New Mexico Mathematics Instructional Scope for Second Grade

June 2020

TABLE OF CONTENTS:

Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)
[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands
[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.
[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text Full text of the standard	**Standard for Mathematical Practices** The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	**Students who demonstrate understanding can:** The cognitive skills students perform to demonstrate to comprehension of a standard. **Depth Of Knowledge:** Correlation of standard to Webb's Depth of Knowledge **Bloom's Taxonomy:** Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction

Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach

Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive) : Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities: Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	**Cross Curricular Connections:** Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

TABLE OF CONTENTS:

NEW MEXICO
Public Education Department

2.OA:OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: A: Represent and solve problems involving addition and subtraction.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.OA.A.1 **Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.**	SMP 1: Students can make sense of problems and persevere in solving them by visualizing what is happening in the problem and how the components are related. SMP 2: Students can reason abstractly and quantitatively by using numbers and symbols to represent quantities. SMP 4: Students can model with mathematics by using pictures, number lines and other representations to model and solve problems.	• Identify the unknown in an addition or subtraction word problem. • Determine operation needed to solve addition and subtraction problems in situations including add to, take from, put together, take apart, and compare. • Use drawings or equations to represent one- and two-step word problems. • Add and subtract within 100 to solve one-step word problems with unknowns in all positions. • Write an addition and subtraction equation with a symbol for the unknown.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand and Apply
<u>Previous Learning Connections</u> • Connect to experiences solving all addition and subtraction problems types. Initially, the meaning of addition is separate from the meaning of subtraction. The problems are limited to numbers within 20 and one-step problems **(1.OA.1)**.	**<u>Current Learning Connections</u>** • Connect to relationships between addition and subtraction. Students apply strategies to solve one- and two-step addition/subtraction problems within 100 to length situations **(2.MD.5, 2.MD.6)** and to problems with bar graphs (2.MD.10). • Connect to the developing understanding of the meaning of operations and computational fluency **(2.OA.2).**	**<u>Future Learning Connections</u>** • Connect to applying strategies to one and two-step problems involving the four operations (addition, subtraction, multiplication, division). Student will learn to use a letter to represent an unknown **(3.OA.8).**

Clarification Statement:
Second Grade students extend their work with addition and subtraction word problems in two major ways. First, they represent and solve word problems within 100, building upon their previous work to 20. In addition, they represent and solve one and two-step word problems of all three types (Result Unknown, Change Unknown, Start Unknown).
As second grade students solve one-and two-step problems, they use manipulatives such as snap cubes, place value materials (groupable and pre-grouped), ten frames, etc.; create drawings of manipulatives to show their thinking; or use number lines to solve and describe their strategies. They then relate their drawings and materials to equations. By solving a variety of addition and subtraction word problems, second grade students determine the unknown in all positions (Result Unknown, Change Unknown, and Start Unknown). Rather than a letter ("n"), boxes or pictures are used to represent the unknown number.
Second Graders use a range of methods, often mastering more complex strategies such as making tens, doubles, and near-doubles for problems involving addition and subtraction within 20. Moving beyond counting and counting-on, second grade students apply their understanding of place value to solve problems.

Common Misconceptions

- Students might assume all separate problems are subtraction and all join problems are addition and not pay attention the part of the problem that is unknown.
- Students may only solve one part of a two-part problem because they are used to only performing one operation to find the answer to a problem.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying representing and solving problems involving addition and subtraction because there a common misconception when students begin to bridge their skills to working within word problems. Students can benefit from analyzing common errors and misunderstandings and connecting them to their own. For example, students can perform and error analysis with an anonymous work sample or mock work sample, looking for patterns in misconceptions

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1 NBT.C4: This standard provides a foundation for work with representing and solving problems involving addition and subtraction because adding and subtracting within 100 fluently will help students move to within 1000. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with representing and solving problems involving addition and subtraction benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives and modeling instruction because students who make relevant connections to their everyday lives may establish appropriate cognitive learning. Proper modeling may assist students in adopting the teacher's thinking process, creating their own reasoning as well as acquiring proper mathematical word problem solving skills.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and solving problems involving addition and subtraction benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because students need to understand the learning goal is not always finding the answer, but sometimes the learning goal is finding the entry point or extracting information.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and solving problems involving addition and subtraction benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students will have a better understanding for extracting information from the problem if they understand the language used within addition and subtraction word problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representing and solving problems involving addition and subtraction benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students will have a better understanding for extracting information from the problem if they understand the language used within addition and subtraction word problems.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with representing and solving problems involving addition and subtraction benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because organizing the information within a real life context allows students to find an entry point and organizational strategy for sorting through pieces of given information in order to determine whether the problem requires addition or subtraction, or a total verses a missing number.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representing and solving problems involving addition and subtraction by providing specific feedback to students on their work through a short mini-lesson because it is imperative that students' understand their own learning, misconceptions and ideas.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a representing and solving problems involving addition and subtraction unit by offering opportunities to understand and explore different strategies because one strategy does not fit all learners. For example, some students may benefit from breaking the numbers apart into place value pieces for spatial disconnects.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying representing and solving problems involving addition and subtraction because students can benefit from looking at numbers in different ways. For example, if a student is finding the sum of 10 and 9 students can be asked other ways to find that sum other than adding the two numbers.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying representing and solving problems involving addition and subtraction the types of mathematical tasks are critical because not all students learn procedurally. Some students need to know the why before the how. Some like to know the how before the why All students should be encouraged to explain the relationship between place value and regrouping and how this applies to the procedures for addition and subtraction.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/2/OA/A/1/tasks/1
This type of assessment question requires students to read word problems, identify unknown numbers in all positions and determine whether to add or subtract to solve. SMP1 is important here as students must make a plan for the solution, and a meaning of their solution**.**
This task allows you to assess solving for unknown in different positions, adding/subtracting 2-digit numbers (place value) and making meaning of problem and solution.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on representing and solving problems involving addition and subtraction consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, letting students experiment with drawing and solving representations that are	Social Studies: Students can connect the idea of solving multi-step problems to analyzing information from tables and graphs.

familiar to them can benefit their understanding of the concept of addition and subtraction.	

NEW MEXICO
Public Education Department

2.OA:OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: Add and subtract within 20.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.OA.B.2 **Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.**	SMP 2: Students can reason abstractly and quantitatively by using number relationships and previously mastered facts to solve more difficult facts. SMP 6: Students can attend to precision by accurately solving the problem.	• Efficiently choose mental strategies for addition and subtraction within 20. • Quickly recall all sums of two one-digit numbers. • Fluently add and subtract within 20.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Remember
<u>Previous Learning Connections</u> • Connecting to adding and subtracting within 20, demonstrating fluency for addition and subtraction within 10 and using strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the	**<u>Current Learning Connections</u>** • Connect to strategies to mentally/fluently solve addition/subtraction problems within 20. **(2.OA.2a)** • Connect to knowing from memory one digit plus one-digit math facts. **(2.OA.2b)**	**<u>Future Learning Connections</u>** • Connect to the future work of fluently adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **(3.NBT.2)**

known equivalent 6 + 6 + 1 = 12 + 1 = 13). **(1.OA.6)**		

Clarification Statement:
This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 20. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Mental strategies help students make sense of number relationships as they are adding and subtracting within 20. The ability to calculate mentally with efficiency is very important for all students. Mental strategies may include the following:

- Counting on
- Making tens (9 + 7 = 10 + 6)
- Decomposing a number leading to a ten (14 – 6 = 14 – 4 – 2 = 10 – 2 = 8)
- Fact families (8 + 5 = 13 is the same as 13 - 8 = 5)
- Doubles
- Doubles plus one (7 + 8 = 7 + 7 + 1)

Common Misconceptions

- Students may assume they must memorize all the facts rather than working to see the patterns and relationships that will help them flexibility and fluently solve a problem.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching mental strategies that support fluently adding and subtracting within 20 by introducing new representations (e.g., number lines) because targeted pre teaching ssist students in making new connections to background knowledge while introducing new content.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.OA.6: This standard provides a foundation for work with fluently adding and subtracting within 20 by focusing first on adding and subtracting within 10.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with fluently adding and subtracting within 20 using mental strategies benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities. It is also easier to access and comprehend for many others when teacher displays information in a flexible format to vary perceptual features. Students may require the use of counting strategies such as mental counting with fingers, memorization, repetition, connections, flashcards, and other tools to facilitate perception then internalization. Total physical response may facilitate this process by cuing the students with physical movement, this may include counting songs, dancing, and jumping when adding to create neurological connections.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with fluently adding and subtracting within 20 using mental strategies benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as prompting or requiring learners to explicitly formulate or restate learning goals because by establishing personalized goals for learning addition of two one-digit numbers, students can work and demonstrate learning at their own pacing with scaffolded guided expectations.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with fluently adding and subtracting within 20 using mental strategies, know from memory all sums of two one-digit numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because students can make connections through anchor charts, cognates, and other pictorial and verbal representations.

