New Mexico Mathematics Instructional Scope for First Grade

June 2020

TABLE OF CONTENTS:

Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

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[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.
[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text Full text of the standard	**Standard for Mathematical Practices** The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	**Students who demonstrate understanding can:** The cognitive skills students perform to demonstrate to comprehension of a standard. **Depth Of Knowledge:** Correlation of standard to Webb's Depth of Knowledge **Bloom's Taxonomy:** Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction

Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach

Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive) : Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities: Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	**Cross Curricular Connections:** Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

TABLE OF CONTENTS:

1.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: A: Represent and solve problems involving addition and subtraction.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.OA.A.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	SMP 1: Students can make sense of problems and persevere in solving them by creating models and connecting those models to pictorial representations, number lines, and other representations. SMP 2: Students can reason abstractly and quantitatively by relating what is happening in a story problem to an add to, take from, put together, take apart, or comparison situation and then represent that situation using mathematical symbols. SMP 3: Students can construct viable arguments by explaining their solving strategy and critique the reasoning of others by comparing their solving strategy to the solving strategy of others.	• Represent word problems involving adding to, taking from, putting together, taking apart, or comparison situations using objects and drawings. • Write equations involving adding to, taking from, putting together, taking apart, or comparison situations with unknown numbers in different positions. • Explain how an equation represents an adding to, taking from, putting together, taking apart, or comparison situation. • Solve word problems representing adding to, taking from, putting together, taking apart, or comparison situations with unknown numbers in different positions. **Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
1.OA.A.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	SMP 1: Students can make sense of problems and persevere in solving them by creating models and connecting those models to pictorial representations, number lines, and other representations. SMP 2: Students can reason abstractly and quantitatively by relating what is happening in a story problem that calls for addition of three whole numbers	• Represent and solve word problems requiring the addition of three whole numbers using objects and drawings. • Write equations involving the addition of three whole numbers representing the unknown using a symbol. • Add three whole numbers whose sum is less than or equal to 20. • Reorder three addends to add efficiently (e.g. 3 + 8 + 7 can be thought of as 3 + 7 + 8 knowing that 3 + 7 = 10).

	and then represent that situation using mathematical symbols. SMP 3: Students can construct viable arguments by explaining their solving strategy and critique the reasoning of others by comparing their solving strategy to the solving strategy of others.	**Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze
Previous Learning Connections • Connect to learning the partners that make 10 for any number and knowing all decompositions for any number below 10. The idea of decomposing numbers (taking apart numbers) lays a foundation for developing strategies based on place value and properties of operations. **(K.OA.3-4)** • Connect to knowing all teen numbers as10 ones and some more. **(K.NBT.1)**	**Current Learning Connections** • Connect to using what students know about making 10 to work with larger numbers and problems with multiple addends. **(1OA.3,6)** • Connect to working to gain confidence and fluency with strategies when solving problems and using these skills to answer questions regarding data in a graph. **(1. MD.4)**	**Future Learning Connections** • Connect to working to become fluent within 100 and to extend their known strategies to larger numbers and two-step word problems. **(2.OA.1)** • Connect to applying this skill with problems in a variety of contexts involving length, picture graphs and bar graphs. **(2.NBT.5)**
Clarification Statement: • 1.OA.A.1: In a **Compare situation**, two **quantities** are compared to find "how many **more**" or "how many **less**." **Addition and Subtraction Situations** by Grade Level.		

sign (=, here with the meaning of "becomes," rather than the more general "equals").

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

- 1.OA.A.2: In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context.

Common Misconceptions

- Students may believe that certain words always indicate a particular operation.
- Students may believe it is not possible to add or subtract more than two numbers.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying how to represent and solve problems involving addition and subtraction because it is building on the foundational skills that students learn in kindergarten such as understanding addition as adding to and subtraction as being taking apart, or taking away from.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.OA.A.1 This standard provides a foundation for work with being able to represent addition and subtraction with objects, fingers, mental images, drawings*, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations because this standard is critical for students to understand what counting through addition and subtraction is. Also, it allows for students to fluently add or subtract within 10 using manipulatives or drawings if needed. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with being able to represent and solve problems involving addition and subtraction benefit when learning experiences include ways to recruit interest such as providing time for self-reflection about the content and activities relating to being able to add and subtract numbers correctly because students may recognize this new understanding and build on skills of decomposing and composing numbers, and comparing numbers while solving for unknown values in all positions.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with being able to represent and solve problems involving addition and subtraction benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors because being able to solve addition and subtraction problems within the number 20 will allow students to improve their fluency skills. Also, students will apply this knowledge as they engage in solving real world word problems when identifying and solving for an unknown.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with being able to represent and solve problems involving addition and subtraction benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students gain knowledge and familiarity with the vocabulary and function of words such as, "sum" and "difference." In addition, students utilize the use of what an "unknown" value is, especially representing a variable. Students can solve problems when the unknown value is in any part of the equation.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with being able to represent and solve problems involving addition and subtraction benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips because this supporting tool will allow all students to be able to communicate their understanding in both verbal and written communication methods. Also, students can demonstrate their understanding of this skill and not be hung up on the initial wording of how to explain their understanding.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with being able to represent and solve problems involving addition and subtraction benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as anchoring instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines), because students can make connections from previously taught skills in kindergarten of number combinations within 10. Students can continue to look for patterns across number combinations including the concept of doubling numbers.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on being able to represent and solve problems involving addition and subtraction by examining tasks from a different perspective through a short mini-lesson because students may benefit from seeing things in a new way and gaining a new perspective to make meaning. In addition, students can benefit from the use of manipulatives and/or drawings to solidify the understanding of this skill.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit in which students are able to represent and solve problems involving addition and subtraction by offering opportunities to understand and explore different strategies because students can better understand the skill if they have multiple strategies to approach and solve a problem. Students may make more connections and see things differently if they have knowledge of multiple strategies. Also, students may recognize patterns within numbers while adding and subtracting, such as looking for and identifying a combination of 10 and patterns with doubling of a number.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying the skill of being able to represent and solve problems involving addition and subtraction because students are now thinking about numbers in a more abstract way. This allows them to shift their thinking from concrete and pictorial to abstract which allows for more depth of

knowledge to take place. Students can also apply this knowledge to multiple problems without having the items in front of them.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying how to represent and solve problems involving addition and subtraction with the use of up to 3 numbers within 20 goal setting is critical because it allows students to take ownership of their own learning related to addition and subtraction. Students can self-reflect on their level of comfort related to this skill and reflect on areas in which they need support. Students can also make goals of making real world connections relevant to their culture using addition and subtraction. Students may benefit from using objects, drawings or equations to solve for the unknown value. Another example is students may use objects or drawings that are significant to them culturally, and this may allow for more student engagement to take place.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://achievethecore.org/coherence-map/1/5/34/34

Daisies in vases

Jasmine has eight daisies and three vases - one large, one medium-sized and one small.

She puts 5 daisies in the large vase, 2 in the medium vase and 1 in the small vase.

Can you find another way to put daisies so that there are the most in the large vase and least in the small vase? Try to find as many ways as you can put the daisies in the vases with the most in the large vase and the least in the smallest vase. If you think you have found them all, explain how you know those are all the possibilities.

This type of assessment question requires students to This task helps illustrate Mathematical Practice Standard 2, Reason abstractly and quantitatively. Students make sense of quantities and how they are related in a problem situation. In the task at hand, students first create a meaningful representation of the problem by using objects, pictures, or equations. Then, they manipulate the objects, pictures, or equations by finding different 3-number combinations of daisies in the vases totaling eight. Lastly, students periodically contextualize the problem by connecting the mathematical objects or symbols back to the context. Thus, students build meaning for the mathematical symbols by reasoning about the problem rather than memorizing an abstract set of rules or procedures. Problems that begin with a context and are represented with mathematical objects or symbols can also be examples of modeling with mathematics (MP.4). The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics.

Relevance to families and communities: During a unit focused on how to represent and solve problems involving addition and subtraction with the use of up to 3 numbers within 20, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, since students are learning to solve addition and subtraction problems, they can relate this skill to real world examples relevant to their city and culture. Students can bring in examples that relate to them and even provide pictures of the items to support their knowledge of adding to and taking away.	**Cross-Curricular Connections:** Social Studies: In first grade the New Mexico Social Studies Standards state students should "understand the concept of goods and services". Consider providing a connection for students to "sell" goods to each other or the larger community (such as in a first-grade market) as a context for math story problems. Language Arts: Writer's Workshop in first grade includes a unit on Small Moments. Consider providing a connection between the beginning, middle and end of the stories students write and making sense of a math story problem using the structure of beginning, middle and end.

NEW MEXICO
Public Education Department

1.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: B: Understand and apply properties of operations and the relationship between addition and subtraction.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.OA.B.3: Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*	SMP 2: Students can reason abstractly and quantitatively by flexibly using properties of operations (such as commutative and associative) without having to name them. SMP 7: Students can look for and make use of structure by seeing and using patterns that help them add and subtract more efficiently.	• Use representations to solve addition and subtraction problems • Describe and make generations regarding properties (adding or subtracting 0 does not change the number) and strategies (making a ten when adding more than two numbers) • Show or explain their thinking **Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
1.OA.B.4: Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*	SMP 4: Students can model with mathematics by representing related addition and subtraction facts using objects, pictures, numbers and words. SMP 7: Students can look for and make use of structure by using the relationship between addition and subtraction to practice and become more automatic as basic facts.	• Explain the relationship between addition and subtraction using objects, pictures, numbers and words. • Represent the relationship between addition and subtraction using objects, pictures, numbers and words. • Rewrite a subtraction equation as an addition equation with a missing addend. **Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to developing the understanding that addition means putting together and subtraction means taking apart and representing and solve word problems within 10. **(K.OA.2)**	• Connect to using patterns that students notice such as place value understanding and properties of operations to add and subtract within100. **(1.NBT.4)** • Connect to becoming fluent adding and subtracting within10, using strategies that make sense to them and explaining the reasoning behind the strategies used. **(1.OA.6)**	• Connect to using place value understanding and properties of operations to add and subtract within100 using up to four 2-digit numbers and explaining why certain strategies work. Connect to using concrete models, drawings and place value strategies to explore addition and subtraction within 1000. **(2.NBT.5-9)**

Clarification Statement:

- 1.OA.B.3: Methods involve **decomposing** an **addend** and **composing** it with the other addend to form an **equivalent** but easier problem. This relies on **properties of operations**. Students do not necessarily have to **justify** their **representations** or **solution** using properties, but they can begin to learn to recognize these properties in action and discuss their use after **solving**.

- 1.OA.B.4: **Put Together/Take Apart problems** with **Addend Unknown** afford students the opportunity to see **subtraction** as the **opposite** of **addition** in a different way than as reversing the action, namely as finding an unknown addend. The meaning of subtraction as an unknown-addend addition problem is one of the essential understandings' students will need in middle school in order to extend arithmetic to **negative rational numbers**.

Common Misconceptions

● Students might believe that subtraction is commutative.

● Students may not realize they can count on to find the difference.

