NEW MEXICO
Public Education Department

3.NF: NUMBER & OPERATIONS-FRACTIONS

Cluster Statement: A: Develop understanding of fractions as numbers.
Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
3.NF.A.1 **Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$.**	SMP4: Model with mathematics: Students will use pictures or tiles to help create & cut equal parts of a fraction and label each part correctly. SMP6: Attend to precision: Students will use fraction vocabulary and relate to models with precision (numerator, denominator, unit fraction, whole, non-unit fraction) SMP7: Look for and make use of structure: Students need to recognize the structure and visual pattern when numerators change to determine how a fraction would be named when more than one-unit part is shaded/considered by adding parts or counting on from the unit fraction.	• Recognize a unit fraction such as ¼ as the quality formed when the whole is portioned into 4 equal parts. • Identify a fraction such as 2/3 and explain that quantity formed is 2 equal parts of the whole portioned into 3 equal parts (1/3 and 1/3 of the whole 3/3). • Express a fraction as the number of unit fractions • Use accumulated unit fractions to represent numbers equal to, less than, and greater than one (1/3 and 1/3 is 2/3; 1/3,1/3 1/3, 1/3 is 4/3). • Explain and represent a unit fraction. • Explain and represent a non-unit fraction. • Identify the numerator and denominator and understand the meaning of each in the fraction • Explain how fraction representations are related (1/b relates to a/b) • Identify a unit fraction and build other fractions from the unit fraction.
		Depth of Knowledge: 1
		Bloom's Taxonomy: Remember, Understand

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Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.NF.A.2 **Understand a fraction as a number on the number line; represent fractions on a number line diagram.** • **3.NF.A.2.A: Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.** • **3.NF.A.2.B: Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.**	SMP4: Model with mathematics: Students will use pictures, tiles, and number lines to represent fractions visually. SMP2: Reason abstractly and quantitatively: Students will reason about what it means when numerators & denominators get larger/smaller and determine how to break up a number line (using the denominator) & where the fraction would be located. SMP5: Use appropriate tools strategically: Students can use fraction tiles, pictures, and number line to help create and represent fractions on a number line diagram.	• Understand fractions as numbers • Interpret fractions with denominators of 2,3,4,6, and 8 using area and length models • Using an area model, explain that the numerator of a fraction represents the number of equal parts of the unit fraction • Using a number line, explain that the numerator of a fraction represents the number of lengths of the unit fraction from 0 • Recognize a unit fraction such as ¼ as the quantity formed when the whole is partitioned into 4 equal parts • Express a fraction as the number of unit fractions • Define the interval from 0 to 1 on a number line as the whole • Divide a whole on a number line into equal parts • Recognize that the equal parts between 0 and 1 have a fractional representation • Represent each equal part on a number line with a fraction • Explain that the endpoint of each equal part represents the total number of equal parts
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply

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Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.NF.A.3 **Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.** • **3.NF.A.3.A: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.** • **3.NF.A.3.B: Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2}=\frac{2}{4}$, $\frac{4}{6}=\frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.** • **3.NF.A.3.C: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = $\frac{3}{1}$; recognize that $\frac{6}{1}$ = 6; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.*** • **3.NF.A.3.D: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.**	SMP2: Students can reason abstractly and quantitatively by applying reasoning skills to analyze what it means when numerators & denominators get larger/smaller and place fractions on a number line that could go larger than 1 whole in order to compare the fractions to benchmark numbers, whole numbers and to each other when the numerators or denominators are the same. SMP3: Students can construct viable arguments and critique the reasoning of others by justifying their answers while comparing fractions with a clear explanation using words, pictures, or numbers.	• Compare fractions by reasoning about their size to determine equivalence • Use number lines, size, visual fraction models, etc. to find equivalent fractions • Recognize whole numbers written in fractional parts on a number line • Recognize the difference in a whole number and a fraction • Explain how a fraction is equivalent to a whole number • Explain what the numerator in a fraction represents and its location • Explain what the denominator in a fraction represents and its location • Explain that a fraction with the same numerator and denominator equals one whole • Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers • Recognize whether fractions refer to the same whole • Determine if comparisons of fractions can be made (if they refer to the same whole). • Compare two fractions with the same numerator by reasoning about their size. • Compare two fractions with the same denominator by reasoning about their size. • Compose and decompose fractions into equivalent fractions using fractions: halves, fourths and eighths; thirds and sixths • Record the results of comparisons using symbols <, =, or >. • Justify conclusions about the equivalence of fractions

		Depth of Knowledge: 2-3 **Bloom's Taxonomy:** Apply, Analyze
Previous Learning Connections • Connect to partition shapes, measure length and solve problems using addition and subtraction. • Connect to equally partitioned circles and rectangles into halves, thirds and fourths, and recognized that equal shares of identical wholes need not have the same shape. (**2.G.3**) • Connect to measure the length of an object twice, using length units of different lengths for the two measurements; described how the two measurements relate to the size of the unit chosen (**2.MD.2**). • Connect to use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (**2.MD.5**) • Connect to represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1, 2..., and represent whole-number sums and differences within 100 on a number line diagram (**2.MD.6**). • Connect to generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a	**Current Learning Connections** • Connect to partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (**3.G.2**). • Connect to generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters (**3.MD.4**)	**Future Learning Connections** • Connect to understanding of fraction equivalence, build fractions from unit fractions and understand and compare decimal fractions. Learners will understand a fraction a/b with a>1 as a sum of fractions 1/b (**4.NF.3**). • Connect to make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction of fractions by using information presented in line plots (**4.MD.4**). • Connect to apply and extend previous understanding of multiplication to multiply a fraction by a whole number (**4.NF.4**) • Connect to explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (**4.NF.1**)

line plot, where the horizontal scale is marked off in whole-number units (**2.MD.9**)		