Internalize

Comprehension: How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with fluently add and subtract within 20 using mental strategies benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because throughout the day. Provide a pictorial support or other tools to assist students in recalling. Use associative memory to connect to previously practiced sums

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on fluently add and subtract within 20 using mental strategies by revisiting student thinking through a short mini-lesson because the teacher can address students' misconceptions, academic and cognitive gaps, and target instruction through a student centered approach in a small group setting.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit during a unit on fluently add and subtract within 20 using mental strategies by offering opportunities to understand and explore different strategies because offering different strategies as an intensive reteach can support students as they internalize the concept of using addition and subtraction and their relationship.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying during a unit on fluently add and subtract within 20 using mental strategies could extend their understanding of the relationship between addition and subtraction to consider patterns in adding and subtracting larger numbers.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, during a unit on fluently add and subtract within 20 using mental strategies, goal setting is critical because while this standard covers a long-term goal, students may set independent goals for academic achievement.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/2/OA/B/2/tasks/1394
This type of assessment question requires students to understand the relationships between numbers, why using a number line or counters helps find the sum or difference and move through different strategies from concrete to abstract. SMP 7 is important to help students look for and make use of structure by using known patterns and facts.
You could use this task to inform and adjust instruction if necessary. This task allows you to assess student's efficiency and flexibility in solving problems using previous understandings. It will also give information on where their understanding is in development of numbers and place value.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on using addition and subtraction within 100 to solve one- and two-step word problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different mental strategies to add and subtract within 100 to solve one- and two-step word problems, students are more successful when strategies include examples that are culturally relevant and familiar. Students will retain more knowledge when interacting with the concept in a more meaningful way.	Music: Students can practice fluency with addition facts through the use of music.

NEW MEXICO
Public Education Department

2.OA:OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: C: Work with equal groups of objects to gain foundations for multiplication.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.OA.C.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	SMP 3: Students can construct viable arguments by proving whether a number is odd or even and critiquing the reasoning of others. SMP 4: Students can model with mathematics by using objects, drawings, numbers and/or equations to show whether a number is odd or even.	Recognize that in groups of even numbers objects will pair up evenly. Determine whether a group of objects is odd or even, using a variety of strategies. Generalize the fact that all even numbers can be formed from the addition of 2 equal addends. Count a group of objects up to 20 by 2s. Write an equation to express a given even number as a sum of two equal addends.
		Depth Of Knowledge: 1
		Bloom's Taxonomy: Understand and Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	SMP 4: Students can model with mathematics by writing an equation to go with an array. SMP 8: Students look for and express regularity in repeated reasoning by using repeated addition to begin thinking about multiplication.	Write an equation with repeated equal addends from an array. Generalize the fact that arrays can be written as repeated addition problems. Solve repeated addition problems to find the number of objects using rectangular arrays.

		Depth Of Knowledge: 1-2
		Bloom's Taxonomy: Understand and Apply
Previous Learning Connections Connect to decomposing numbers within 10 into pairs. (K.OA.3) and with addition of numbers within 20, specifically with doubles. (1.OA.6)	**Current Learning Connections** Connect to their understanding of number groupings to skip count within 1000. (2.NBT.2) and partitioning rectangles into rows and columns of same-size squares and count to find the total number of them. (2.G.2)	**Future Learning Connections** Connect to future work with understanding of odd and even to develop and apply more sophisticated mathematical arguments and proofs (MP3, MP4) and relating arrays to equal group situations. (3.OA.1)

Clarification Statement:

Second graders apply their work with doubles to the concept of odd and even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends or doubles addition facts (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Second graders use rectangular arrays to work with repeated addition, a building block for multiplication in third grade. A rectangular array is any arrangement of things in rows and columns, such as a rectangle of square tiles. Students explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Due to the commutative property of multiplication, students can add either the rows or the columns and still arrive at the same solution.

Common Misconceptions

- Students may struggle with determining if a number greater than 9 is even or odd if they simply memorize a rule about digits because they won't understand whether 23 is even or odd based on the digits of 2 and 3 in the number.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying equal groups of objects to gain foundations for multiplication because working with equal groups of objects can be used for students to understand how these facts are derived and allows them to make those connections to multiplication.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.OA.D.7: This standard provides a foundation for work with equal groups of objects to gain foundations for multiplication because students need the skill of adding and subtracting to understand the connection how repeated addition can represent the action of adding equal groups or rows/columns.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with working with equal groups of objects to gain foundations for multiplication benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in

creative ways because working with equal groups of objects can be used with novel and relevant problems in a way that is meaningful for students to understand how these facts are derived and allows them to make those connections to multiplication.

Build

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with working with equal groups of objects to gain foundations for multiplication benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because developing a good understanding of what multiplication is and means is a prerequisite for moving onto basic multiplication facts. Students can use vocabulary and symbols to better understand that multiplication is repeated addition by making equal groups of objects with the help of pictures.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with working with equal groups of objects to gain foundations for multiplication benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) because building connections to prior understandings strengthens number sense and expands their thinking processes to see numbers in groups. These connections use prior knowledge to relate addition to multiplication.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on working with equal groups of objects to gain foundations for multiplication by clarifying mathematical ideas and/or concepts through a short mini-lesson because clarifying ideas or concepts are a targeted re-engagement that can support students understanding of the relationship between operations and clarifies their distinction as they internalize the content while still maintaining the flow of the unit.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit working with equal groups of objects to gain foundations for multiplication by offering opportunities to understand and explore different strategies because offering different strategies as an intensive reteach can support students as they internalize the concept of grouping equal groups in using repeated addition for multiplication.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying working with equal groups of objects to gain foundations for multiplication because students could extend their understanding of the relationship between multiplication to use their understanding of the properties of operations to multiply and divide.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying working with equal groups of objects to gain foundations for multiplication the pattern of questions within the classroom is critical because students are gaining knowledge of conceptual understanding, procedural skills, and fluencies that lead into multiplication. Students need to make that connection to the relationship between operations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/2/OA/C/3/tasks/1418
This type of assessment question requires students to understand numbers and the patterns that make them even or odd and use the patterns to help solve problems more efficiently. You can assess if a student can write an equation for an even number using two equal addends and if equation for odd number has 2 equal addends plus 1.
You could use this task to inform and adjust instruction if necessary. This task allows you to assess students' understandings of even odd and patterns.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on Working with equal groups of objects to gain foundations for multiplication, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example developing real world connections that are culturally relevant to students make more meaningful interactions for students to engage and conceptualize new ideas and vocabulary.	Art: Students can study the use of arrays and even and odd in the creation of pieces of art.

NEW MEXICO
Public Education Department

2.NBT:NUMBER & OPERATIONS IN BASE TEN

Cluster Statement: A: Understand place value.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.NBT.A.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones, e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: • 2.NBT.A.1.A: 100 can be thought of as a bundle of ten tens — called a "hundred." • 2.NBT.A.1.B: The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	SMP 7: Students look for and make use of structure that each hundred is that number of hundreds (100 is 1 hundred) with 0 tens and 0 ones. SMP 8: Students look for and express regularity in repeated reasoning by understanding that every time they have 10 of a particular item, they group it to make the next place value unit.	• Count by 10s, ten 10s equals 100, ten 1's equals 10. • Explain the names of places (ones, tens, hundreds) and how the place impacts the value of the digit. • Explain the difference and relationship between value and place. • Use base ten blocks to model numbers. • Represent place values with pictures or representations. • Utilize a place value chart to determine and identify places and values for digits in a three-digit number. **Depth Of Knowledge:** 1 **Bloom's Taxonomy:** understand
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s.	SMP 7: Students look for and make use of structure by using known patterns and facts. SMP 8: Students look for and express regularity in repeated reasoning when skip counting as a pattern of regularity.	• Count using a 100 chart or number line to explain patterns and to skip count to 1000 by 5s, 10s, and 100's. • Write in or verbally say missing numbers in a skip counting pattern • Describe place value patterns when skip counting **Depth of Knowledge:** 1