● Students confuse the parts of addition and subtraction equations.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying the skill of being able to understand and apply properties of operations and the relationship between addition and subtraction because students are learning about the commutative property along with the associative property to support the understanding of addition. Students are also looking for combinations to make ten and may benefit from having additional time with this new thinking.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.OA.A.2: This standard provides a foundation for work with having access to solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem because this skill allows students to fully understand that addition is adding parts together and subtraction is taking

apart from a total. Students can use manipulative and/or drawings to support their thinking and development of the skill. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engage with being able to understand and apply properties of operations and the relationship between addition and subtraction benefit when learning experiences include ways to recruit interest such as setting personal academic goals because students can be given time to self-reflect on their understanding of the content based on an individual goal that was set. Students would be given the opportunity to self-reflect, ask questions and learn from peers in order to better improve their understanding.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with being able to understand and apply properties of operations and the relationship between addition and subtraction benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because this can allow students to recognize and better understand common misconceptions related to the task and content being taught. Students can learn multiple strategies to explain their own thinking, but also be able to solve a problem if an error is present.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with being able to understand and apply properties of operations and the relationship between addition and subtraction benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students can learn new math vocabulary language such as "addend," to better explain and demonstrate their understanding. In addition, students can mathematically communicate with peers about the content being taught.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with being able to understand and apply properties of operations and the relationship between addition and subtraction benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models,

base-ten blocks), because the use of manipulatives can support all student learners to better understand the skill, verify their answer through the use of manipulatives, or even present a concrete working model for students to physically move as they add or subtract numbers.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with being able to understand and apply properties of operations and the relationship between addition and subtraction benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as posting goals, objectives, and schedules in an obvious place because students can benefit from visual objectives and goals to keep them on track and understand what they are learning and why. In addition, students can see how previous and upcoming skills being taught connect over time and relate to one another.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on being able to understand and apply properties of operations and the relationships between addition and subtraction by providing specific feedback to students on their work through a short mini-lesson because the students can then receive immediate feedback to support and correct any misconceptions the students may have. It is important to ensure that students are following the correct procedure when adding or subtracting, and immediate feedback and help support students to follow the correct order of steps.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on being able to understand and apply properties of operations and the relationships between addition and subtraction by offering opportunities to understand and explore different strategies because this allows students to rely on multiple strategies to approach and solve a problem rather than one that may be causing confusion. Also, students can benefit from knowing multiple strategies because it can deepen their understanding and allow them to make connections about addition and subtraction.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying how to understand and apply properties of operations and the relationships between addition and subtraction because students can move from concrete and pictorial representations of addition and subtraction to more abstract concepts. This allows students to build on their skills and understand mathematical relationships, specifically related to addition and subtraction.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying the skill of being able to understand and apply properties of operations and the relationship between addition and subtraction the types of mathematical tasks are critical because students are building the foundation and understanding of numbers and the relationship between them, while also solving for an unknown value. Also, students are developing multiple strategies associated with the commutative property and associative property. As students are developing skills related to addition and subtraction, they can relate to real world examples that can be important to their culture and language. Students can also incorporate their language and how to count in their language as they work to solve math problems.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://achievethecore.org/coherence-map/1/5/37/37

Domino Addition
Materials

- A large set of dominoes to affix to a whiteboard or place in a pocket chart, or a regular set to use on a document projector.
- One set of dominoes for each student or pair of students
- Domino addition worksheets

Actions
The teacher asks a child to choose a domino from a stack or bag. As the teacher holds up the domino, the students call out how many dots are on the domino altogether.

Next the class counts the number of dots on each end of the domino to check their responses. Then the class names an addition equation that represents the relation between total number of dots and the number of dots on each end. For example, if the domino has 4 dots on one side and 2 dots on the other, the teacher can show the domino with the 4 on the left and the 2 on the right and the class names the equation 4+2=6. The teacher then writes the equation.

Then the teacher rotates the domino so the 2 is on the left and the 4 is on the right, and the class can name the equation 2+4=6. The teacher then writes the equation. The teacher then draws the dots from the chosen domino on a blank domino.

Once the students understand the task, they can work on their own. Students should have a set of dominoes to explore individually or with a partner, along with the domino addition worksheet. There are two variants of this task.

Students can choose dominoes at random, draw the dot pattern, and write the two related equations.

Students can find all the dominoes that have a sum, and then draw all the related dot patterns and equations.

For example, they could look for all the dominoes that have 6 dots all together, then draw the dot patterns for those dominoes and write the corresponding equations.

This type of assessment question requires students to understand the commutative property of addition. Because the total number of dots is the same regardless of how a domino is oriented, the domino reinforces the idea that the addends can be written in any order. Using dominoes in this way can help bridge between using moveable manipulatives such as cubes and using only symbolic representations of numbers and equations. This task is especially appropriate once students have internalized the dot patterns shown on dominoes and dice. If the teacher does not already have dominoes, they are easily found online and can be printed onto colored construction paper and cut out.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on the skill of being able to understand and apply properties of operations and the relationship between addition and subtraction, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can share ways to count or refer to numbers in their language, or a language spoken at home, to support their learning and extension of peers' learning. Also, this can promote engagement will all students and provide a more robust understanding of numbers as students make connections to different structures of number-names in other languages.	Language Arts: Literature can offer connections to help students move from counting to addition such as: *Math Fables* by Greg Tang and *Math Fables Too* by Greg Tang. Art: Even though it is not explicit in the standard, the clarification statement makes it clear that it is important for students to share, discuss and compare their strategies as a class. Consider providing a connection where they can work together to make posters that illustrate each strategy.

NEW MEXICO
Public Education Department

1.OA: OPERATIONS & ALGEBRAIC THINKING		
Cluster Statement: C: Add and subtract within 20. **Major Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text **1.OA.C.5: Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).**	**Standard for Mathematical Practices** SMP 5: Students can use tools by using a variety of materials as they continue to work on counting strategies to find sums and differences. SMP 7: Students can look for and make use of structure by looking for patterns as they use counting strategies, including for which facts counting is an efficient strategy.	**Students who demonstrate understanding can:** • Represent addition and subtraction with base ten blocks, counters, ten frames, number lines, and drawings. • Add by counting all and counting on. • Explain that one can count on from either addend in an addition equation. • Recognize that +1 means the next number and that +2 means the next-next number in the counting sequence. • Subtract by counting back or counting on. • Explain that one can count back the total amount being subtracted (i.e. in 9-7, one can count back 7) or that one can count back to the number being subtracted (i.e. in 9-7, one can count back to 7). • Recognize that -1 means the number before and that -2 means the number that is two numbers before in the counting sequence. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand and Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.OA.C.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	SMP 6: Students can attend to precision by accurately, automatically and flexibly knowing their addition and subtraction facts within 10. SMP 7: Students can look for and make use of structure by extending their use of addition and subtraction strategies to sums within 20.	• Consistently add within 10 with accurate and efficient results • Consistently subtract within 10 with accurate and efficient results • Use strategies to find sums and differences when they can't be recalled quickly, including counting on, make a ten and doubles • Show or explain their thinking **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand and Apply
<u>Previous Learning Connections</u> • Connect to counting to 100 and count on from any given number within 100. **(K.CC.1)** • Connect to reading and writing to 20 and demonstrating one-to-one correspondence when counting objects. **(K.CC.3, K.CC.4a)**	**<u>Current Learning Connections</u>** • Connect to developing fluency (working flexibly, accurately, efficiently and appropriately) when adding and subtracting within 10 but continuing to use strategies to solve within 100 and explaining their reasoning. **(1.NBT.4)**	**<u>Future Learning Connections</u>** • Connect to fluently adding and subtracting within 20 using mental strategies and mentally adding two 1-digit numbers. **(2.OA.2)**

Clarification Statement:

- 1.OA.C.5: Unlike **counting down**, **counting on** reinforces that **subtraction** is an **unknown-addend problem**. Learning to think of and **solve** subtractions as unknown addend problems makes subtraction as easy as **addition** (or even easier), and it emphasizes the relationship between addition and subtraction.

- 1.OA.C.6: Students might use the **commutative property of addition** to change ? + 6 = 15 to 6 +? = 15, then count on or use methods to **compose** 4 (to **make ten**) plus 5 (**ones** in the 15) to find 9. Students might reverse the action in the **situation** represented by ? - 6 = 9 so that is becomes 9 + 6 = ?. Or they might use their knowledge that the **total** is the first **number** in a **subtraction equation** and the last number in an **addition equation** to rewrite the **situation equation** as a **solution equation**: ? - 6 = 9 becomes 9 + 6 = ? or 6 + 9 = ?. Appendix from K, Counting and Cardinality; K–5, Operations and Algebraic Thinking: Methods used for solving **single-digit** addition and subtraction problems

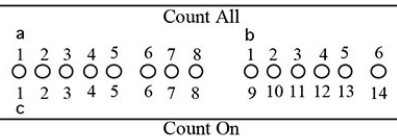
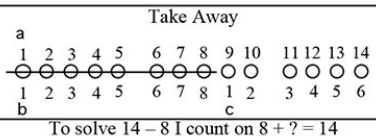
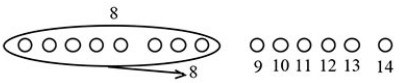
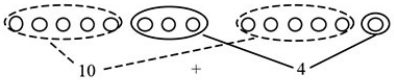
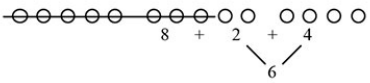
Appendix. Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away.

Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Adding ($8 + 6 = \square$): Represent each addend by a group of objects. Put the two groups together. Count the total. Use this strategy for Add To/Result Unknown and Put Together/Total Unknown.

Subtracting ($14 - 8 = \square$): Represent the total by a group of objects. Take the known addend number of objects away. Count the resulting group of objects to find the unknown added. Use this strategy for Take From/Result Unknown.

Levels	**8 + 6 = 14**	**14 – 8 = 6**
Level 1: Count all	Count All  	Take Away 
Level 2: Count on	Count On  	To solve 14 – 8 I count on 8 + ? = 14 I took away 8  8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten   	14 – 8: I make a ten for 8 + ? = 14  8 + 6 = 14
Doubles ± *n*	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Common Misconceptions

● Students might double count a number when adding or subtracting (For example, starting with the 6 when adding 4 to get 6, 7, 8, 9 rather than 10.)

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying add and subtract within 20 because the foundation of this cluster is to understand that counting leads to understanding addition and subtraction. Students can practice counting forward and/or backward from a given number. This can be a group activity.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.CC. A2: Counting to tell the number of objects, is the prior standard. This standard provides a foundation for work with adding and subtracting numbers within 20 because it provides the understanding that you can a given number of objects within 20. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with adding and subtracting within 20, benefit when learning experiences include ways to recruit interest such as providing choices in their learning (ex. which strategy to use, and what tools to use) because students will reflect on what strategies they understand and can utilize the best. This promotes autonomy of student learning, metacognition, while increasing fluency in the topic.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with adding and subtracting within 20 benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because the purpose of this standard it to increase fluency using strategies chosen by the student. It is important for students to receive feedback on the strategy they chose and whether it was effective and why or why not. Sample tasks for this standard can be found at: http://tasks.illustrativemathematics.org/1.OA.C-2

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with adding and subtracting within 20 benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where

appropriate (e.g., formulas, word problems, graphs) because students might need different modalities to demonstrate understanding such as use of counters, ten frames, number bonds, or mental math strategies.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with adding and subtracting within 20 benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) because the overall goal of this standard is to increase student fluency through understanding.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with adding and subtracting within 20 benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because this skill will be used in later advanced math skills. See Recognizing Repetition (link below). As students gain fluency, this opens paths for more critical thinking.
  http://www.fosteringmathpractices.com/routinesforreasoning/recognizing-repeptition/

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on add and subtract within 20 by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to know that counting objects leads to the total number of objects given. This is a foundational skill to adding numerical amounts.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on add and subtract within 20 by addressing conceptual understanding because they need to connect the idea of counting objects in groups and adding to what they have already counted without starting from 1.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying adding and subtracting within 20 because students may be ready for larger numbers by using units of 10 or previously learned strategies. This is also related to the next standard in the progression.