Clarification Statement:

• Students use area and length models to compose and decompose fractions into **equivalent fractions** using related fractions: **halves, fourths, eighths, thirds,** and **sixths**.
• **Related fractions** are fractions in which one denominator is a multiple of the others; thirds and sixths are related fractions, while fourths and sixths are not related fractions.
• Students should be able to explain that fractions with the same **numerator** and **denominator** equal one whole.
• Renaming fractions with the same numerator and denominator as one whole without a model is not sufficient for this standard.
• The standard also expects students to express **whole numbers** as **fractions**. This work is limited to whole numbers less than 4.
• Expressing whole numbers as fractions lay the groundwork for seeing a fraction as a division problem, e.g., the fraction 4/2 represents 4 pieces that are a half each that equal 2 wholes.
• This standard is the building block for later work in Grade 5 where students divide a set of objects into a specific number of groups.

Common Misconceptions

• Students may not use benchmark numbers like 0, 1/2, and 1 to compare fractions because they have restricted their understanding of fractions to part-whole situations and do not think of the fractions as numbers.
• Students may overgeneralize and think that "all 1/4 s (for example) are equal".
• Students may not understand that the size of the whole determines the size of the fractional part.
• Students may struggle with the idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying develop understanding of fractions as numbers because while using different representations, such as pictures, students are introduced to appropriate labels to communicate the meaning of their representation.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.G.A.3: This standard provides a foundation for work with develop understanding of fractions as numbers because student need the foundational understanding of equal share partitioning, how to accurately describe the shares, and the understanding that decomposing into more equal shares creates smaller shares in order to make the connection between concrete (e.g., pictures) and abstract (e.g., fractions as numbers). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with develop an understanding of fractions as numbers benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because students may recognize the underlying mathematical relationships in representations quickly but may need support perceiving them in a different representation. By showing several representations students can connect and build from one representation to another allowing for greater long-term access to all representations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because the external environment must provide options that can equalize accessibility by supporting learners who differ in initial motivation, self-regulation skills, etc.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because text is a particularly weak form of presentation for learners who have text- or language-related disabilities. Providing alternatives—especially illustrations, simulations, images or interactive graphics—can make the information more comprehensible for any learner and accessible for some who would find it completely inaccessible in text.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-10 blocks, number lines, fraction bars) because virtual and concrete manipulatives provide a more flexible and accessible toolkit with which learners can more successfully take part in their learning and articulate what they know. Unless a lesson is focused on learning to use a specific tool (e.g., learning to draw with a compass), curricula should allow many alternatives. Learners should learn to use tools that are an optimal match between their abilities and the demands of the task.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to students by intentionally building connections to prior

understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections (e.g., teaching literacy strategies in the social studies classroom) because all learners can benefit from assistance in how to transfer the information they have to other situations, as learning is not about individual facts in isolation, and students need multiple representations for this to occur.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on developing understanding of fractions as numbers by revisiting student thinking through a short mini-lesson because when students can explain their thought process, they also understand the possibility of different interpretations and therefore the necessity for precision in their work.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on developing understanding of fractions as numbers by confronting student misconceptions because it takes time and multiple passes to develop understanding, so students need regular opportunities to think about, talk through, and refine ideas

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying develop understanding of fractions as numbers because in order to develop and solidify ideas, students need to be able to connect what they are learning to multiple disciplines and real-world connections through the productive struggle of open-ended tasks.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying understanding fractions as numbers supporting productive struggle is critical because building and internalizing fractions as parts of a whole that can be represented by numbers is foundational to all future comprehension of fraction math problems. It is through productive struggle that students develop their own understanding of math concepts in a mental context that they own, that has meaning to them, and that they can easily access and manipulate for future procedural tasks and problem solving.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 3 Number and Operations Fractions
STANDARD: Develop understanding of fractions as numbers: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (03.NF.01.03)
LEARNING TARGET: I can find two equivalent fractions and compare them to a third fraction. DOK: 2

1. Look at these fractions.

$$\frac{2}{3}, \frac{1}{2}, \frac{4}{6}$$

a. Which two of the fractions are equivalent to each other? Show or explain how you know.

b. Is the remaining fraction greater than or less than the two equivalent fractions? Show or explain how you know.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding fractions as numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example: increasing or decreasing ingredients in a recipe; using statistics from current events, relevant to a family or community, to determine their impact (converting a whole number statistic into a fraction to show its impact as part of a greater whole population); using a given amount of time, determining how it is used, and comparing the portions of that time for time management goals, using a given amount of money, determining how it is used, and comparing the portions of that money to set a budget; dividing a plot of land into equal portions to plan and plant a garden, etc.	Social Studies: Based on current events and topics, create a survey, collect data, and represent results in each category as fractions of the whole survey population. Music: Reading the value of musical notes 1/4, 1/2, 3/4, 1 and the relation of note count to measure.