		Bloom's Taxonomy: understand
Standard Text 2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**Standard for Mathematical Practices** SMP 7: Students look for and make use of structure of base-ten numeral, number name, and expanded form patterns.	**Students who demonstrate understanding can:** • Explain the difference between expanded form and standard form. • Write numbers out with words. • Read and write numbers up to 1,000 using base-ten numerals (e.g., 234) • Read and write numbers up to 1,000 using number names (e.g., two hundred thirty-four). • Read and write numbers using expanded form (e.g., 200 + 30 + 4). • Decompose numbers using expanded form. • Record number decompositions in various ways (i.e. 234 as 230 + 4, 199 + 35, 200 + 34, or 225 + 14)
		Depth of Knowledge: 1
		Bloom's Taxonomy: understand, apply
Standard Text 2.NBT.A.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively through comparisons and recording with symbols. SMP 3: Students construct viable arguments and critique the reasoning of others by defining place value in the digits. SMP 7: Students look for and make use of structure of place value for three-digit numbers.	**Students who demonstrate understanding can:** • Compare two three-digit numbers. • Use inequality symbols to write comparisons about two three-digit numbers. • Explain how two numbers compare based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols.
		Depth of Knowledge: 1-2

		Bloom's Taxonomy: understand, apply
Previous Learning Connections • Connect to students will count to 120, starting with any number less than 120. **(1.NBT.1)** • Connect to understand place value of ones and tens in two-digit numbers. **(1.NBT.2)**	**Current Learning Connections** • Connect the skills from within this cluster to represent and solve addition and subtraction, 2-step word problems. **(2.OA.1)**	**Future Learning Connections** • Connect to interpret the products of whole numbers, such as interpreting 7 x 5 as the total number of objects in 7 groups of 5 objects each. **(3.OA.1**) • Connect to use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. **(3.OA.3)**

Clarification Statement:
In second grade, students continue to develop a deep understanding of **place value** and use that understanding to add and subtract within 1,000. This cluster focuses on the development of place value up to and beyond 100. Students should use the structure of building **tens** out of 10 ones, building **hundreds** out of 10 tens, and building a **thousand** out of 10 hundreds. This is the structure of our **base-ten** place value system. It is built on repeated reasoning that every time you have 10 of a particular item, you group it to make the next place value **unit**. Students use precision in describing their work with appropriate vocabulary and reading numbers accurately. They explain their reasoning to classmates throughout the cluster and compare their thinking with that of their peers.

Common Misconceptions
Students may struggle with grouping or bundling, one-to-one correspondence, or skip-counting which will impact their work with place value.

Students may confuse directionality of symbols, thus leading to an incorrect comparison of numbers.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding place value because many students struggle with understanding the connection between the place and the value. For example, students can name that the 5 is in the tens place in 653, but then when asked about values they still might say 5. Strategic and purposeful instruction regarding the most common misconceptions can benefit students.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1NBT.B2: This standard provides a foundation for work with understanding place value because it teaches the difference between ones and tens. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with understanding place value will benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as text or spoken for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because allowing students to manipulate objects will help students understand the idea of bundles. Students can manipulate base ten block or mock money.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding place value will benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students will begin to see connections between the different places and values which will benefit them as they approach higher grade level work that requires the foundation of place value understanding.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding place value will benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures> because the number system is based on the number 10 and when students understand that concept they are able to make connections into decimal systems later.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding place value benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because students can manipulate groups of 10 and 100 to assist in understanding the value of the next place.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with understanding place value benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" (e.g., portfolio review, art critiques) because students need to be able to explain their

understanding of the whole place value system and go beyond naming places from memorization.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding place value by examining tasks from a different perspective through a short mini-lesson because students may need to view the number as money, or view the number as a quantity so that they can truly understand the meaning of all digits in a number.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding place value by addressing conceptual understanding because students need to be clear that the number system is based on the number 10. Extra time can be spent showing the relationship between the number one, ten and hundred.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying understanding place value because students can link their ideas to real word situations with comparing numbers.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding place value, the use of mathematical representations within the classroom is critical because although money is a good representation for place value, it may not be the entry point for all students. Students should be encouraged to use a variety of representations for place value including drawing bundles, grouping items, blocks or tiles.

Standards Aligned Instructionally Embedded Formative Assessment Resources:	
Source: http://tasks.illustrativemathematics.org/content-standards/2/NBT/A/1/tasks/192 This type of assessment question requires students to understand the value of digits, not just the place they are in. They need to understand the difference and relationship between value and place. SMP 7 Look for and make use of structure is important because students need to understand structure of 3-digit number; place of digit and value of each. This task allows you to assess student's ability to break apart and put together numbers (ones to tens and tens to hundreds). They also must understand comparing numbers, equal to hundreds (fill a box). The tasks increase in difficulty and therefor give a breakdown of where struggling student needs support.	
Relevance to families and communities: During a unit focused on understanding place value consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about different places that students see numbers in their communities and real life situations can help them connect to the true meaning of the value of a number.	**Cross-Curricular Connections:** Physical Education: Students can practice skip counting while keeping track of their repetitions when doing activities such as jumping rope.

2.NBT: NUMBER & OPERATIONS IN BASE TEN

Cluster Statement: Use place value understanding and properties of operations to add and subtract.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.NBT.B.5 **Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.**	SMP 2: Students reason abstractly and quantitatively by looking at properties of operations and the relationships of addition and subtraction. SMP 6: Students attend to precision when using knowledge of place value, properties of operations, and/or the relationship between addition and subtraction. SMP 7: Students look for and make use of structure of place value and/or the relationship between addition and subtraction.	• Apply different strategies to add and subtract within 100. • Explain the relationship between addition and subtraction to solve problems. • Explain different strategies for adding and subtracting. • Use place value to add and subtract. • Create concrete models, or drawings and the following strategies to add within 100; place value, properties of operations and the relationship between adding and subtracting. • Utilize number lines, blocks, or items to find unknown numbers.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: understand, apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.NBT.B.6 **Add up to four two-digit numbers using strategies based on place value and properties of operations.**	SMP 6: Students attend to precision when using knowledge of place value, properties of operations, and/or the relationship between addition and subtraction. SMP 7: Students look for and make use of structure when adding based on place value and properties of operations.	• Model addition using number lines, base ten blocks or other objects, and hundreds charts. • Add up to four two-digit numbers • Explain strategies for adding multiple two-digit numbers. • Explain why addition and subtraction strategies work for adding four two-digit numbers, by citing place value and the properties of operations.

		Depth of Knowledge: 1-2
		Bloom's Taxonomy: understand, apply
Standard Text **2.NBT.B.7** **Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.**	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively as adding and subtracting within 1000 using models, drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. SMP 3: Students construct viable arguments and critique the reasoning of others when justifying the reasoning used with a written explanation. SMP 6: Students attend to precision when using knowledge of place value, properties of operations, and/or the relationship between addition and subtraction. using concrete models or drawings to add and subtract within 1000**.**	**Students who demonstrate understanding can:** • Use models to add and subtract within 1000. • Explain the relationship between addition and subtraction and use the relationship to solve problems. • Apply place value understanding to add and subtract each place value. Explain different strategies for adding and subtracting. • Utilize concrete models or drawings and the following strategies to add within 1000; place value, properties of operations and the relationship between adding and subtracting.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: understand, apply
Standard Text **2.NBT.B.8** **Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.**	**Standard for Mathematical Practices** SMP 7: Students look for and make use of structure that the only place value that changes is the tens or hundreds place when mentally adding 10 or 100 to a given number. SMP 8: Students look for and express regularity in repeated reasoning when the only place value that changes is the tens or hundreds place when mentally adding 10 or 100 to a given number.	**Students who demonstrate understanding can:** • Use place value to mentally add 10 or 100 from any number 100-900. • Mentally subtract 10 or 100 from any number 100-900 using place value.
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: understand, apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.NBT.B.9 **Explain why addition and subtraction strategies work, using place value and the properties of operations.**	SMP 3: Students construct viable arguments and critique the reasoning of others to explain why addition and subtraction strategies work**.**	• Explain using place value • Apply properties of addition and subtraction • Explain why the addition and subtraction strategies work **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
Previous Learning Connections • Connect to add with one- and two-digit numbers and multiples of 10 within 100 using models and strategies and use the concepts of place value, properties of operations, and the relationship between addition and subtraction. (**1.NBT.4**) (**1.NBT.5**) • Connect to subtract by units of 10 using properties of operations, place value, and addition/subtraction relationship. (**1.NBT.6**) • Connect to apply properties of operations as strategies to add and subtract. (**1.OA.3**) • Connect to understanding subtraction as an unknown-addend problem. (**1.OA.4**)	**Current Learning Connections** • Connect to represent and solve addition and subtraction, 2-step word problems within 100. (**2.OA.1**) • Connect to fluently add and subtract within 20 using mental strategies. (**2.OA.2**) • Connect to understanding that the three digits of a three-digit number represent specific amounts (**2.NBT.1**). • Connect to fluently and subtract within 100 using strategies, place value, and relationship between addition and subtraction (**2.NBT.5**). • Connect to using addition and subtraction within 100 to solve word problems involving lengths (**2MD.5)**	**Future Learning Connections** • Connect to solve two-step word problems. (**3.OA.8**) • Connect to fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (**3.NBT.2**)

Clarification Statement:
In 2nd grade, students use place value understanding and properties of operations to add and subtract. They also use strategies based on number sense, mental mathematics, and the relationship between addition and subtraction to solve problem situations with sums to 100.