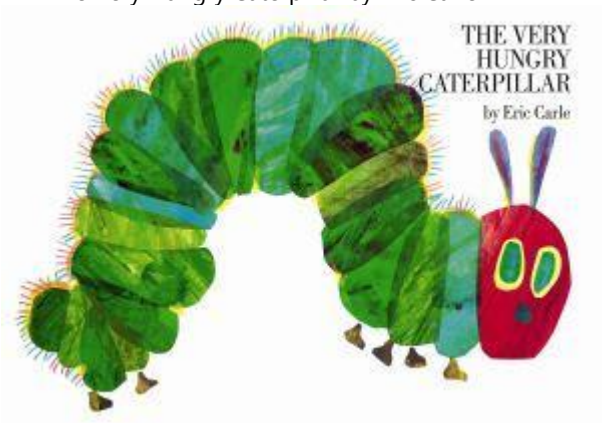
Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying adding and subtracting within 20 the types of mathematical tasks are critical because it allows students flexibility to work within components of their own culture and language. They can use their home language skills to help understand what the problem is asking. Students can also choose items such as counters lines or methods of counting that might be available in the home setting.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
The Very Hungry Caterpillar
Provided by Illustrative Mathematics
Materials

- The Very Hungry Caterpillar by Eric Carle



- The students work individually or in pairs. Each student or pair needs:
- Three ten-frames for each student or pair of students (download PDF for black line master)
- 30 counters or unifix cubes per pair of students
- One small dry-erase board and dry-erase maker per pair of students

Actions
The teacher reads the book to the class and asks, "How many things do you think the caterpillar ate in this story?" The students take a minute to share their estimate with a partner. Next, the teacher reads The Very Hungry Caterpillar again. After each page, the teacher pauses so that the students can add counters or unifix cubes to the ten-frame to represent the number of things the caterpillar ate, and then write an equation on the dry-erase board connecting addition to the number of counters used. After each ten-frame is filled in the students move to the next one. If the students are working in pairs, one student can add the counters/unifix cubes to the ten-frame while the other student writes the equation. By the end of the story, there should be a total of 25 food items eaten and 1 leaf eaten. (The students can decide as a class whether to count the leaf as a food). There will be two ten-frames completed with 5 or 6 counters/unifix cubes on the third ten-frame. If students come up with different, but correct, equations, then discuss the different equations and ask students, "Can all of these be correct?"

This type of assessment question requires students to solve word problems that call to relate counting on to addition (1.OA.5) https://achievethecore.org/coherence-map/1/5/39/39	
Relevance to families and communities: During a unit focused on adding and subtracting within 20, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, count, add and subtract in your home language. Use items from around the house to model addition and subtraction problems.	**Cross-Curricular Connections:** Science: In first grade the NGSS state students should "make observations at different times of year to relate the amount of daylight to the time of year." Consider providing a connection for students to find the difference in the number of hours daylight during different times of the year. Language Arts: Letters or digraphs are something that students can count. Consider providing a connection where students "hunt" for particular letters or digraphs on 2 different pages of their reading book and then add or subtract to find the total or difference, being mindful that all answers are within 20.

1.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: D: Work with addition and subtraction equations.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.OA.D.7: Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.	SMP 2: Students can reason abstractly and quantitatively by transitioning from using concrete objects that represent equations to using symbols. SMP 6: Students can attend to precision by relating mathematical symbols to their meaning in developmentally appropriate ways (= represents the concept of "same as", + represents addition, — represents subtraction and □ [or a similar symbol] represents finding the unknown in an equation).	• Explain the meaning of equal using models and drawings. • Determine if two quantities are equal. • Represent equal quantities with an equation with operations on either side, neither side, or both sides of the equal sign. • Determine whether an equation is true or false. **Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Analyze and Evaluate
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
1.OA.D.8: Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.*	SMP 3: Students can construct viable arguments by explaining how they found the unknown value in an equation and critique the reasoning of others comparing how others' solutions are like and different from their own. SMP 7: Students can look for and make use of structure by solving for the unknown in various positions in addition and subtraction equations.	• Determine the unknown in various positions in an addition equation. • Determine the unknown in various positions in a subtraction equation. • Explain how an unknown in an equation was determined. **Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to comparing the number of objects in one group to the number of objects in another group to decide if they are equal, as well as comparing two written numbers between 1 and 10 and discussing if they are equal. **(K.CC.6-7)**	• Connect to changing the structure of problems (e.g., changing a subtraction problem to an addition problem) and being flexible with the position of unknowns and the location of the equal sign in equations (e.g., 5 + 4 = 9 or 9 = 5 + 4) **(1.OA.3-4)**	• Connect writing equations to express equivalent groups and the ideas of even numbers, equal parts, skip counting, etc. **(2.OA.3-4)** • Connect to writing equations to solve word problems. **(2.OA.1)** • Connect to thinking about inequalities and students continuing to use their understanding of the equal sign. **(1.NBT.4)**

Clarification Statement:

- In this standard, students develop an understanding of the meaning of the **equal sign** and apply their understanding in order to determine whether an equation is true. Students learn that the equal sign does not mean "the answer comes next", but that the symbol signifies an equivalent relationship. Students need to understand that an **equation** needs to "**balance**", with **equal quantities** on both sides of the equal sign. Once students understand the meaning of the equal sign, they can determine if an equation is true (9 = 9) or not true (9 = 8).

Common Misconceptions

● Students thinking that the equals sign means that an operation must be performed on the numbers on the left and the result of this operation is written on the right.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying work with addition and subtraction equations because OA.D is a new concept for first graders there are no kindergarten standards that link to it, therefore going into detail about what an equal sign is and the importance of it, is very important for a deep understanding.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.OA.D.7: This standard provides a foundation for work with addition and subtraction equations because first graders are introduced to this standard for the first time in first grade and in order to be successful in the rest of their computing careers in school they need to master this first skill of understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through*

vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with work with addition and subtraction equations benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because Students must use mathematical symbols correctly and describe the meaning of the symbols they use. In this case, they understand that the equal sign denotes that the quantities on either side have the same value and use this understanding flexibly to identify and express equivalences. When crafting their explanations, they learn to communicate their reasoning by using precise mathematical vocabulary describing each quantity accurately.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with work with addition and subtraction equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because OA.D is a major cluster therefore students need to meet mastery in order be proficient in the grade level. Providing feedback that allows for identifying mistakes and turning them into positive strategies makes them goal driven learners.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with work with addition and subtraction equations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because in order for students to really understand the equal sign they must know what it means. Therefore, connecting it to prior learning allows for deeper understanding.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with work with addition and subtraction equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g. blocks, 3D models, base-ten blocks) because first graders are still at conceptual understanding and need to move to a more application level using manipulatives to help them achieve this.

Internalize:

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with work with addition and subtraction equations benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" (e.g., portfolio review, art critiques) because it is important for students to see their work and others work.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on work with addition and subtraction equations by critiquing student approaches/solutions to make connections through a short mini-lesson because in order for students to truly be successful in addition or subtraction fluidly they need to be proficient in understanding what the equal sign means and be able to solve.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit work with addition and subtraction equations by helping students move from specific answers to generalizations for certain types of problems because students should solve addition and subtraction equations with different structures so that they are able to see the connections between addition and subtraction more easily. Examples should be presented with the sum or difference on either side of the equal sign in order to dispel the notion that it means "compute."

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying work with addition and subtraction equations because as student become more fluent with adding and subtracting they need to be able to solve no matter where the equal sign is to have them build their fluency skills.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and

maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying work with addition and subtraction equations. Supporting productive struggle is critical because when solving addition and subtraction students need consistent fluency practice without a lot of guidance from teachers. Allowing them to productively struggle with these equations will encourage perseverance no matter the culture of the child.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Find the Missing Number

Find the missing number in each of the following equations:

9-3=_ 8+_=15 16-_=5 _=7-2 13=_+7 6=14-_

This type of assessment question requires students to solve addition and subtraction equations with different structures so that they can see the connections between addition and subtraction more easily. Examples should be presented with the sum or difference on either side of the equal sign in order to dispel the notion that means "compute."

Source: https://achievethecore.org/coherence-map/1/5/45/45

Relevance to families and communities:

During a unit focused on work with addition and subtraction equations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, Continue to practice adding and subtracting within 20 or Practice "counting on" as a strategy for addition, e.g. if you have 7 LEGO pieces, and then you get 3 more, encourage your student to start with the number 7 and count "8...9...10" to find the total. Discuss various ways to take apart a given number, e.g. 6 is made of 1 and 5, 2 and 4, 3 and 3, etc.

Cross-Curricular Connections:

Language Arts: Literature can offer connections to help students find the unknown in various positions in addition and subtraction equations such as: *Safari Park* by Stuart J. Murphy.

Physical Education: Keeping score during a team game can offer connections to help students understand that equal means the "same as" or "tied". Basketball, football or another game where the number of total number of points could be scored in a variety of different ways (e.g. 2 touchdowns + 1 safety = 2 touchdowns + 2 conversions) is especially helpful in developing this idea.

NEW MEXICO
Public Education Department

1.NBT: NUMBER & OPERATIONS IN BASE TEN

Cluster Statement: A: Extend the counting sequence.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.NBT.A.1: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent several objects with a written numeral.	SMP 6: Students can attend to precision by recognizing that 24 is different than 42. SMP 7: Students can look for and make use of structure by recognizing patterns with numerals on a hundred chart.	• Count to 120. • Count to 120 starting at any number. • Read any number name up to 120. • Write any numeral up to 120. • Label a set of objects up to 120 with a written numeral.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Remember and Apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to counting from 1 to100 by ones and tens beginning with any number and reading, writing and representing objects with a range of numbers from 0-20. (**K.CC.1-3)**	• Connect to understanding that the two-digits in the two-digit number represent tens and ones. **(1.NBT.2)**	• Connect to skip counting within 1000 (by 5s,10s and 100s) and using base ten numerals, number names, and expanded form to read and write numbers within 1000. **(2.NBT.1-3)**

Clarification Statement:
Students build on their counting to 100 by ones and tens and start a count at any number less than 120 and continue to 120. Students should be able to count and represent their counting in many ways; hundred charts and number lines are useful tools.

Common Misconceptions
● Students may reverse digits in writing numerals and believe that 24 and 42 have the same value.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines or number chart) when studying extending the counting sequence because students will be exposed to the written and oral representation of counting on from any given number to 120. Also, students are expected to represent several objects with the written form

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.CC.A.1 This standard provides a foundation for work with extending the counting sequence because students begin to count forward to 100 by ones and tens. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with extending the counting sequence benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students need to know how and why they will use the math in their lives. With counting, students need to be able to start counting from any number in written and oral form in order to be able to solve real-world problems.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with extending the counting sequence benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because students will be able to make connections with objects that are related to their cultural background or interests when counting in sequence from any given number.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with extending the counting sequence benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students can make connections to recognizing the numbers and their names when they are start counting from any given number.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with extending the counting sequence benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because it will help students to be able to count on from any given number when using physical manipulatives as they continue to count in sequence.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with extending the counting sequence benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need multiple opportunities to be able to count in sequence from any given number using different modalities, such as written and oral.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on extending the counting sequence by providing specific feedback to students on their work through a short mini-lesson because as students are able to give feedback to other students, then they are able to do some critical thinking to determine where there might have been an error as the students was counting from any given number.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit extending the counting sequence by offering opportunities to understand and explore different strategies because students need to be exposed to multiple opportunities to be able to get a concrete understanding of counting on from any given number. It would be helpful to provide students with a number chart or number line to provide a visual support.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying extending the counting sequence because an open ended task would allow students more practice to become more fluent with counting on from any given number.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying extending the counting sequence the types of mathematical tasks are critical because students need to be able to count on or backwards from any given number as they get into higher grades. It is important for students to be fluent and have a good understanding of the order of numbers. Students can also count in their home language, if possible.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://achievethecore.org/coherence-map/1/4/16/16

Choral Counting II

Provided by Illustrative Mathematics

Materials

- 100 chart or large number line, preferably one that extends beyond 100.
- A pointer
- Setup
- Have students sit in the whole group meeting area.

