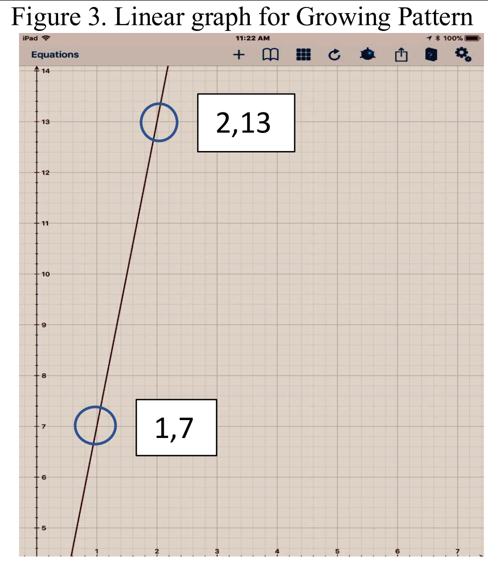
The MPS:
(1st Compression) sees 90 miles divided by 70 mi./hr. in fraction notation rather than long division.
(2nd Compression) decomposes 90/70 into 70/70 plus 20/70, and
(3rd Compressions) removes zeros from both numerators and denominators (i.e., mentally dividing each by 10), leaving, in effect, 7/7hr. (1 hour) plus 2/7 hr. as the solution.
(4th Compression) visualizes 2/7 as part of an hour and as a fraction operating on 60 minutes to determine 2/7 of 60 minutes, that is, first shrinking 60 minutes to ≅ 8.5 by dividing by 7, then expanding 8.5 minutes to 17 minutes by multiplying by 2. (SMP.4, 5, 6, 7)</td>
<td>

Figure 2. Value Table for Growing Pattern</td>
<td>Figure 3. Linear graph for Growing Pattern


$y = 6x + 1$
As the pattern grows from one stage to the next, six squares and the original hexagon remain constant, so stage 15 would be 6 x 15 + 1 = 91.</td>
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<td colspan="2">I am driving to Bloomfield, NM from Cuba, NM – a distance of 90 miles. If I set my cruise control at 70 miles per hour, how long before I arrive at Bloomfield?

$$\frac{90mi.}{\frac{70mi.}{hr.}} = \frac{70mi.}{\frac{70mi.}{hr.}} + \frac{20mi.}{\frac{70mi.}{hr.}} = \frac{9mi.}{\frac{7mi.}{hr.}} = \frac{7mi.}{\frac{7mi.}{hr.}} + \frac{2mi.}{\frac{7mi.}{hr.}} = 1hr. + \frac{2}{7}hr.$$

$$\frac{2}{7}hr * \frac{60min.}{1hr} \cong 17min. \quad \underline{So, 1hr. + 17\ min.}$$

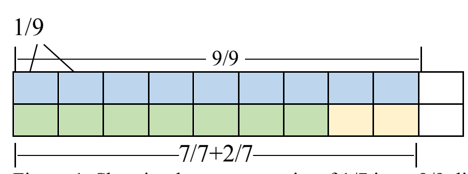

Figure 1. Showing how many units of 1/7 in or 9/9 divided by 7 = 7/7 +2/7 or one with remainder 2.

I see 90 miles as a container and 70 mi./hr. as an object, and I am determining how many measures of 70 miles per hour are in 90 miles. So, by dividing 90 miles by 70 miles per hour, I find that it will take about 7/7 (1hr) and 2/7 (17 min) to get to Bloomfield from Cuba.