Common Misconceptions
Students with difficulties with place value reasoning skills may have difficulty regrouping when solving addition and subtraction algorithms.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some students may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying place value and properties of operations to add and subtract because making the jump from identifying places and values to adding and subtracting may take additional time. In

addition, students need time to find entry points for word problems and real-life contexts.

Pre-teach (intensive): What critical understandings will prepare students to access the mathematics for this cluster?

- 1 NBT.C 4: This standard provides a foundation for work with place value understanding and properties of operations to add and subtract because if students can add within 100 their connections to adding within 1,000 will be smoother. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, students engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences ensure information is accessible to students through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because some students may need to draw out representations of the place value to assist with addition and subtraction. It is essential student understand beyond "crossing off and borrowing" and drawing it out or physically manipulating place value tools may help the concepts of understanding the process of regrouping.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, students engaging using place value understanding and properties of operations to add and subtract benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because misunderstandings frequently practiced can result in students needing to "unlearn" a procedure or understanding.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all students? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for students from differing cultural or familial backgrounds)*

- For example, students engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all students such as making connections to previously learned structures because students that understand the value of the places and bundles can make a stronger connection to the concept of regrouping.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, students engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because there are different ways to add and subtract

numbers within 1000 such as traditional, breaking apart place value and decomposing numbers.

Internalize
Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, students engaging with using place value understanding and properties of operations to add and subtract benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), elevating the frequency of self-reflection and self-reinforcements because students can benefit from knowing and understanding the most common errors students make within this cluster, and reflecting on their most common errors.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding properties of operations to add and subtract by revisiting student thinking through a short mini-lesson because it will be helpful to determine if students have a grasp of the basic place value system before expecting addition and subtraction with larger numbers.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on using place value understanding and properties of operations to add and subtract by confronting student misconceptions because students may still need clarification regarding the meaning of regrouping. It is imperative students understand what they are taking or giving when they apply the process of regrouping.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some students may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying understanding and properties of operations to add and subtract because students can benefit from the chance to determine reasonability of their answers using what they know about place value. For example, students can explore estimation strategies.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves

mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying using place value understanding and properties of operations to add and subtract because there are multiple ways to add and subtract including traditional method, breaking apart place value and decomposing numbers. Students should be encouraged to explore different methods, compare methods and choose methods that promote their success. Students should be encouraged to articulate how methods are related.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: http://tasks.illustrativemathematics.org/content-standards/2/NBT/B/7/tasks/1063

This type of assessment question requires students to understand different strategies can be used to fluently add and subtract. Numbers can be decomposed into many values. Students can use the relationship between addition and subtraction to solve problems. Students can explain different strategies for solving. SMP 6 Attend to precision is important when using knowledge of place value, properties of operations, and/or the relationship between addition and subtraction

This task allows you to assess students understanding of place value and understanding of addition and subtraction.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on using place value understanding and properties of operations to add and subtract consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning when in real life situations students may be required to add and subtract can give students a sense of connection and understanding.	Science: Students can mentally add and subtract measurements rounded to the nearest ten or one hundred.

2.MD: MEASUREMENT & DATA

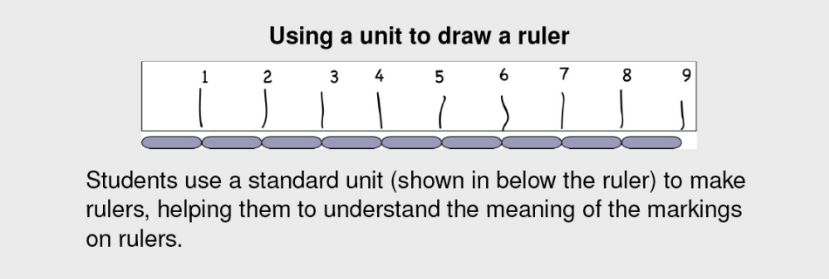
Cluster Statement: A: Measure and estimate lengths in standard units.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.MD.A.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	SMP 5: Students can use tools strategically by selecting an appropriate measurement tool (rulers, yardsticks, meter sticks, and measuring tapes) for various contexts. SMP 6: Students can attend to precision by measuring accurately to the designated unit.	• Identify the location of zero, not always at the end of the measuring tool, and explain the role of zero in measuring an object. • Recognize that the numbers on measurement tools are equally spaced. • Identify that the length of an object is the number of same size units the object spans with no gaps or overlaps. • Choose an appropriate tool to measure the length of objects. • Explain why a specific measurement tool is best for a given situation. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand and Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.MD.A.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	SMP 3: Students can construct viable arguments by describing how the quantity of a measurement relates to the size of the unit chosen and critique the reasoning of others as they explain these ideas. SMP 5: Students can use tools strategically by selecting an appropriate measurement tool (rulers, yardsticks, meter sticks, and measuring tapes) for various contexts.	• Use the terms length and measure precisely. • Measure an object using two different tools. • Describe two different measurements of one object and the relationship between the measurement and the unit length. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Remember, Understand and Apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.MD.A.3: Estimate lengths using units of inches, feet, centimeters, and meters.	SMP 2: Students can reason abstractly and quantitatively by using units to help reason about estimates.	• Use mental and visual information to determine a measurement without using a measurement tool. • Justify the reasonableness of an estimate. • Choose an appropriate unit of measurement to estimate. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and Evaluate
Standard Text 2.MD.A.4: Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit.	**Standard for Mathematical Practices** SMP 5: Students can use tools strategically by selecting an appropriate measurement tool (rulers, yardsticks, meter sticks, and measuring tapes) for various contexts**.**	**Students who demonstrate understanding can:** • Choose an appropriate tool to measure the length of objects. • Compare two quantities by finding their difference in length. • Explain how many units longer one object is compared to another object. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply, Analyze and Evaluate
<u>Previous Learning Connections</u> • Connect to the process of measuring objects using non-standard units, such as cubes or paperclips. **(1.MD.1)** • Connect to the concept of comparing objects, limited to comparative terms such as shorter, longer, shortest, and longest. **(1.MD.2)**	**<u>Current Learning Connections</u>** • Connect to the transition from use of non-standard units to standard units of measure in this cluster. • Connect to the use addition and subtraction strategies as they compare the length of objects and determine the difference. **(2.NBT.5)** • Connect to the understanding of linear measurement to solve word problems. **(2.MD.5)**	**<u>Future Learning Connections</u>** • Connect to generate measurement data by measuring objects and showing the measurements on a line plot **(2.MD.9)** • Connect to sub-divide lengths as they use rulers marked with halves and fourths of an inch. **(3.MD.3)** • Connect to applying linear measurement to measure perimeter and area. **(3.MD.5-8)**
Clarification Statement: • 2.MD.A.1: It is vital that students learn that "one" represents the space from the beginning of the ruler to the hash mark, not the hash mark itself.		

- 2.MD.A.1: To learn measurement concepts and skills, students might use both simple rulers (e.g., having only whole units such as centimeters or inches) and physical units (e.g., manipulatives that are centimeter or inch lengths).