Actions

Lead the students in chanting the counting sequence starting with one to one hundred; use the pointer to follow the number sequence. Then start counting at various numbers other than one that are randomly selected from 1-120. Have a student take over the job of pointing out the number sequence. Highlight the multiples of ten using a marker or a colored screen and have students chant the counting sequence by 10s, by 5s and by 2s. This should be done daily.

This type of assessment question requires students to count the days in the month on the classroom calendar to practice the counting sequence; first count the number of days total and then count from the current date to the end of the month to get practice starting at numbers other than one. It is important start from different numbers each day when counting by ones to increase student flexibility with the number sequence.

Relevance to families and communities:

During a unit focused on extending the counting sequence, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, students make connection of number sense by counting objects within 120, counting to 100 is easier than counting to 120 because of number sense. "What number comes after 100?"

Cross-Curricular Connections:

Social Studies: In first grade, the New Mexico Social Studies Standards state students should "describe different ways to determine a decision (e.g., majority rule, consensus, authoritarian [parent, teacher, principal])". Consider providing a connection for students to have 120 people or less (maybe the entire first grade) vote on something and then count the votes.

Classroom Jobs (or other similar routine): Consider providing a connection to counting or taking inventory of various items around the classroom.

1.NBT: NUMBER & OPERATIONS IN BASE TEN

Cluster Statement: B: Understand place value.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.NBT.B.2: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: • **1.NBT.B.2.A: 10 can be thought of as a bundle of ten ones — called a "ten."** • **1.NBT.B.2.B: The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.** • **1.NBT.B.2.C: The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).**	SMP 4: Students can model with mathematics by using concrete materials to represent numbers including 10, 20, 30, ..., 90 as groups of ten with no ones. SMP 7: Students can look for and make use of structure by connecting words to the written numeral, such as 4 tens and 2 ones is written as 42.	• Represent 10 as ten ones. • Represent numbers 11 to 19 as a ten and some ones. • Represent two-digit numbers using physical tools, drawings, and number names (2 tens is 20, 2 tens and 7 ones is 27). • Explain the value of each digit in a two-digit number (place value). • Locate a two-digit number on a hundred chart and number line. **Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
1.NBT.B.3: Compare two two-digit numbers based on meanings of the tens and one's digits, recording the results of comparisons with the symbols >, =, and <.	SMP 4: Students can model with mathematics by using materials such as objects on place value charts, tens frames, hundreds chart and number lines to compare two 2-digit numbers. SMP 6: Students can attend to precision by describing the comparison using terms including greater than, more than, less than, fewer than, equal to, and same as.	• Determine when a two-digit number is greater than, less than, or equal to another two-digit number. • Explain why a two-digit number is greater than, less than, or equal to another two-digit number using physical models, hundred charts, number lines, and drawings. • Compare two two-digit numbers using place value understanding. • Record the comparison using the symbols >, <, and =. **Depth of Knowledge:** 2

		Bloom's Taxonomy: Apply and Analyze
Previous Learning Connections • Connect to composing and decomposing using ten ones and some more ones. **(K.NBT.1)** • Connect to working with values between 1 and 10 and identifying whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, as well as comparing two numbers between 1 and 10. **(K.CC.6-7)**	**Current Learning Connections** • Connect to applying place value strategies of breaking apart numbers into tens and ones to help with adding and subtracting within 100. **(1.NBT.4,6)** • Connect to using the concept of tens and ones to mentally find 10 more or 10 less. **(1.NBT.5)**	**Future Learning Connections** • Connect to applying place value concepts to a larger range of numbers to include numbers to 1000. **(2.NBT.1-4)**

Clarification Statement:

- 1.NBT.B.2: More generally, first graders learn that the two **digits** of a **two-digit number** represent amounts of **tens** and **ones**, e.g., 67 represents 6 tens and 7 ones. Saying 67 as "6 tens, 7 ones" as well as "sixty-seven" can help students focus on the tens and one's structure of written **numerals**.
- 1.NBT.B.3: Grade 1 students use their **base-ten** work to help them recognize that the digit in the **tens place** is more important for determining the size of a two-digit number. Correctly placing the **<** and **>** **symbols** is a challenge for early learners. Accuracy can improve if students think of putting the wide part of the symbol next to the **larger** number.

Common Misconceptions

● Students may count tens and ones separately, such as 10, 20, 1, 2, 3 instead of 10, 20, 21, 22, 23.

● Students may not recognize that in two-digit numbers the position of the digit determines its value.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying understanding place value because place value is a foundational skill that students need to have a strong understanding, so allowing students time to explore any confusions would help them to clear up those confusions and build a deeper understanding of place value.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.NBT. A.1: This standard provides a foundation for work with understanding place value because students begin to decompose numbers from 11 to 19 into tens and ones. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding place value benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because students need to be able to discuss and talk about their learning with their peers, especially with the foundational skill of place value. Students need to be able to complete social tasks that involve comparing two numbers as they continue to understand the concept of place value.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding place value benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because place value is a foundational skill that students need to have a strong understanding of so they need to receive frequent feedback to ensure they are progressing and being able to demonstrate their understanding of place value while comparing numbers two two-digit numbers..

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding place value benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as <pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because it is important for students to know and understand the meaning of a "ten" and "one". Also, students need to know and understand the symbols when comparing two two-digit numbers. The connection of more than or less than can be made to the student's background knowledge of counting in sequence from any given number.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding place value benefit when learning experiences attend to the multiple way's students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because it is important for students to use manipulatives when building their understanding of place value. Students need to be able to see how many ones make a ten. Also, when comparing numbers, it would be important for students to see the numbers with base-ten blocks as they compare.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with understanding place value benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because students need to be able to have access to various scaffolds as they work to understand place value. Some students might need to use a place value chart, while others will need base-ten blocks to support their understanding.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding place value by revisiting student thinking through a short mini-lesson because students might have some minor confusions on how to decompose numbers into tens and ones. Students should be using manipulatives, such as base-ten blocks, while building their understanding of place value. The use of a place value chart with base-ten blocks can help to solidify the student's understanding.

Re-teach (intensive) : *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding place value by confronting student misconceptions because students need to have a strong understanding of place value so it is important to clarify any misconceptions the student might have. It is important to utilize manipulatives so students can visually see the concept as well.

Extension Ideas

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying understanding place value because an open ended task would allow students to explore and have a deeper understanding of place value, which will benefit them in the future.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying understanding place value, the types of mathematical tasks are critical because students need to have a strong understanding of place value before they begin to use numbers procedurally. It is important to utilize manipulatives when building a student's understanding of place value, so they can visually see the concept as well. Engage students in learning by building on their experiences and provide multiple options for how students can interact with instructional

content. Use consistent spoken and body language with all students to avoid unconscious bias in verbal or nonverbal cues. A teacher randomly draws from popsicle sticks with student names when asking questions. This ensures all students have an equal chance of participating in the whole class discussion.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://achievethecore.org/coherence-map/1/4/17/17
Roll & Build
Provided by Illustrative Mathematics
Materials
For each pair:

- 2 ten-sided dice with the numbers 0 to 9 or two spinners with the numbers 0 to 9
- Base-10 blocks, linking cubes, or bundled and loose popsicle sticks
- Paper and pencil

Play
Student A rolls the dice.

Student B makes a number using the values on the dice as digits and both students write it on the paper. For example, if student A rolled a 3 and a 4, the number can be 34 or 43.

Student A represents the number with the tens and one's blocks/popsicle sticks.

Student B counts the blocks to check that they correctly represent the number.

Both students draw a picture of the tens and ones on the paper.

The students should take turns.

This type of assessment question requires students to practice representing two-digit numbers with concrete objects to reinforce the meaning of the tens digit and the ones digit. This task works best in partners; however, it can be played individually. The teacher should show the students how to play using an overhead projector or the white board before the students start. This is just an example.

Relevance to families and communities:
During a unit focused on understanding place value, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, students can make the connection that two two-digit numbers have ones and tens, so 5 tens and 4 ones is 54. Families can play "I spy" number game. Make connections with two two-digit numbers, use vocabulary words greater than, less than, and equal to instead of bigger or smaller numbers. Connect the vocabulary words with the symbol greater than >, less than <, and equal to = whenever possible.

Cross-Curricular Connections:

Science: In first grade the NGSS state students should "make observations at different times of year to relate the amount of daylight to the time of year." Consider providing a connection for students to compare number of hours daylight during different times of the year.

Art: Even though there is a difference between illustrating or drawing and pictorial representation in math, students do need significant experience with concrete representations to develop the idea of place value. Consider providing a connection where they can create pictures that include groups of ten objects together and then single ones (such as a bouquet of flowers and single flowers or a bunch of balloons and single balloons).

NEW MEXICO
Public Education Department

1.NBT: NUMBER & OPERATIONS IN BASE TEN		
Cluster Statement: C: Use place value understanding and properties of operations to add and subtract. **Major Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text **1.NBT.C.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.**	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by asking themselves if their answers are reasonable by reflecting on the value of the numbers and using strategies based on number sense. SMP 4: Students can model with mathematics by modeling additional examples with sums to 100 using concrete materials, pictures, and lastly numerals.	**Students who demonstrate understanding can:** • Explain addition within 100 adding a two-digit number and a one-digit number using physical models, drawings, hundred charts, and number lines. • Explain addition within 100 adding a two-digit number and a multiple of ten number using physical models, drawings, hundred charts, and number lines. • Explain addition within 100 adding a two-digit number and a two-digit number using physical models, drawings, hundred charts, and number lines. • Use partial sums by decomposing both addends to add within 100. • Use partial sums by decomposing one addend to add within 100. • Explain why a new ten is sometimes made when adding numbers. **Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Analyze and Evaluate

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.NBT.C.5: Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	SMP 3: Students can construct viable arguments by explaining their reasoning using place value understanding and patterns on the hundreds chart. SMP 8: Students look for and express regularity in repeated reasoning by by mentally calculating 10 more or 10 less than a given number.	• Determine 10 more and 10 less of any two-digit number using physical tool, hundred charts, and number lines. • Recall 10 more for any two-digit number (e.g., 32 + 10 = 42) without using a tool or representation. • Recall 10 less for any two-digit number (e.g., 32 - 10 = 22) without using a tool or representation. • Explain why the tens digit changes and why the ones place does not change when finding ten more or ten less. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand, Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
1.NBT.C.6: Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	SMP 3: Students can construct viable arguments by explaining their reasoning using place value understanding and patterns on the hundreds chart. SMP 7: Students can look for and make use of structure by looking for and describing patterns they find as they work with various representations.	• Determine the difference of two multiples of 10 (e.g. 90 - 40) using models, drawings, hundred charts, and number lines. • Subtract a multiple of 10 from a multiple of 10. (e.g., subtract 90 - 40). • Explain the difference between two multiples of 10 by relating it to subtracting the tens digit. • Explain why the ones place does not change when subtracting multiples of 10. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to composing and decomposing numbers into tens and ones and students using what they know to solve word problems within 10. **(K.NBT.1, K.OA.2)**	• Connect to relating counting to addition and subtraction. (1.OA.5) • Connect to starting to generalize addition and subtraction strategies to numbers within 100 and focusing on multiples of 10 to encourage the use of place value concepts/strategies. **(1.NBT.2)**	• Connect to fluently adding and subtracting within 100 and solving word problems using strategies based on place value properties of operations, and/or the relationship between addition and subtraction. **(2.NBT.5)**

Clarification Statement:

- 1.NBT.C.5: Students may explain their reasoning by saying that they have **one more** or **one less ten** that before.
- 1.NBT.C.6: **Differences** of **multiples** of 10, such as 70 – 40 can be viewed as 7 **tens minus** 4 tens and represented with **concrete models** such as objects **bundled** in tens or drawings. Children use the relationship between **subtraction** and **addition** when they view 80 – 70 as an **unknown addend addition problem**, 70 + <box> = 80, and reason that 1 ten must be **added** to 70 to make 80, so 80 –70 = 10.