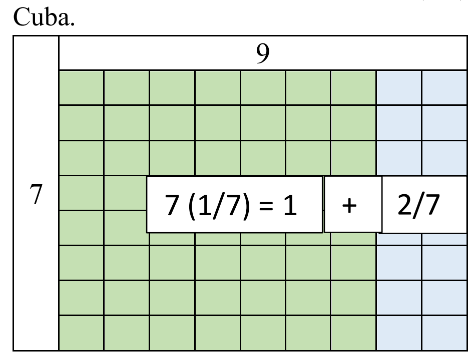

Figure 2. Area model illustrating how many 1/7 in 9/7.</td>
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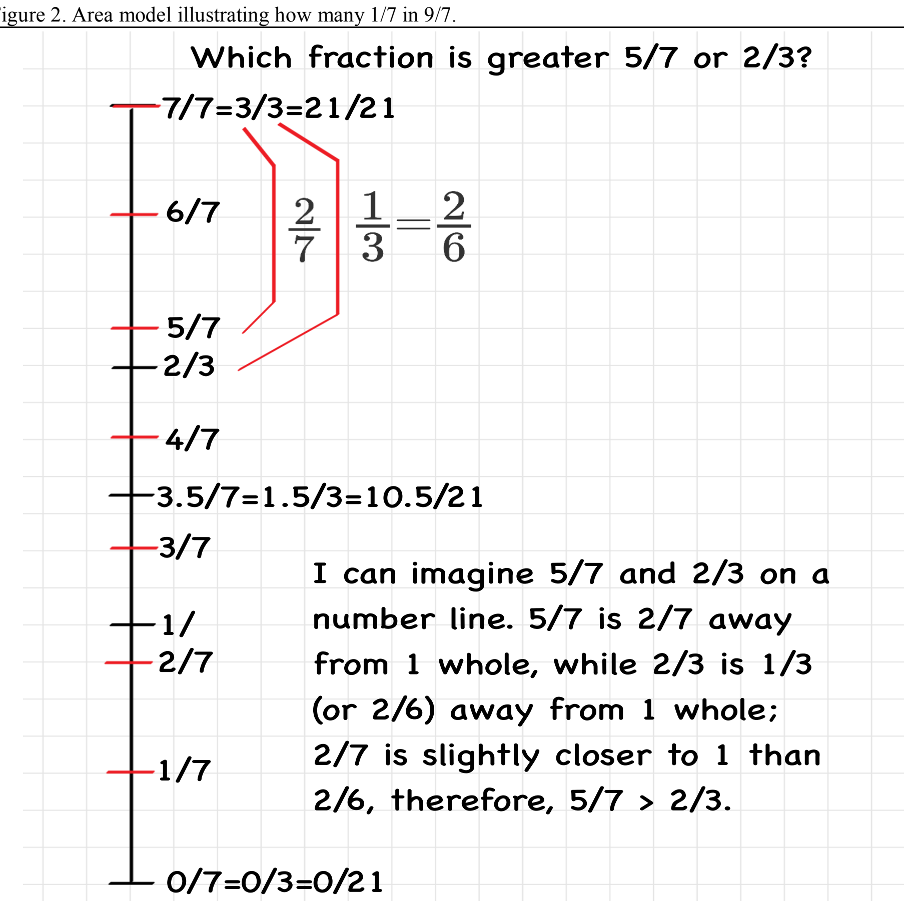
<td>Making Sense of Mathematical Practices
MPS have developed a fraction intuition that allows them to determine size, order, equivalence, and patterns that helps them make connections to various problem-solving strategies and overarching mathematical ideas. For example, the student (See Figure 1, right) immediately understands that the denominator of both 5/7 and 2/3 are factors of 21; therefore, 21 is a multiple of 7 and 3, so mentally constructs a number line between 0/21 and 21/21 to readily compare the closeness of 5/7 and 2/3 to 21/21 or 1 whole. In addition, the MPS sees that both 5/7 and 2/3 are greater than ½, because 3.5/7, 1.5/3, and 10.5/21 are all equal ½. This fraction fluency allows the MPS to quickly see that when 5/7 is closer to 1 then 1/3 or its equivalent, 2/6. That is, by comparing the numerators, 2 parts of each sevenths and sixths, respectively; the MPS sees sevenths as being smaller than sixths, which means sevenths is closer to 1, so 5/7 is greater than 2/3. (SMP.5,7)</td>
<td colspan="2">

Figure 1. Number line illustrates 2/3 is further from 1 whole than 5/7, therefore, 5/7>2/3.</td>
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