- 2.MD.A.2: Students can learn that the larger the unit, the fewer number of units in a given measurement. That is, for measurements of a given length there is an **inverse relationship** between the size of the unit of measure and the number of those units. This is the time that measuring and reflecting on measuring the same object with different units, both **standard and nonstandard**, is likely to be most productive.
- 2.MD.A.2: Students can also use the concept of unit to make inferences about the relative sizes of objects; for example, if object A is 10 regular paperclips long and object B is 10 jumbo paperclips long, the number of units is the same, but the units have different sizes, so the lengths of A and B are different
- 2.MD.A.3: Although **"guess and check"** experiences can be useful, research suggests explicit teaching of **estimation** strategies (such as iteration of a mental image of the unit or comparison with a known measurement) and prompting students to learn **reference or benchmark lengths** (e.g., an inch-long piece of gum, a 6-inch dollar bill), order points along a continuum, and build up mental rulers.
- 2.MD.A.4: Second graders learn to combine and compare lengths using arithmetic operations. That is, they can add two lengths to obtain the length of the whole and subtract one length from another to find out the difference in lengths.

Common Misconceptions

- Students beginning the measurement count at the "1" mark on a ruler rather than the "0" and not accounting for it.
- Students not iterating the unit accurately, leaving gaps or overlapping
- Students selecting the incorrect unit of measure (e.g., when asked for centimeters, learner measures in inches)
- Students believing the numbers on a ruler are counting the marks as opposed to the space between the marks.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying measuring and estimating lengths in standard units (inches, feet, centimeters, and meters) by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes; describing how two measurements relate to the size of the unit chosen and measuring to determine how much longer one object is than another because students used non-standard units of measurement in prior grades. Students often misread measurement tools. When provided additional time for confusion to happen with new mathematical ideas, students could experience less stress.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.MD.A.2: This standard provides a foundation for work with measuring and estimating lengths in standard units because students have the opportunity to use non-standard tools and practice lining objects up with no gaps or overlaps. This will help them when using standard tools. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with measuring and estimating lengths in standard units benefit when learning experiences that ensure information is accessible to learners through a variety of methods for navigation, such as physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because as students engage with measuring and estimating lengths some may need alternatives measuring tools provide greater accessibility for students.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with measuring and estimating lengths in standard units benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because as students work in a community of learners, they can discover measurement properties and engage in discussions relevant to the learning outcome.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with measuring and estimating lengths in standard units benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because key vocabulary, labels, icons, and symbols should be linked to, or associated with, alternate representations of their meaning to ensure accessibility for all. Students cultural and linguistic backgrounds may not include the units of measurement customarily used within the United States (e.g., inch, foot), and all students benefit from making all key information available in English as well as first languages.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with measuring and estimating lengths in standard units benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because allowing for multiple modalities for students to express their understanding allows for stronger formative assessment of what students are learning.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with measuring and estimating lengths in standards units benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because the measurement standards for length provide a critical stepping stone to future learning for all types of measurement in later grades, helping students to set goals and learn from their mistakes will lead to stronger long term learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on measuring and estimating lengths in standard units by providing specific and immediate feedback to students on their work through a short mini lesson because standard units and estimation require practice and precision.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit measuring and estimating lengths in standard units by addressing conceptual understanding because measurement involves spatial reasoning. Some learners may need additional guidance and practice in measurement and estimating measurement.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying measuring and estimating lengths in standard units (inches, feet, centimeters, and meters) by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes; describing how two measurements relate to the size of the unit chosen and measuring to determine how much longer one object is than another. This is helpful in the instruction of science and social studies mapping for students to be proficient in estimating lengths for possible projects and for distances on a map.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Goal Setting): Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which

can create a vicious cycle of low expectations and low achievement. For example, when studying measuring and estimating lengths in standard units (inches, feet, centimeters, and meters) by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes; describing how two measurements relate to the size of the unit chosen and measuring to determine how much longer one object is than another goal setting is critical because society and cultures often negatively portray students' ability to measure and estimate lengths. For example, the media belittles and creates jokes regarding the fast food employee who doesn't comprehend that a "foot long" menu item and a "twelve-inch" menu item are the same length. As learners set challenging, but attainable goals, they can engage with interesting and rigorous mathematical content of measurement and estimating lengths to achieve in mathematics.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/2/MD/A/tasks/1313
The linked assessment question addresses 2.MD.A.2, specifically describing two different measurements of one object and the relationship between the measurement and the unit length. Students can demonstrate their understanding of the idea of a standard unit of measure and why we use them. The tasks also reinforce the idea that we measure lengths by placing units end-to-end. Teachers can use this formative assessment task early in a unit on measurement to assess if students understand the importance of measurement and common units.

Relevance to families and communities:
During a unit focused on measuring and estimating lengths in standard units (inches, feet, centimeters, and meters) by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes; describing how two measurements relate to the size of the unit chosen and measuring to determine how much longer one object is than another, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways measurement and estimating lengths is used in the home and community can a be a great way to connect schools tasks with home tasks. Students may have different non-standard units of measurement that are used in the home and community. Learners may also have examples of how measurement and estimation of measurements is used in the home and the community (e.g., the length of a garden, the linear length of material for fencing or sewing).

Cross-Curricular Connections:
Science: In second grade the NGSS states students should "plan and conduct an investigation to determine if plants need sunlight and water to grow." Consider providing a connection for students to measure plant growth.

Language Arts: Literature can offer connections about measurement such as: *How Big is a Foot* by Rolf Myller and *How Short, How Far Away?* by David A. Adler.

NEW MEXICO
Public Education Department

2.MD: MEASUREMENT & DATA

Cluster Statement: Relate addition and subtraction to length.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.MD.B.5: Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	SMP 1: Students can make sense of problems and persevere in solving them by interpreting and solving word problems involving lengths using manipulatives or drawings. SMP 4: Students can model with mathematics by applying addition and subtraction to solve problems.	• Recognize that when adding or subtracting lengths, the measurement units must be the same. • Represent measurement problems using drawings and equations. • Solve length word problems by adding or subtracting measurements within 100. • Write an equation for a word problem involving length and represent the unknown number with a symbol. **Depth of Knowledge: 2** **Bloom's Taxonomy:** Remember, Apply and Evaluate
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.MD.B.6: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	SMP 2: Students can reason abstractly and quantitatively by representing whole numbers and whole number sums and differences on a number line diagram. SMP 4: Students can model with mathematics by using a number line diagram to solve addition and subtraction problems.	• Create a number line to show whole numbers using equally spaced marks. • Represent addition and subtraction problems on a number line diagram. • Solve addition and subtraction problems using a number line diagram.

		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Remember, Understand and Apply
Previous Learning Connections • Connect to using open number lines to add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10. (**1.NBT.4)**	**Current Learning Connections** • Connect to solving one- and two-step measurement problems. **(2.OA.1)** • Connect to using open number lines to add and subtract within 1000. Connect to understanding number lines to include a distance model for subtraction. **(2.NBT.7)** • Connect to generating measurement data by measuring objects and showing the measurements on a line plot. **(2.MD.9)**	**Future Learning Connections** • Connect to understanding a fraction as intervals between whole numbers on a number line. **(3.NF.2)**

Clarification Statement:
2.MD.B.5: As an arithmetic example, students might measure all the sides of a table with unmarked (foot) rulers to measure how much ribbon they would need to decorate the perimeter of the table.

2.MD.B.6: To use a number line diagram to understand number and number operations, students need to understand that number line diagrams have specific conventions: the use of a single position to represent a whole number and the use of marks to indicate those positions. They need to understand that a number line diagram is like a ruler in that consecutive whole numbers are 1 unit apart, thus they need to consider the distances between positions and segments when identifying missing numbers.

Common Misconceptions
● Students may have misconceptions about how to express the way they solved a problem using an equation.