Common Misconceptions

● Students may subtract the digits in the tens place but ignore value of the ones place.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources when studying use place value understanding and properties of operations to add and subtract because the majority of this cluster will work from the hundreds chart. If students can review this chart and the numbers on it, this will help towards understanding.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.NBT.A: Work with numbers 11-19 to gain foundations for place value This standard provides a foundation for work with place value because it combines the concept of ones and tens. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because the concept can be presented in multiple ways visually, increasing the likelihood of students gaining understanding of the concept (ex. place value blocks, hundreds charts, place value charts, coloring coding place value).

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as displaying the learning goals in multiple ways because there are various tools available in regards to this standards. Students should have access and the ability to explore these tools to find a method or methods that work best for their individual learning needs. It will also provide students with autonomy for their own learning.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because the concept presented in this cluster can be performed in various mediums using different tools. Students should have access to multiple representations to support generalizing the concept across domains. Here is a link showing various tasks that use different representations to complete the concept of the cluster. http://tasks.illustrativemathematics.org/1-3

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because it makes the concept of adding and subtracting larger numbers more concrete, thereby increasing the understanding.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as offering devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the mathematical practices for the purpose of monitoring and improving because students need access to see how numbers can be made up using various tools demonstrating place value. Once they can see how the quantities are derived, they might have better understanding when combining these qualities or taking away.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using place value understanding and properties of operations to add and subtract by critiquing student approaches/solutions to make connections through a short mini-lesson because exploring where they went wrong in their approach to understanding place value for adding and subtracting, but also be reintroduced to tools and strategies that work better for their particular needs.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit using place value understanding and properties of operations to add and subtract by offering opportunities to understand and explore different strategies because these students might need more individualized support with the different strategies introduced. The various strategies might be helpful to the student; however, they need step by step directions on how to use the strategies and tools to increase familiarity.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. When studying using place value understanding and properties of operations to add and subtract because they could consider how adding and subtracting 100 more or less or 1000 more or less would differ from working with 10 more or less.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of

low expectations and low achievement. For example, when studying use place value understanding and properties of operations to add and subtract goal setting is critical because this is a foundational standard where so many skills are built from. Students need to feel comfortable with the tools and language needed to perform the tasks. There might need to be some added reflection time to encourage students to talk about what is confusing or what they understand.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://achievethecore.org/coherence-map/1/4/22/22
Addition within 100
Provided by Learning Heroes, a project of New Venture Fund

$$\begin{array}{r} 5\ 6 \\ +\ 2\ 6 \\ \hline \end{array}$$

Correct if student writes the number 82.

The best way to find the sum is to take advantage of place value, adding ones to ones and tens to tens.

There are several ways this can be done. Some first graders might use the standard algorithm to organize their work.

This type of assessment question requires students to first combine the tens (50 + 20 = 70), next combine the ones (6 + 6 = 12), and finally add these results to obtain the final answer: 70 + 12 = 82. This approach is fine in grade 1. As problems get more complicated, students will learn increasingly efficient procedures culminating in the standard algorithms.

Place Value Tens
Provided by Learning Heroes, a project of New Venture Fund
Fill in the missing number:

50 = 2 tens + _____ tens

Solution
Correct if student writes the number 3.

50 is 5 tens, and 5 tens can be broken down into parts as 2 tens plus 3 tens. So, the missing number is 3.
This type of assessment question requires students to understand the place value units. This helps students learn how to "borrow" and "carry" in the standard algorithms for adding and subtracting multi-digit numbers. It also prepares students to learn about decimal fractions in later grades. Using tens as a unit can also help with mental math (for example, in problems like 60 + 20 = 80).

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on using place value understanding and properties of operations to add and subtract, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school	Science: In first grade the NGSS recommend studying light, transparency, and shadows. Consider providing a connection for students to add the lengths of the

connections for students, to work on some problem solving activities together. They can use home language for corresponding vocabulary and numbers.	shadows of two items, where at least one length is in double-digits and the sum is within 100. Social Studies: Social Studies: In first grade the New Mexico Social Studies Standards state students should "Understand the purpose of rules and identify examples of rules and the consequences of breaking them". Consider providing a connection for students to "earn" and "lose" points for following or breaking various rules. Earning can be in groups of 1s, 2s and 5s, and losing can be in groups of 10.

NEW MEXICO
Public Education Department

1.MD: MEASUREMENT & DATA		
Cluster Statement: A: Measure lengths indirectly and by iterating length units. **Major Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text **1.MD.A.1: Order three objects by length; compare the lengths of two objects indirectly by using a third object.**	**Standard for Mathematical Practices** SMP 6: Students can attend to precision by using correct and appropriate vocabulary when comparing two or three objects, referencing words such as "shorter," "longer," "shortest," and "longest." SMP 7: Students can look for and make use of structure by ordering the same three objects in two different orders (longest to shortest or shortest to longest).	**Students who demonstrate understanding can:** • Identify which of two objects is longer or shorter. • Order three objects by length (longest to shortest or shortest to longest). • Decide how the lengths of two objects relate to one another by comparing them both to a third object (e.g. the crayon is shorter than the pencil because it is shorter than the marker and the marker is shorter than the pencil). **Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Analyze and Evaluate
Standard Text **1.MD.A.2: Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.***	**Standard for Mathematical Practices** SMP 5: Students can use tools strategically by using non-standard tools to measure and estimate length. SMP 6: Students can attend to precision by measuring accurately (no gaps or overlaps and with the correct number of length units lined up from end-to-end) using non-standard tools.	**Students who demonstrate understanding can:** • Illustrate how to use multiples copes of a shorter objects to find the length of a longer object. • Connect the length of the longer object to the total number of shorter objects used and express the length of the longer object as the whole number of shorter objects used. • Describe why gaps and overlaps are not allowed when measuring the length of an object.

		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Knowledge, Understand and Apply
Previous Learning Connections • Connect to describing measurable attributes of objects, such as length (**K.MD.1)** • Connect to directly comparing two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute and describing the difference. For example, directly comparing the heights of two children and describing one child as taller/shorter. **(K.MD.2)**	**Current Learning Connections** • Connect to non-standard objects and more standard measurement tools to help students state length units in whole numbers and make direct and indirect comparisons of objects.	**Future Learning Connections** • Connect to making decisions about the tool's students use to measure the length of an object by selecting and using tools such as a yardstick, meter stick, rulers, tape measures, etc. **(2. MD.1)** • Connect to measuring to determine how much longer one object is than another and expressing the length difference in terms of a standard-length unit **(2. MD.4)**

Clarification Statement:

- 1.MD.A.1: This standard focuses on the property of **transitivity**: If A is longer than B and B is longer than C, then A must be longer than C as well. Students will revisit this idea in future grades. Students should apply the principle of transitivity of measurement to make indirect comparisons of the length of objects, but they need not use this technical term.
- 1.MD.A.2: Measuring length or distance consists of two aspects, choosing a **unit** of measure and subdividing (mentally and physically) the object by that unit, placing that unit end to end (**iterating**) alongside the object. The **length** of the object is the number of units required to iterate from one end of the object to the other, without gaps or overlaps.

Common Misconceptions

● Students may not accurately line up the objects being compared.

● Students may believe they need to sort a second time to move from shortest to longest or longest to shortest rather than realizing the inverse relationships will hold.

● Students may not accurately line up the multiple copies of the shorter object being used to measure the larger object.

● Students may not realize that the shorter object needs to be iterated and use multiple types or sizes of shorter objects.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying measure lengths indirectly and by iterating length units. because in kindergarten children are exposed to comparing objects based on

attributes, referring to this will help students understand measurement by noticing how long or short something is.

Pre-teach (intensive): What critical understandings will prepare students to access the mathematics for this cluster?

- K.MD.A.1: This standard provides a foundation for work with measure lengths indirectly and by iterating length units because students often initially hold undifferentiated views of measurable attributes, saying that one object is "bigger" than another whether it is longer, or greater in area, or greater in volume, and so forth. For example, two students might both claim their block building is "the biggest." Conversations about how they are comparing—one building may be taller (greater in length) and another may have a larger base (greater in the area)—help students learn to discriminate and name these measurable attributes. As they discuss these situations and compare objects using different attributes, they learn to distinguish, label, and Describe several measurable attributes of a single object. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade-level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with measure lengths indirectly and by iterating length units. This benefits when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because students should measure something that interests them (namely themselves) by laying multiple copies of a shorter object that represents the length unit end to end. This task provides students an opportunity to discuss the need to be careful when measuring as it is very likely that some of them will get incorrect comparisons of their leg length with their partner's leg length. With some body parts, it's not clear where to begin and end the measurements. This is a good opportunity to help the students think more precisely about how to define the "beginning and endings" more precisely.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with measure lengths indirectly and by iterating length benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because the use of a variety of different length units, before students understand the concepts, procedures, and usefulness of measurement, may actually deter students' development. Instead, students might learn to measure correctly with standard units, and even learn to use rulers, before they can successfully use nonstandard units and understand relationships between different units of measurement.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with measure lengths indirectly and by iterating length benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because it is important for students to know specific measurement terms in order to measure accurately.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with measure lengths indirectly and by iterating length benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) because as students are learning to measure, they may need guidance and support to make sure they are measuring accurately.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with measure lengths indirectly and by iterating length benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections (e.g., teaching literacy strategies in the social studies classroom) because measurement directly correlates with science and students can use measuring in science during different experiments.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on measure lengths indirectly and by iterating length units by examining tasks from a different perspective through a short mini-lesson because sometimes students may need to be taught in a different way. To realize that arbitrary (and especially mixed size) units result in the same length being described by different numbers, a student must reconcile the varying lengths and numbers of arbitrary units.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit measure lengths indirectly and by iterating length units by confronting student misconceptions because children should engage in experiences that allow them to connect number to length, using manipulative units that have a standard unit of length, such as centimeter cubes. These can be labeled "length-units" with the students. Students learn to lay such physical units end-to-end and count them to measure a length. They compare the results of measuring to direct and indirect comparisons.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying measure lengths indirectly and by iterating length unit because measurement can be used across multiple areas and allowing students to connect this way will help them deepen their understanding.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying measure lengths indirectly and by iterating length units the types of mathematical tasks are critical because it will allow students to measure using different tools that may be associated with their culture. They could also measure various objects that are important to their culture. For example, having students bring in an item that may be pertinent in their family such as native jewelry or sombrero etc.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://tasks.illustrativemathematics.org/content-standards/1/MD/A/2/tasks/1086
Growing Bean Plants
Provided by Illustrative Mathematics
Materials

- 2 clear plastic cups for each pair of students
- 4 bean seeds for each pair
- soil
- Uni fix cubes
- a plant or math journal to record data in

Actions
Students in pairs grow bean plants from seed. Students should label the first cup with an A and the second cup with a B and write their names on the cups. Then they fill their plastic cups 2/3 full of soil and place two bean seeds in each cup about one inch below the surface of the soil. Water the seeds and place the cups on a window ledge where they will receive sun light. The teacher should grow several cups to be "class plants" and as backups.
The beans will sprout within 7-10 days. When the bean seeds start to grow, choose two plants to be the class plants. Every few days, the class should water the plants. As a whole group, they measure the height of the class plants with unifix cubes. The first few times the teacher can do it, then the students can take turns measuring and recording the height in a table.