● Students may not understand they could jump by more than ones on a number line to add or subtract.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying and relating addition and subtraction to length because it helps with students' understanding by allowing and helping them make connections to the new information. Students can access their schema and experiences to understand the relationship of addition and subtraction to length.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.MD.A.1: Order three objects by length; compare the lengths of two objects indirectly by using a third object. This standard provides a foundation for work with relating addition and subtraction to length because students need to understand comparing to help with addition and subtraction using length measurements. If students have unfinished learning within this standard, based on assessment data, consider ways to provide

intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with relating addition and subtraction to length benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as physically interacting with materials by hand because using a ruler to measure or add/subtract is more concrete and hands-on than looking at picture of a ruler and trying to measure or add/subtract.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with relating addition and subtraction to length benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because it allows for students to make mistakes, figure out how to fix their mistakes, learn new strategies, and allows for peer interactions to enhance learning.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with relating addition and subtraction to length benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge. Pre-teaching vocabulary gives students the opportunity to identify words and then be able to place them in context and remember them and apply it to what they are working on.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with relating addition and subtraction to length benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing text-to-speech software (voice recognition), human dictation, recording because while students may be able to solve a math equation and have the skill set to do so, some students may be low literacy readers and by removing the stress of reading the equation they can focus on solving the equation.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with relating addition and subtraction to length benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because visuals such as graphic organizers are a helpful learning tool for students of all ages to organize, clarify, or simplify complex information—they help

students construct understanding through an exploration of the relationships between concepts. They also are a useful scaffold to support student learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on relating addition and subtraction by providing specific feedback to students on their work through a short mini lesson because providing students with meaningful feedback can greatly enhance their learning and achievement. Re-teaching by providing students with information on what exactly they did well, and what may still need improvement.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on relating addition and subtraction to length by confronting student misconceptions because students will shut down if they think they are doing it wrong. Focusing on the facts and why 'right is right' gives them the detailed steps again to follow to get the answer right. We help them and re-teach them until they get the steps right.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying and relating addition and subtraction to length because it gives students opportunities to teach themselves new skills of how to learn and apply to other content areas. Self-selected topics and learning can improve thinking skills, research skills, self-management skills, social skills, and communication skills and allows for further independent practice.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying and relating addition and subtraction to length the types of mathematical tasks are critical because with the increasing diversity in our classrooms, it is important for teachers to implement culturally relevant pedagogy and challenge students since these tasks are the basis of opportunity for students to learn.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/2/MD/B/6/tasks/1081
This type of assessment question requires students to understand number lines can be used to represent sums from addition and or differences from subtraction. SMP 2, reason abstractly and quantitatively is important because students make sense of quantities and how they are related in a problem situation. During this process, students periodically contextualize the problem by connecting the mathematical symbols back to the context.

You could use this task to inform and adjust instruction if necessary. This task allows you to assess students' understanding of the number line as tool to help solve problems. It can also assess their understanding of equations and 2 different equations being equal even if addends are different (3+5 = 2+6)

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on relating addition and subtraction to length, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways you can measure items in your home and community and then compare lengths (adding/subtracting) is a great way to connect schools tasks with home tasks. To ensure students have the opportunity to engage in high-level thinking and culturally relevant tasks, teachers must select and implement tasks that promote reasoning and problem solving (lower-level tasks are classified as memorization and procedures without connections to understanding, meaning or concepts. Both represent limited opportunities for students to understand important concepts.	Science: In second grade the NGSS states students should "plan and conduct an investigation to determine if plants need sunlight and water to grow." Consider providing a connection for students to measure plant growth and then pose addition and subtraction problems within the context of their growth. Language Arts: Literature can offer connections about measurement such as: *How Big is a Foot* by Rolf Myller and *If You Hopped Like a Frog* by David Schwartz.

2.MD: MEASUREMENT & DATA

Cluster Statement: Work with time and money.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.MD.C.7: Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	SMP 6: Students can attend to precision by using specific vocabulary to describe and tell time to the nearest five minutes. SMP 7: Students can look for and make use of structure by understanding that there are 24 hours in each day consisting of two 12-hour cycles separated into a.m. and p.m.	• Explain the difference between a.m. and p.m. and identify activities appropriate for both. • Read or say the time given a digital or analog clock (minutes displayed as a multiple of 5). • Translate time on an analog clock to a digital clock and vice-versa, including drawing the appropriate hands on an analog clock (minutes displayed as a multiple of 5). • Write the time and draw the appropriate hands on an analog clock when verbally told what time it is to the nearest 5 mi. • Use specialized language such as half past, quarter after/past, quarter to, minutes after/past and minutes to when telling time.
		Depth of Knowledge: 1
		Bloom's Taxonomy: Remember and Understand

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.MD.C.8: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	SMP 1: Students can make sense of problems and persevere in solving them by interpreting what the information in a money word problem means and determining how to solve the problem. SMP 2: Students can reason abstractly and quantitatively by making sense of the value of the coins, e.g., three dimes and two pennies is thirty-two cents. SMP 7: Students can look for and make use of structure by interpreting the addition of mixed coins.	• Identify and name dollar bills, quarters, dimes, nickels and pennies and their values. • Solve word problems involving money. • Use money symbols such as $ and ¢ correctly. • Show the same money value in different ways. **Depth Of Knowledge:** 2 **Bloom's Taxonomy:** Remember and Apply
<u>Previous Learning Connections</u> • Connect to telling time to the hour and half-hour using analog and digital clocks. **(1.MD.3)**	**<u>Current Learning Connections</u>** • Connect to skip count by 5s and 10s. **(2.NBT.2)** • Connect to applying knowledge of one- and two-step word problems to a variety of contexts and domains, including money. **(2.OA.1), (2.NBT.5) (2.MD.5)**	**<u>Future Learning Connections</u>** • Connect to telling and writing time to the nearest minute. **(3.MD.1)** • Connect to solving word problems using all four operations with measurement quantities. **(3.OA.3, 3.OA.8)**

Clarification Statement:
The mathematics in this cluster focuses on telling time and solving word problems involving money. Students are introduced to time in 1st grade and they capstone their work with time in third grade.

Common Misconceptions

● Students might mix up the hour and minute hands.

● Students may not understand they need to "move" the hour hand when drawing the hands on an analog clock for a time other than "o'clock" (e.g., representing 2:00, 2:15, and 2:45 with the same hour hand).

● Students may focus on counting objects rather than value when working with coins.

● Students may believe that a larger coin has a larger value.

● Students may confuse using the $ and ¢ symbols.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying time and money. because money and clocks are items they have seen and have around them daily. Giving them connection and vocabulary will help them talk about the math they are doing.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1MD.B.3: This standard provides a foundation for work with time to the hour and half hour because students need to have an understanding of hour and minute hands so they can move from hour and half hour to now 5-minute intervals. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with time and money benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as physical manipulatives because math manipulatives make learning math interesting, fun, and enjoyable. Giving students the choice of working on money problem with hands-on coins is motivating while helping students learn.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with time and money benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because peer support increases student levels of self-esteem and allows them to take risks in their learning. It builds confidence and positive feelings that they are doing good work.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with time and money benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because if math language isn't explicitly taught, students learn to disregard math words and only pay attention to the numbers.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with time and money benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., coins, bills, clocks) because physically interacting with materials by hand because using actual coins/clocks to solve money problems or tell time is more concrete and hands-on than looking at pictures or listening to a lecture.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with time and money benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because visuals such as graphic organizers are a helpful learning tool for students of all ages to organize, clarify, or simplify complex information—they help students construct understanding through an exploration of the relationships between concepts. They also are a useful scaffold to support student learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on time and money by providing specific feedback to students on their work through a short mini lesson because providing students with meaningful feedback can greatly enhance their learning and achievement. Re-teaching by providing students with information on what exactly they did well, and what may still need improvement.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on time and money by confronting student misconceptions because students will shut down if they think they are doing it wrong. Focusing on the facts and why 'right is right' gives them the detailed steps again to follow to get the answer right. We help them and re-teach them until they get the steps right.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying time and money because it gives students opportunities to teach themselves new skills of how to learn and apply to other content areas. Self-selected topics and learning can improve thinking skills, research skills, self-management skills, social skills, and communication skills and allows for further independent practice.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying working with time and money the types of mathematical tasks are critical because students need real life experience engaging with money and time. They need practice looking at and talking about it within its everyday uses.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/2/MD/C/8/tasks/1071
This type of assessment question requires students to articulate their addition strategies and explain their thinking. SMP 4 model with mathematics is important as students apply the mathematics, they know to solve problems arising in everyday life.
This task allows you to assess students understanding of coins and their values, and addition of numbers equal to 100 or less. It will also assess their understanding of numbers and values when they show other ways to show the same value.

Relevance to families and communities:

During a unit focused on working with time and money, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways money is used in the home and community can a be a great way to connect schools tasks with home tasks. Creating opportunities for tasks such as open-ended money word problems which might involve a series of steps and representations including symbolism, graphs, and manipulatives (coin and clocks).

Cross-Curricular Connections:

Social Studies: In second grade the New Mexico Social Studies Standards state students should "sequence historical events". Consider providing a connection for students to look at the value of common household items in different years.

Language Arts: Literature can offer connections about measurement such as: *The Penny Pot* by Stuart J. Murphy and *The Clock Struck One* by Trudy Harris.

2.MD: MEASUREMENT & DATA

Cluster Statement: D: Represent and interpret data.

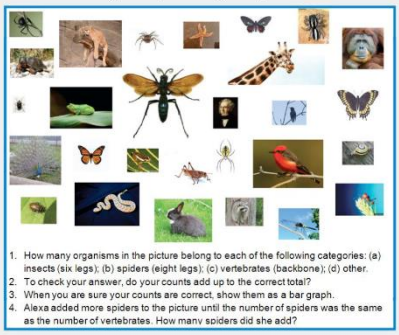
Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.MD.D.9: Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	SMP 4: Students can model with mathematics by using a line plot to organize and represent their data. SMP 5: Students can use tools by generating measurement data to the nearest whole unit using rulers, yardsticks, meter sticks and measuring tapes.	• Measure and record the lengths of several objects to the nearest whole-number. • Create a line plot with a horizontal scale marked off in whole-number units. • Record length measurements on a line plot.
		Depth of Knowledge: 2
		Bloom's Taxonomy: Remember, Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.MD.D.10: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simply put-together, take-apart, and compare problems using information presented in a bar graph.	SMP 1: Students can make sense of problems and persevere in solving them by applying what they know about put-together, take-apart, and compare problems to information presented in a bar graph. SMP 2: Students can reason abstractly and quantitatively by making sense of the quantities counted in each category in picture and bar graphs.	• Collect data representing up to 4 categories • Draw a picture graph and a bar graph. • Answer questions regarding graphs up to 4 categories. • Solve simply put-together, take-apart and compare problems using the information presented in a bar graph.

	SMP 4: Students can model with mathematics by using a picture or bar graph to organize and represent a data set with up to four categories.	**Depth of Knowledge:** 2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Previous Learning Connections • Connect to developing the skills to accurately measure objects using non-standard units of measure **(1.MD.2)** • Connect to organizing and represent data in ways that make sense to students and asking and answering questions about the data. **(1.MD.4)**	**Current Learning Connections** • Connect to solving simply put-together, take-apart, and compare problems. **(2.OA.1)** • Connect to other measurement work, as the count scale in a bar graph is a segment of a number line diagram and can be used to represent sums and differences. **(2.MD.6)**	**Future Learning Connections** • Connect to collecting and representing measurement data to include fractions. **(3.MD.4**)

Clarification Statement:
2.MD.D.10:

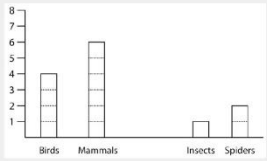
Activity for representing categorical data

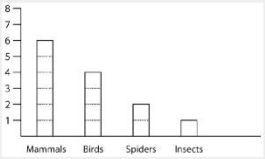


1. How many organisms in the picture belong to each of the following categories: (a) insects (six legs); (b) spiders (eight legs); (c) vertebrates (backbone); (d) other.
2. To check your answer, do your counts add up to the correct total?
3. When you are sure your counts are correct, show them as a bar graph.
4. Alexa added more spiders to the picture until the number of spiders was the same as the number of vertebrates. How many spiders did she add?

Students might reflect on the way in which the category counts in part 1 of the activity enable them to efficiently solve the word problem in part 4. (The word problem in part 4 would be difficult to solve directly using just the array of images.)

Different bar graphs representing the same data set





• To minimize potential confusion, it might help to avoid presenting students with examples of categorical data in which the categories are named using numerals, e.g., "Candidate 1," "Candidate 2," "Candidate 3." This will ensure that the only numbers present in the display are found along the count scale.

The illustration shows an activity in which students make a **bar graph** to represent **categorical data**, then **solve addition** and **subtraction** problems based on the data.

Common Misconceptions

- Students may think marking X's on the line plot in different sizes means different quantities.
- Students may not understand how the graph represents the data they collected.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories because students may need practice in generating categories, reading a tape measure, ruler, yardstick, or meterstick. Students also may need to be explicitly guided to the concept that a line plot with a horizontal scale is like what they used in prior grades as a number line with whole number units.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- K.MD.B.3: This standard provides a foundation for work with drawing picture and bar graphs to represent a data set with up to four category because students must be capable of first categorizing items to be able to successfully draw picture graphs and bar graphs to represent the data set with up to four categories . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because as students have contextualized examples of how measurement and data are relevant outside the classroom in a real-world aspect, the learning will be more meaningful.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories benefit when learning experiences attend to students attention and affect to support sustained

effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because children come from a variety of backgrounds and interests. A student from an agricultural background may see the relevance of measurement data within the context of land or produce, while a student who is interested in constructing, may see the validity of measurement in the form of building, etc.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because a second grader has been exposed to number lines and can extend that previously learned structure to the creation of a line plot, where the horizontal scale is marked off in whole-number units.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., pre-made number lines that gradually lead to self-created number lines) because second graders' fine motor skills are not always developed to independently construct straight lines with equidistant horizontal scales.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as offering devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the mathematical practices for the purpose of monitoring and improving because second graders are new to the mathematical skills involved in measurement and charting data. Students may require multiple opportunities in this cluster to master the desired learning goal.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories by providing specific feedback to students on their work through a short mini-lesson because often students do not attend to precision when reading measurements on a ruler, tape measure, yardstick, or meterstick. Students need reinforcement that they are correctly and accurately reading measurements to be successful in this cluster. Students must be shown often where to begin and end the measurement when using the tools; for example, students often begin reading at the beginning of the tool instead of where the 0 mark is indicated.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories by confronting student misconceptions because each standard within this cluster necessitates attending to precision. Students may need one-on-one instruction or peer-tutoring to successful read measurement tools. They may also require additional time and practice to accurately read measuring tools. Students may need differentiated scaffolding in the creation of line plots and bar graphs, perhaps beginning with a template and working toward individual construction of the line plot or bar graph with a horizontal scale marked off in whole-number units.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories because some students are naturally prone to investigate data. Students could measure more objects as they predict which items would be the same measurement or the same difference of measurement from other objects investigated.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories facilitating meaningful mathematical discourse is critical because as learners are given the opportunity to have connections with real-life situations in which measurement and data can be used in a variety of situations without the negative images and stereotypes around mathematics, they are validating that this skill is legitimate in all cultures. Students may experience many ways that measurement is used from the units (feet, inches, yards, meters, centimeters) to tasks such as sewing, carpentry, design. Line plots, picture graphs, and bar graphs to represent a data set is not only useful in academia, but also in business and society.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/2/MD/D/9/tasks/493
This type of assessment question requires students to measure and record growth data accurately.
This task allows you to assess student's ability to measure using standard tool. This activity also assesses student's ability to add an unknown (new growth since last measurement) to find total height.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on representing and interpreting data by generating measurement data by measuring the lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units and drawing picture and bar graphs to represent a data set with up to four categories, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the mathematics used within the different careers of your family and community can provide a strong connections between school and careers.	Science: In second grade the NGSS states students should "plan and conduct an investigation to determine if plants need sunlight and water to grow." Consider providing a connection for students to measure plant growth and record and analyze their data. Social Studies: In second grade the New Mexico Social Studies Standards state students should "compare similarities of the history of peoples in North America through literature (e.g., story-telling, fables, folktales, fairy tales)". Consider providing a connection for students to compare these similarities by displaying and analyzing the data.

2.G: GEOMETRY

Cluster Statement: Reason with shapes and their attributes.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
2.G.A.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	SMP 4: Students model with mathematics when recognizing and drawing shapes with specified attributes. SMP 6: Students attend to precision to draw shapes with specified attributes such as a given number of angles or a given number of equal faces.	• Classify shapes (Two-dimensional shapes as flat or plane shapes. Three-dimensional shapes are solid shapes.) • Identify attributes that define shapes include angles, faces, and sides. • Understand two- and three-dimensional shapes can be described and classified by their attributes. • Recognize shapes with specified attributes. • Draw shapes to represent given attributes. • Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
		Depth of Knowledge: 1
		Bloom's Taxonomy: understand
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	SMP 4: Students model with mathematics to partition a rectangle. SMP 6: Students attend to precision when partitioning a rectangle into rows and columns of same-size squares**.**	• Explain how rectangles can be divided into rows and columns. • Understand how to explore geometric relationships • Understand how shapes can be used to explain parts of a whole or totals. • Divide a rectangle into equal rows and columns to create an array. • Determine the number of shapes in an array

		Depth Of Knowledge: 1 **Bloom's Taxonomy:** understand, apply
Standard Text 2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	**Standard for Mathematical Practices** SMP 4: Students model with mathematics to partition circles and rectangles and identify their fractional parts SMP 6: Students attend to precision when partitioning circles and rectangle into two, three, or four equal shares.	**Students who demonstrate understanding can:** • Know that equal shares are made by dividing a shape evenly into multiple parts. • Understand that exploring geometric relationships relates to reasoning skills. Shapes can be used to explain parts of a whole or totals. • Partition circles into equal shares of two, three, or four. • Identify equal shares using different terms. • Explain how equal shares can have different shapes. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply
<u>Previous Learning Connections</u> • In first grade, students reasoned about shapes. They described and classified shapes, including drawings, manipulatives and real-world objects, in terms of their geometric attributes (**1.G.1**). Within that standard, students also distinguished between defining attributes (sides, vertices) versus non-defining attributes (color, size, orientation).	**<u>Current Learning Connections</u>** • Connect to their previous knowledge of addition from earlier in 2nd grade (**2.OA.4**) to find the total number of objects arranged in rectangular arrays.	**<u>Future Learning Connections</u>** • Connect to their work with rectangular arrays to the concept of equal groups as multiplication (**3.OA.1**). • Connect to shape categories to help form relationships among categories and subcategories (**3.G.1**). • Connect to understanding of fractions to name equal parts using written fraction notation **(3.NF.1, 3.G.2).**
Clarification Statement: Students build understandings of two- and three-dimensional shapes and their properties and compose and decompose shapes. They develop connections among spatial structures.		
Common Misconceptions Students may assume a shape with a changed orientation is no longer the same shape.		