Date	Plant A	Plant B
March 1	2	1
March 4	4	3
March 8	8	5
March 11	11	7

The measurements should be recorded to the closest whole number.
Once students have done the measuring and recording as a whole group, the pairs of students can measure their own bean plants. They should measure the tallest plant in each cup if more than one bean sprouts. Students record each measurement in a table with the date in a math journal (or a plant journal for this project only if the class does not keep math journals).
After the students have measured their plants, they answer these two questions:
Which plant is tallest today?
How much taller is it?
On the last day of the activity, the students can compare the tallest plant in the room with the shortest plant. This type of assessment question adds some rigor to the activity, by collecting actual growth data, providing practice for students in measuring and recording length measurements. In addition to measuring and recording plant height, students can also count and record how many leaves their plant has on each date.

Relevance to families and communities:
During a unit focused on measure lengths indirectly and by iterating length unit, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, order family members as tallest to shortest of shortest to tallest, give your student many opportunities to measure objects using other, smaller objects, e.g., "How many Lego pieces long is your book? How many blueberries long is this notebook?"

Cross-Curricular Connections:

Science: In first grade the NGSS recommend studying light, transparency, and shadows. Consider providing a connection for students to order the lengths of the shadows of various items.

Social Studies: A map and strings is often used to track Flat Stanley's travels (*Flat Stanley* by Jeff Brown). Consider providing a connection for students to measure and/or compare the lengths of the strings.

NEW MEXICO
Public Education Department

1.MD: MEASUREMENT & DATA

Cluster Statement: B: Tell and write time.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
1.MD.B.3: Tell and write time in hours and half-hours using analog and digital clocks.	SMP 7: Students can look for and make use of structure by making connections between fractions and time and using phrases such as "half past" to tell time. SMP 8: Students look for and express regularity in repeated reasoning by noticing that each hour is comprised of 60 minutes and that the minute hand is read the same no matter what the hour is.	• Identify the difference between an analog and digital clock. • Identify the hour and minute hand on an analog clock. • Remember how many minutes are in an hour and a half-hour. • Observe time to the hour and half-hour. • Write the time in hours and half-hours when given a time verbally. • Draw hands on a clock to show a given time in hours and half-hours. • Relate time on both digital and analog clocks. • Explain what "o'clock" and "thirty" mean.
		Depth of Knowledge: 1
		Bloom's Taxonomy: Remember and Understand
<u>Previous Learning Connections</u> • Connect to the fact that while Kindergarten does not have specific standards for time, over the course of a day they are exposed to concepts of time such as the morning, or afternoon, etc.	**<u>Current Learning Connections</u>** • Connect telling time to the nearest half-hour to partition circles into halves. **(1.G.3)**	**<u>Future Learning Connections</u>** • Connect to telling and writing time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **(2. MD.7)** • Connect to telling and writing time to the nearest minute and measure time intervals in minutes. Connect to solving word problems involving addition and subtraction of time intervals in minutes, for example, by representing the problem on a number line diagram. **(3. MD.1)**

Clarification Statement:
Students need to experience a progression of activities for learning how to tell time from a one-handed **clock** to tell time in **hour** and **half-hour** intervals to clocks with the hour and minute **hand**. Students should also make connections between **digital** and **analog** clocks.

Common Misconceptions
● Students may have difficulty noticing the differences between the two hands on an analog clock and how they work.

● Students may have difficulty identifying the time on an analog clock when the hour hand is not directly pointing to a number.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying tell and write time because when learning how to tell time to the hour and half hour students need to know the proper language and vocabulary attached to accurately tell time.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.MD.B.3: This standard is the foundation for work with tell and write time because telling time to the hour and half hour in first grade is an additional cluster and is introduced in first grade to continue work in 2nd grade. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with tell and write time benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because when teaching time students need to physically see how the hands move and how to count the minutes on the clock. They need to be able to see the time change and to create a model of one.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with tell and write time benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because as students are learning

how to tell and write time they need constant feedback to know that they are doing so correctly. In second grade students quickly move onto telling time to the nearest 5 minutes. They need to be able to tell time to the hour and half-hour.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with tell and write time benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because when telling time in first grade there are many vocabulary words that can be difficult to understand. For example: half past, half hour, etc.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with tell and write time benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because students need to physically see how the hands move and how to count the minutes on the clock. They need to be able to see the time change and to create a model of one.

Internalize:

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with tell and write time benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing interactive representations that guide exploration and new understandings because it is important for first graders to see and comprehend how a clock works in order for them to abstractly tell time.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on tell and write time by clarifying mathematical ideas and/or concepts through a short mini-lesson because when telling time in first grade they frequently misinterpret how the numbers on the clock are supposed to be read. Therefore, re-teaching this skill in a small group will benefit them, providing a lot of time for practice.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions*

- For example, some students may benefit from intensive extra time during and after a unit tell and write time by confronting student misconceptions because students frequently misinterpret the numbers on the clock. They may say it is two, six; instead of two thirty. Giving students more time for practice will help clear up these misconceptions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students, when studying telling and writing time because students who already grasp the concept of telling time to the hour and half hour may be ready explore the connections between time and astronomy.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying tell and write time facilitating meaningful mathematical discourse is critical because telling time is an abstract skill requiring students to be able to count, determine the meaning between hours and minutes and knowing all the required vocabulary to make sense of the clock, allowing discussion around this will allow for misunderstandings to be cleared and peer to peer.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks..org/cillustrativemathematicsontent-standards/1/MD/B/tasks/992
Making a Clock
Materials

- 12 sheets of laminated paper or tag board with the numbers 1-12
- 12 sheets of laminated paper or tag board with the numbers 0, 5, 10, ... 55
- 1 long arrow (minute hand)
- 1 short arrow (hour hand)
- White board, dry erase marker
- 1 large clock face with hands that can be moved to different positions
- Small clock faces with hands that can be moved to different positions (1 for each pair of students)
- A cut-out circle with the sun on one side and moon on the other.

Actions
The students sit in a large arc/rainbow. The teacher explains that they are going to make a giant clock to practice telling time.

The teacher shows students a clock and asks what numbers they see on it. After students respond, the teacher has students help position the tag board sheets with the numbers 1-12 in a circle on the floor to represent a clock. Students count from 1 o'clock through 12 o'clock as the numbers are laid out.

Then the teacher says,

There are 60 minutes in an hour. When the minute hand goes from one number to the next, that means five minutes have passed. It starts on the 12; that means 0 minutes have passed.
Then the teacher has a student put the paper with the 0 below the paper with the 12 on it. Next, the teacher models counting by 5's, having a student place the corresponding number of minutes below each of the numbers 1-11.

Then the teacher says,

When the minute hand reaches 12 again, that is 60 minutes, and one hour has passed. Now the count starts over for the next hour.
The students and teacher can count by 5s together pointing to each of the numbers 1-11.

The teacher then discusses the hour and minute hands and shows how they move in a clockwise direction. The teacher models moving the hands to show time to the hour and half hour.

In pairs, the students create the same time on their mini clocks. The teacher guides the students in telling the times to their partner and shows them how to write the times on the whiteboard in both word format (i.e. eight o'clock) and digital format (8:00).

Once students have demonstrated proficiency in identifying what time it is, the teacher uses the moon/sun icon to show that the same time can occur at night or during the day. For example, she can model 12:00 lunchtime and hold up the sun icon, and model 12:00 while the students are in bed and hold up the moon icon. This can be extended with other events that are relevant to the students' lives.

This type of assessment question introduces students to the concept of reading an analog clock. Students would then tell time to the hour and half-hour. This task can be extended by having students take turns moving the hour and minute hands on the class clock or mini clocks, as well as figuring out the times. The teacher should use times that are on the hour and half hour (example: 12:00, 1:30, 4:00, 6:30, etc). Students can also write the times on paper or mini whiteboards. To teach the vocabulary of "clockwise," students can practice a clock dance with their partner. The partners face each other, and one gently moves the other's arm in a clockwise motion through each imaginary number on a clock, chanting one o'clock, two o'clock, etc. Or, the teacher can lead the entire class through a clock dance, with the students moving their own arms.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on tell and write time, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, Identify the time frame of favorite TV shows, Identify the length of time for sporting event (break up halves or quarters into time lengths, Discuss with parents the times they do important things during the day (wake-up, go to school, get out of school, eat dinner, go to bed). They can then make a schedule of their day writing the times in analog or digital format.	Social Studies: Different map skills are often explored in first grade, including classroom and neighborhood maps. Consider providing a connection for students to read or write times related to visiting different locations on the map. Language Arts: Literature can offer connections about measurement such as: *The Grouchy Ladybug* by Eric Carle.

NEW MEXICO
Public Education Department

1.MD: MEASUREMENT & DATA

Cluster Statement: C: Represent and interpret data.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

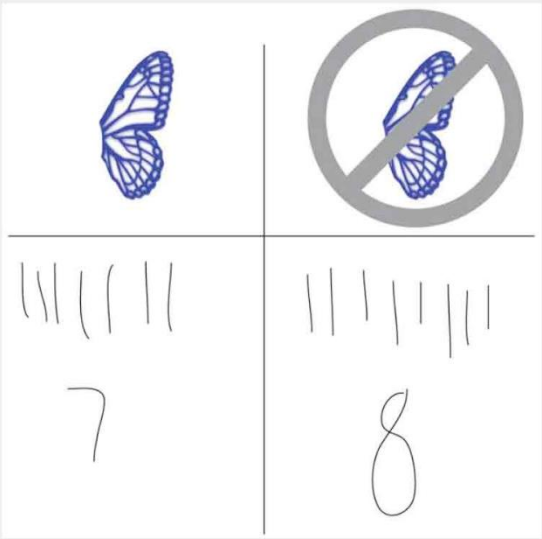
Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.MD.C.4: Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	SMP 2: Students can reason abstractly and quantitatively by making sense of the data and answering questions about it. SMP 4: Students can model with mathematics by collecting and using data to answer questions about their everyday lives. SMP 6: Students can attend to precision by organizing and representing data.	• Organize a given data set with up to three categories into a chart or other display. • Ask and answer questions about data points. • Compare data from up to three categories.
		Depth of Knowledge: 2, 3
		Bloom's Taxonomy: Apply, Analyze and Evaluate
<u>Previous Learning Connections</u> • Connect to classifying objects into given categories and sorting the categories. Connect to counting the number of objects in each category and understanding the relationship between numbers and quantities in order to answer questions about how many up (to 20). **(K.MD.3) (K.CC.3-5)**	**<u>Current Learning Connections</u>** • Connect to using addition and subtraction within 20 to solve word problems that may use up to three whole numbers. **(1.OA.1-2)** Using tally marks to represent data collected provides an opportunity to have more practice with groups of tens and ones.	**<u>Future Learning Connections</u>** • Connect to representing a data set with up to four categories by drawing a picture graph and a bar graph with single-unit scale. (**2. MD.10)**

Clarification Statement:

- 1.MD.C.4: Students' **data** work in Grade 1 has important connections to **addition** and **subtraction**. Students in grade 1 can ask and answer questions about **categorical data** based on a **representation** of the data. Students can also ask and answer questions leading to other kinds of addition and subtraction problems (1. OA), such as **compare problems** or problems involving the addition of three numbers (for situations with three categories).

There is no single correct way to represent categorical data-and the Standards do not require Grade 1 students to use any specific format. However, students should be familiar with **mark schemes** like the one shown in the figure. Another format that might be useful in Grade 1 is a **picture graph** in which one picture represents one object. (Note that picture graphs are not an expectation in the Standards until Grade 2.)

Sorting categorical data



The marks represent individual data points. The two category counts, 7 and 8, are a numerical summary of the data.

Common Misconceptions

- Students may incorrectly read or record data when transferring between different displays.
- Students may pose a question that is too open-ended or has too many choices.
- Students may not collect data from more than one person to answer the questions.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying the skill of being able to organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another because students are working with new mathematical vocabulary words that include, category, categories, and data for example. It is important to ensure that students have a clear understanding of these new math terms as they work to solve problems associated with being able to represent and interpret data.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.MD.B.3: This standard provides a foundation for work with students being able to classify objects and count the number of objects in each category because it will then

set up the foundation of students to be able to count correctly, and associate a given total to a specific category. As students' progress in first grade, they will be expected to work within a maximum of three categories and be able to identify totals, and one more or one less of an amount in any given category. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with being able to represent and interpret data benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features. This can include larger and more visible categories to represent data, graphs and charts that are easy to read without conflicting colors, and easily recognizable symbols to demonstrate values in a chart because all students should have the same access to understanding, inferring and solving questions related to the data presented in the chart. Students may recognize the categories listed and the data that is being represented in each.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with being able to represent and interpret data benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because when students have opportunities to communicate with peers they can cooperatively set goals and learn from each other. This allows students to build on their knowledge and teach their peers.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with being able to represent and interpret data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf, because it is necessary for all learners to be able to access data points, categories and symbols related to being able to interpret data with up to three categories.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with being able to represent and interpret data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students can express their understanding in numerous ways especially when they have a variety of strategies to pull from. Students can improve

their depth of knowledge by learning new strategies and approaching problems in different ways.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with able to represent and interpret data benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as elevating the frequency of self-reflection and self-reinforcements because when students are given the opportunity to do self-reflect they can identify where their understanding is and where they might need more support to clear up misconceptions. Students should be able to self-reflect as they organize, represent and interpret data within given categories in order to be able to ask and answer questions.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on being able to organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another by clarifying mathematical ideas and/or concepts through a short mini-lesson because students will be able to receive intervention on the skills associated with organizing, representing and interpreting data within up to three categories. This is important because it can clear up any misconception's students may have either associated with the categories or data that are being represented, as well as any mathematical counting errors of one more or one less of a category.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit in which they are able to organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another by helping students move from specific answers to generalizations for certain types of problems because students will be exposed to data in multiple ways as they continue with first grade and prepare for second grade concepts. Students gain new understanding that data exists all around them and with cross-curricular topics and can be used to organize data points and information into categories, and based on that organization it allows for students to make meaning of data and information while comparing the totals of various categories.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying the skill of being able to organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another because this allows students to make connections related to data and connections. When given choice

	and the opportunity to explore topics of choice related to this mathematical skill, students may be more engaged in the work and in making connections related to the organization and representation of data. Also, students are given the opportunity to see more real-world connections associated with data, categories, recognizing one more or one less and being able to compare categorical values.

Culturally and Linguistically Responsive Instruction:

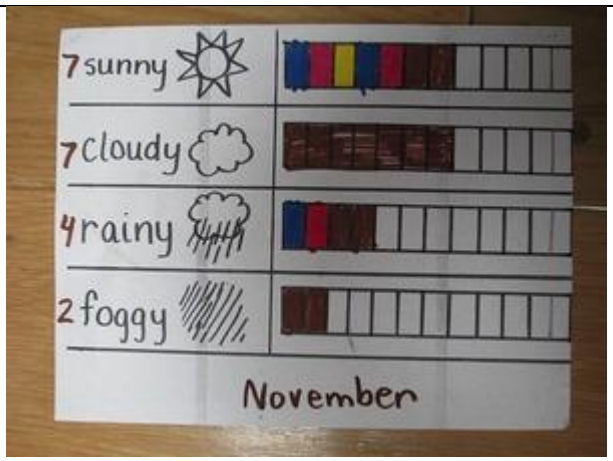
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying the math skill on being able to organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another the use of mathematical representations within the classroom is critical because students are transitioning and learning more representations of math through the use of data and categories. Students may also be utilizing symbols when interpreting data and can even bring in their own symbols related to their culture when creating data. Students can build a bridge between mathematics and culture, along with language as they interpret data and answer questions related to the data and information provided. Also, students can represent not only their learning, but category information as well to make cultural connections and build on this skill.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://achievethecore.org/coherence-map/1/3/13/13

Weather Graph Data
Materials
- Completed monthly weather recording sheet

- 

- Crayons
- Sentence strips with frames (see below)
- Student worksheet

Actions
Every day for a month the students records the weather by shading in an appropriate box on the recording sheet (attached).
At the completion of a month of school, the teacher projects the completed recording sheet using a document projector (if working with the whole class) or shows it to the students (if working in a small group setting).
The teacher asks the students to count how many days in the month each type of weather occurred, writing the number by the type of weather.
The teacher or another student asks questions like these:

How many rainy days did we have this month?
How many more rainy days did we have than sunny days?
How many days did we record the weather?
Students compose answers to the questions using sentence frames like these:

In September there were ___ _____ days
There were ___ more _____ days than _____ days
There were ___ fewer _____ days than _____ days
We recorded ____ days of weather altogether.
Both numbers and types of weather are used to fill in the blanks. Students pair up and share an idea for each sentence frame; then the teacher chooses a student to fill in one of the frames orally, while it is recorded onto the sentence frame.

The students copy the data from the recording sheet onto their worksheet to create their own graph. The teacher may need to model this for the students the first time they do it.
The students choose a sentence based on the completed frames to match each of the weather pictures given. So, for the sunny picture a student could write, "There were 5 more sunny days than rainy days," or "There were 12 sunny days in September." The frame of "We recorded ___ days of weather altogether" could be stated orally, or added at the bottom of the recording sheet to be filled in. The teacher may need to model this for the students the first time they do it.

This type of assessment question requires students to represent and interpret weather data, as well as answer questions about the data they have recorded. Teachers could modify this task to include the most common weather in their local area.

Relevance to families and communities: During a unit focused on 1.MD.C, represent and interpret data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different structures for the number names across the languages in your classroom can lead to a more robust understanding of number for all students by making connections to the different structures of number-names in other languages.	**Cross-Curricular Connections:** Language Arts: Consider providing an opportunity for students to survey each other regarding the books they are reading (favorite book, favorite character, what do you think will happen next) and then graph and analyze the results. Physical Education: Consider providing an opportunity for students to time or count the number of reps they can do for a certain fitness task (sprint, sit-ups, jumping jacks) and then graph and analyze the results.

1.G: GEOMETRY

Cluster Statement: A: Reason with shapes and their attributes.

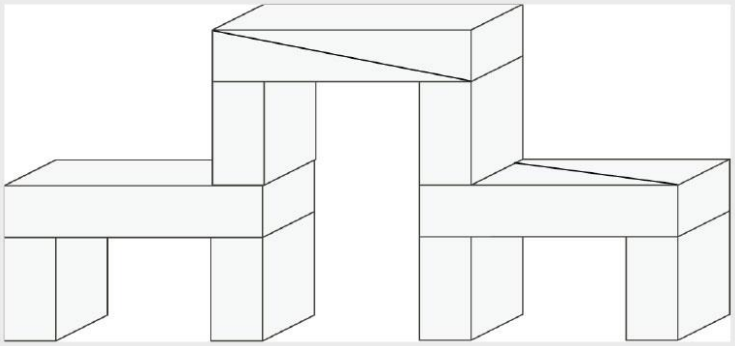
Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
1.G.A.1: Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	SMP 6: Students can attend to precision by using clear, specific definitions to define attributes. SMP 7: Students can look for and make use of structure by identifying similarities and differences based on defining and non-defining attributes.	• Explain the difference between defining attributes (e.g., sides, angles, faces) and non-defining attributes (e.g., color, orientations, overall size). • Identify two-dimensional shapes including rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles • Identify three-dimensional shapes cubes, right rectangular prisms, right circular cones, and right circular cylinders. • Construct and draw a shape when given defining attributes. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
1.G.A.2: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.	SMP 1: Students can make sense of problems and persevere in solving them by recognizing created composite shapes as a combination of single shapes. SMP 5: Students can use tools by using manipulatives, such as pattern blocks, to build composite shapes.	• Create new shapes using two-dimensional and/or three-dimensional shapes. • Identify the name of the composite shape as well as the names of each shape that forms it. • Solve shape puzzles, create shape designs, and maintain a shape as a unit. **Depth of Knowledge:** 2-3

		Bloom's Taxonomy: Apply, Analyze and Create
Standard Text 1.G.A.3: Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of,* and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**Standard for Mathematical Practices** SMP 6: Students can attend to precision by using terms such as halves, fourths and quarters to describe the partitioning of shapes. SMP 7: Students can look for and make use of structure by recognizing that as they create more parts the parts get smaller.	**Students who demonstrate understanding can:** • Partition (divide) a circle and rectangle into two and four equal parts. • Describe the equal parts of a circle and rectangle with words (halves, fourths, and quarters). • Describe the whole by the number of equal parts (e.g., two halves make a whole). • Explain the more equal parts in circle or rectangle, the smaller the parts.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply and Analyze
<u>Previous Learning Connections</u> • Connect to naming regular shapes (squares, circles, rectangles, triangles, hexagons, cubes, cones, cylinders and spheres) and analyzing and comparing these shapes using formal and informal language. **(K.G.1-3)** • Connect to composing simple shapes to form larger shapes. **(K.G.6)**	**<u>Current Learning Connections</u>** • Connect to telling and writing time to the hour and to the half-hour and thinking about equality, including the idea of equal shares. **(1. MD.3)**	**<u>Future Learning Connections</u>** • Connect to working with shapes, drawing and analyzing shapes with a given number of angles and faces and identifying triangles, quadrilaterals, hexagons and cubes. **(2.G.1)** • Connect to partitioning shapes into equal shares, adding in thirds and deepening understanding of part and whole relationship by stating that a whole can be made up of three thirds, four fourths, etc. and that the equal shares of identical wholes do not have to be the same shape. **(2.G.2-3)**
Clarification Statement:		

- 1.G.A.1:

Arches created from prisms



Right rectangular prisms are composed with prisms with right triangle bases. Note that the dimensions of the triangular prism on the top arch differ from the dimensions of that on the right.

[Students] differentiate between **geometrically defining attributes** (e.g., "**hexagons** have six **straight sides**") and **nondefining attributes** (e.g., color, overall **size**, or **orientation**). For example, they might say of this **shape**, "This must go with the **squares**, because all four sides are the **same**, and these are **square corners**. It doesn't matter which way it's turned" (MP3, MP7). They explain why the variants shown earlier (p. 6) are members of familiar shape **categories** and why the difficult distractors are not, and they draw examples and nonexamples of the shape categories (MP7, MP8).

- 1.G.A.2: From the early beginnings of informally matching shapes and **solving** simple shape puzzles, students learn to intentionally **compose** and **decompose plane** and **solid figures** (e.g., putting two **congruent isosceles triangles** together with the explicit purpose of making a **rhombus**), building understanding of **part-whole relationships** as well as the **properties** of the original and **composite shapes**. In this way, they learn to perceive a combination of shapes as a single new shape (e.g., recognizing that two isosceles triangles can be combined to make a rhombus, and simultaneously seeing the rhombus and the two triangles).

Common Misconceptions

● Students may find the terms closed and unclosed (open) confusing.

● Students may have difficulty visualizing or filling in shape puzzles.

● Students may believe the size of the shares is directly related to the number of shares. For example, since there are four fourths in a whole and only two halves in a whole, fourths must be bigger.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying reasoning about shapes and their attributes because students are being introduced to new language that is connected to shapes, such as partition, fourths, halves, and quarters. Rehearsing the new language prior to teaching the concepts will allow students to have the opportunity to be exposed to it prior to it being taught to them.