Some students may fail to recognize a partitioned shape as a total of equal shares and think of the partition only as part of the whole.

Students may not be familiar with the vocabulary of halves, thirds, etc.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying reasoning with shapes and their attributes because some learners may have difficulty with the names of shapes based on the attributes of the number of angles or the number of equal faces. There are additional shapes added in this grade level from previous grades, and students may need multiple exposures to the attributes and names using images/resources.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying reasoning with shapes and their attributes because some learners may have difficulty with the names of shapes based on the attributes of the number of angles or the number of equal faces. There are additional shapes added in this grade level from previous grades, and students may need multiple exposures to the attributes and names using images/resources.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging and reasoning with shapes and their attributes benefit when learning experiences include ways to recruit interest such as providing choices in their learning such as using tangrams or shape manipulatives to see and feel the angles, faces, sides, etc. allows for all students to learn in their own way through touching and seeing.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging and reasoning with shapes and their attributes benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because it allows for students to make mistakes, figure out how to fix their mistakes, learn new strategies, and allows for peer interactions to enhance learning.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging and reasoning with shapes and their attributes benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching

vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because pre-teaching vocabulary gives students the opportunity to identify words and then be able to place them in context and remember them and apply it to what they are working on.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging and reasoning with shapes and their attributes benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing text-to-speech software (voice recognition), human dictation, recording because while students may be able to solve a math problem of how many angles does a hexagon have and have the skill set to do so, some students may be low literacy readers and by removing the stress of reading the problem they can focus on solving the problem.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging and reasoning with shapes and their attributes benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because visuals such as graphic organizers are a helpful learning tool for students of all ages to organize, clarify, or simplify complex information—they help students construct understanding through an exploration of the relationships between concepts. They also are a useful scaffold to support student learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- If students exhibit one more of these misconceptions, consider addressing the misconception by for example, students may benefit from re-engaging with content during a unit on reasoning with shapes and their attributes by clarifying mathematical ideas and/or concepts through a short mini-lesson because students will need to gain mastery of the attributes of shapes to be successful in following grades.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas, for example, some students may benefit from intensive extra time during and after a unit reasoning with shapes and their attributes by offering opportunities to understand and explore different strategies because students may be visualizing shapes differently. When offered opportunities to understand and explore different strategies (composing and decomposing shapes, manipulatives that can be used to create shapes, etc.) students gain a better understanding of reasoning with shapes and their attributes.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying reasoning with shapes and their attributes because "Students learn to combine their composition and decomposition competencies

	to build and operate on composite units (units of units), intentionally substituting arrangements or composites of smaller shapes or substituting several larger shapes for many smaller shapes, using geometric knowledge and spatial reasoning to develop foundations for area, fraction, and proportion. They recognize that the hexagonal faces of these constructions have equal area, that each trapezoid has half of that area, and each rhombus has a third of that area."
Culturally and Linguistically Responsive Instruction: **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages? **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? Task: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying reasoning with shapes and their attributes the types of mathematical tasks are critical because learners from a variety of cultural backgrounds may identify a variety of strategies and visual representations of the shapes and their attributes through spatial reasoning, mathematical knowledge, and artistic aptitudes.	
Standards Aligned Instructionally Embedded Formative Assessment Resources: *Source:* http://tasks.illustrativemathematics.org/content-standards/2/G/A/2/tasks/2063 This type of assessment question requires students to divide a rectangle into squares and explore the different ways to count the area squares. This task allows you to assess student's ability to reason with shapes and skip count.	
Relevance to families and communities: During a unit focused on reasoning with shapes and their attributes, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different structures for the shape names across the languages in your classroom can lead to a more robust understanding of shapes for all students by making connections to the different structures of shape-names in other languages.	**Cross-Curricular Connections:** Science: Students can describe and classify different kinds of materials by their observable properties.

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest:** ? What do you anticipate in the range of student interest for this lesson? ☐ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ⍰ How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ⍰ What do you anticipate about the range in how students will perceive information presented in this lesson? ☐ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ⍰ How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ⍰ What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ☐ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT ⍰ How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ⍰ What do you anticipate about the range in student effort? ☐ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication:** ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation:** ? What do you anticipate about barriers to student engagement? Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension:** ? What do you anticipate about barriers to student comprehension? Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions:** ⍰ What do you anticipate about barriers to students demonstrating what they know? ⍰ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

> **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
>
> **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

> **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
>
> **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
>
> **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

---

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.

Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

CAST (2018). *Universal Design for Learning Guidelines version 2.2.* Retrieved from http://udlguidelines.cast.org

Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

Gifted and talented | NZ Maths. (n.d.). Retrieved from https://nzmaths.co.nz/gifted-and-talented

Gojak, L. M. (Ed.) (2015-2018). *The Common Core Mathematics Companion: The Standards Decoded: What They Say, What They Mean, How to Teach Them (Corwin Mathematics Series)* (1st ed.). Thousands Oak, CA: Corwin.

Gonzales, Lorenzo (2004). *Building personal knowledge of rational numbers through mental images, concepts, language, facts, and procedures.* Unpublished Doctoral Dissertation, New Mexico State University, College of Education, Las Cruces, New Mexico.

Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

Howard County Public School System: Mathematics. (n.d.). Mathematics – HCPSS. Retrieved from https://www.hcpss.org/academics/mathematics/

Illustrative Mathematics. (2014, February 14). Standards for Mathematical Practice: Commentary and Elaborations for K–5. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf

Illustrative Mathematics. (2014b, May 6). Standards for Mathematical Practice: Commentary and Elaborations for 6–8. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/05/2014-05-06-Elaborations-6-8.pdf

Kansas State Department of Education. (2018). 2018 Kansas Mathematics Flip Books. Retrieved from https://community.ksde.org/Default.aspx?tabid=5646

Kaplinsky, R. (n.d.). SIOP Content and Language Objectives for Math. Retrieved from http://robertkaplinsky.com/wp-content/uploads/2017/04/SIOP-Content-and-Language-Objectives-for-Math.pdf

Louisiana Department of Education . (n.d.). K-12 Math Planning Resources. Retrieved from https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Los Angeles, CAUSA. (n.d.). VABB™. Retrieved from https://www.culturallyresponsive.org/vabb

New Visions for Public Schools. (2020). Re-engagement Lessons. Retrieved from https://curriculum.newvisions.org/math/nv-math-team/curriculum-components/re-engagement-lessons/

North Carolina Department of Instruction. (2016–2019). North Carolina Unpacked Content for Mathematics. Retrieved from https://www.dpi.nc.gov/teach-nc/curriculum-instruction/standard-course-study/mathematics

Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD.

South Dakota Department of Education. (n.d.). South Dakota Content Standards - Math. Retrieved from https://doe.sd.gov/contentstandards/math.aspx#ratios+

Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
Student Achievement Partners. (n.d.-a). Coherence Map. Retrieved from https://achievethecore.org/coherence-map/

Trundley, R. (2018). Changing lives and providing equity through pre-teaching and assigning competence. *Association of Teachers of Mathematics (ATM)*, *July*, 31–34. Retrieved from https://www.atm.org.uk/write/MediaUploads/Journals/MT262/MT26213.pdf

Utah Education Network. (2016). Mathematics Core. Retrieved from https://www.uen.org/core/math/

Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

---

[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

---

[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.