Pre-teach (intensive) : *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.G.B.4: This standard provides a foundation for work with reasoning about shapes and their attributes because students have been exposed to analyzing and comparing shapes according to their attributes. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with reasoning about shapes and their attributes benefit when learning experiences include ways to recruit interest such as creating an accepting and supportive classroom climate because shapes and attributes can be a challenging concept for some students, so it is important for students to feel safe to take risks as they begin to explore new shapes and their attributes.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with reasoning about shapes and their attributes benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because this will allow students to work together as they begin to learn about different shapes and their attributes. Also, students would be able to work together as they discover how to partition shapes into half, fourths and quarters.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with reasoning about shapes and their attributes benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.) because it would be helpful for students to see the various shapes they are learning and their attributes. Also, students need to see visuals of the different ways a shape can be partitioned into halves, fourths and quarters.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with reasoning about shapes and their attributes benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips <sentence frames for using in an example> because this will provide a level of support for students to start to describe various types of shapes according to their attributes..

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with reasoning about shapes and their attributes benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as offering devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the mathematical practices for the purpose of monitoring and improving because it would be helpful for students to organize the various types of shapes and their attributes in a chart to display their understanding. Also, a chart will be able to provide support to students that need to see how shapes can be partitioned in different ways (halves, fourths and quarters).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on reasoning about shapes and their attributes by revisiting student thinking through a short mini-lesson because students might not have a strong understanding of the various types of attributes the different shapes have. Also, students might need to be exposed to partitioning shapes into fourths, halves and quarters as this is a new concept for 1st graders.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit reasoning about shapes and their attributes by confronting student misconceptions because students might be confused on the various types of attributes of the shapes. Also, students may have misconceptions on how to partition shapes into halves, fourths and quarters depending on the wording that is used.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying reasoning about shapes and their attributes because students can get a deeper understanding of shapes and partitioning shapes when working with an open ended task.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

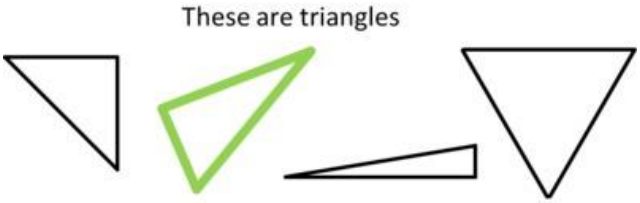
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

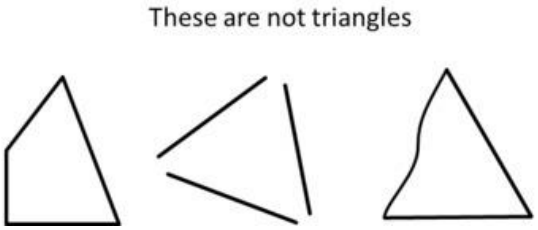
Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and

varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying reasoning about shapes and their attributes the use of mathematical representations within the classroom is critical because promoting collaborative teaching and learning with student-to-student and student-to-teacher dialogues to encourage students' participation. For example, a teacher might plan "turn and talks" during a math lesson to help students discuss their understanding of the content using appropriate terminology.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://tasks.illustrativemathematics.org/content-standards/1/G/A/1/tasks/752
All vs. Only some
First pose the question:

Here are four triangles. What do all these triangles have in common? What makes them different from the figures that are not triangles? What is true for some but not all these triangles?

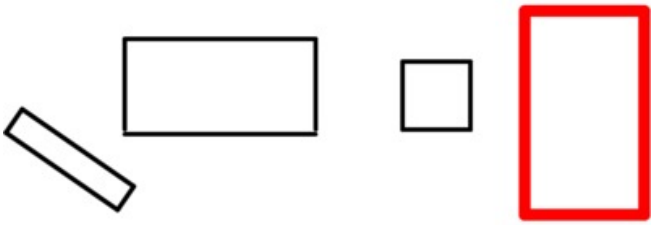




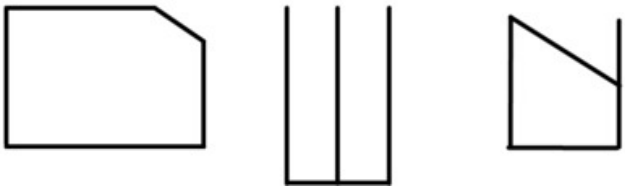
If students come up with a statement that is true about all the triangles that they see but not true of all triangles in general, the teacher should ask students if they can imagine a triangle without that attribute. For example, if a student says, "All of the triangles are white on the inside," the teacher can ask, "Would it be possible for a triangle to have a different color on the inside?" When the class comes up with an attribute that is truly shared by all triangles, then the class can complete the sentence frame: All triangles _____, but only some triangles _____. When the students have written (or composed) their sentences based on the sentence frames, the class can write the definition of a triangle together:

A triangle is a closed shape with three straight sides that meet at three corners.
The teacher will repeat the process for rectangles and then squares. Each time, the class should complete the appropriate sentence frame once they have settled on a universal attribute. Then the teacher can help them compose a definition for the shape.

These are rectangles

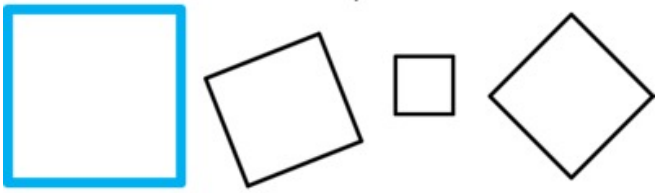


These are not rectangles

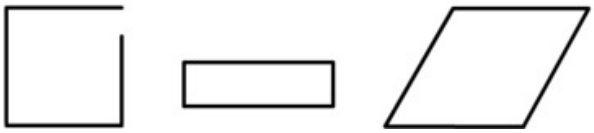


A rectangle is a closed shape with four straight sides and four-square corners.

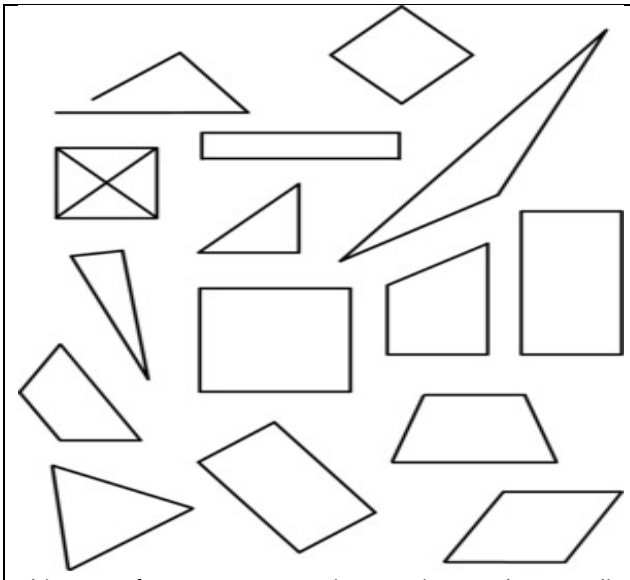
These are squares



These are not squares



A square is a closed shape with four straight sides and four-square corners. The four sides are the same length. Once the class has working definitions in grade appropriate language for these shapes, students can identify the triangles, rectangles, and squares below. * Color all the triangles blue. * Color all the squares red. * Color all the rectangles green.



This type of assessment question requires students to discuss and come to understand what constitute defining attributes for triangles, squares, and rectangles. Students start by looking for attributes shared by all the instances of a shape. Some, but not, all these attributes will be defining attributes. For example, all rectangles have opposite sides parallel, but this isn't a defining attribute--it is something you can show starting only with the defining attributes that a rectangle is a quadrilateral with four right angles. The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics.

Relevance to families and communities:

During a unit focused on reasoning about shapes and their attributes, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, Identify 2D and 3D shapes found at home, in the community, and in nature. Identify the shapes of culturally significant places or even items in the world, what shapes make up that structure/item or that you can derive from the structure. Ex. The Pentagon: what shape is it, and what shapes can you make from it? Pyramids: what shapes are they; what shapes make up the pyramids? How are these different from the Temples in South America? Making connections between the world outside of school and the math classroom. Traffic signs such as a stop sign. Shapes in nature, including a turtle shell and honeycomb.

Cross-Curricular Connections:

Social Studies: In first grade the New Mexico Social Studies Standards state students should "identify and compare celebrations and events from the United States, Mexico, and Canada". Consider providing a connection for students to create images to represent different celebrations and events using only shapes that combine to form the larger image.

Language Arts: Literature can offer connections to help students begin to understand part-whole relationships such as: *Give Me Half* by Stuart J. Murphy and *Picture Pie* by Ed Emberley.

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest:** ? What do you anticipate in the range of student interest for this lesson? ☐ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ⍰ How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ⍰ What do you anticipate about the range in how students will perceive information presented in this lesson? ☐ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ⍰ How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ⍰ What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ☐ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT ⍰ How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ⍰ What do you anticipate about the range in student effort? ☐ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication:** ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation:** ? What do you anticipate about barriers to student engagement? ❑ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension:** ? What do you anticipate about barriers to student comprehension? ❑ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions:** ⍰ What do you anticipate about barriers to students demonstrating what they know? ⍰ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

> **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
>
> **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

> **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
>
> **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
>
> **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

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[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.

Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

CAST (2018). *Universal Design for Learning Guidelines version 2.2.* Retrieved from http://udlguidelines.cast.org

Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

Gifted and talented | NZ Maths. (n.d.). Retrieved from https://nzmaths.co.nz/gifted-and-talented

Gojak, L. M. (Ed.) (2015-2018). *The Common Core Mathematics Companion: The Standards Decoded: What They Say, What They Mean, How to Teach Them (Corwin Mathematics Series)* (1st ed.). Thousands Oak, CA: Corwin.

Gonzales, Lorenzo (2004). *Building personal knowledge of rational numbers through mental images, concepts, language, facts, and procedures.* Unpublished Doctoral Dissertation, New Mexico State University, College of Education, Las Cruces, New Mexico.

Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

Howard County Public School System: Mathematics. (n.d.). Mathematics – HCPSS. Retrieved from https://www.hcpss.org/academics/mathematics/

Illustrative Mathematics. (2014, February 14). Standards for Mathematical Practice: Commentary and Elaborations for K–5. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf

Illustrative Mathematics. (2014b, May 6). Standards for Mathematical Practice: Commentary and Elaborations for 6–8. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/05/2014-05-06-Elaborations-6-8.pdf

Kansas State Department of Education. (2018). 2018 Kansas Mathematics Flip Books. Retrieved from https://community.ksde.org/Default.aspx?tabid=5646

Kaplinsky, R. (n.d.). SIOP Content and Language Objectives for Math. Retrieved from http://robertkaplinsky.com/wp-content/uploads/2017/04/SIOP-Content-and-Language-Objectives-for-Math.pdf

Louisiana Department of Education . (n.d.). K-12 Math Planning Resources. Retrieved from https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Los Angeles, CAUSA. (n.d.). VABB™. Retrieved from https://www.culturallyresponsive.org/vabb

New Visions for Public Schools. (2020). Re-engagement Lessons. Retrieved from https://curriculum.newvisions.org/math/nv-math-team/curriculum-components/re-engagement-lessons/

North Carolina Department of Instruction. (2016–2019). North Carolina Unpacked Content for Mathematics. Retrieved from https://www.dpi.nc.gov/teach-nc/curriculum-instruction/standard-course-study/mathematics

Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD.

South Dakota Department of Education. (n.d.). South Dakota Content Standards - Math. Retrieved from https://doe.sd.gov/contentstandards/math.aspx#ratios+

Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
Student Achievement Partners. (n.d.-a). Coherence Map. Retrieved from https://achievethecore.org/coherence-map/

Trundley, R. (2018). Changing lives and providing equity through pre-teaching and assigning competence. *Association of Teachers of Mathematics (ATM)*, *July*, 31–34. Retrieved from https://www.atm.org.uk/write/MediaUploads/Journals/MT262/MT26213.pdf

Utah Education Network. (2016). Mathematics Core. Retrieved from https://www.uen.org/core/math/

Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.