

3.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: B: Understand properties of multiplication and the relationship between multiplication and division.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text

3.OA.B.5

Apply properties of operations as strategies to multiply and divide. *2 Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

Standard for Mathematical Practices

SMP2 Students can reason abstractly and quantitatively by expressing their understanding and applying "if, then" logic.

SMP8 Students can look for and express regularity in repeated reasoning by identifying and using repeated reasoning to apply commutative, associative, and distributive properties, i.e., realizing that a multidigit number can always be broken into a group of tens and ones. Students will use and explain their use of properties to multiply and divide.

Students who demonstrate understanding can:

- Explain the commutative property of multiplication where two factors can be multiplied in either order and have the same product.
- Explain the associative property of multiplication states that the position of three or more factors are grouped before multiplying does not affect the product
- Explain that the distributive property allows you to separate numbers into parts so that the numbers are easier to work with, and apply properties to use with basic facts or multiply with multiples.
- Multiply and divide within 100 and explain how the properties of operations work. Apply properties of operations as strategies to multiply or divide
- Utilize visuals that support their understanding of the application of properties as strategies to multiply and divide.

Depth of Knowledge: 2-3

Bloom's Taxonomy: Apply

<p>Standard Text</p> <p>3.OA.B.6 Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.</p>	<p>Standard for Mathematical Practices</p> <p>SMP5: Students can use appropriate tools strategically by using concrete models, pictures, words, and numbers to explain their thought process and justify their solutions, identifying the patterns they find in multiplication and division.</p> <p>SMP6: Students can attend to precision by using the language of multiplication and division to communicate their thinking about the individual operations and how they are related to each other.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Describe how multiplication and division are related. Identify the unknown factor in the related multiplication problem. Identify the multiplication problem related to the division problem. Use multiplication and division to solve division problems. Recognize multiplication and division as related operations and explain how they are related. Solve an unknown-factor problem, by using division strategies and/or changing it to a multiplication problem. Use visuals such as an array with related multiplication problems to demonstrate their understanding of division as an unknown factor problem. <p>Depth of Knowledge: 1-2</p> <p>Bloom's Taxonomy: Remember, Understand</p>
<p><u>Previous Learning Connections</u></p> <p>Connect to previous work understanding equal groups, skip counting by 2, 5, 10, 100's, work with arrays up to 5 rows and 5 columns. (2.OA.3, 2.OA. 4 and 2.NBT.2)</p>	<p><u>Current Learning Connections</u></p> <p>Connections to multiplication and division exist across the standards in third grade. (3.OA.1) (3.OA.2)(3.OA.3) (3.MD.7) (3.OA.5) (3.OA.6) (3.NBT.3) (3.OA.4) (3.OA.7) (3.OA.8) (3.OA.9)</p>	<p><u>Future Learning Connections</u></p> <p>Connect to 4th grade, where students use multiplication and division with larger numbers. (4.NBT.5) (4.NBT.6)</p>
<p>Clarification Statement:</p> <ul style="list-style-type: none"> Requires students to apply properties of operations as strategies to multiply and divide Requires students to explain and represent the commutative property (two factors can be multiplied in either order and still have the same product) Students need to explain and represent the associative property (the way in which three or more factors are grouped before multiplying does not affect the product) Students need to explain and represent the distributive property (breaking numbers into parts so that the numbers are easier to work with) Students need to apply properties to recall basic facts or multiply with multiples of 10 These standards require students to explain the relationship between multiplication and division Students need to use multiplication to find an unknown in a division equation, as well as use division to find an 		

unknown in a multiplication equation.

- Students need to understand the relationship between multiplication and division and explain their processes of solving multiplication and division problems using an inverse operation.

Common Misconceptions

- Students may think that division is commutative. $5 \div 3 = 3 \div 5$
- Students may see multiplication and division as different and unrelated operations.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying understanding properties of multiplication and the relationship between multiplication and division because when students review their understanding of arrays and multiplication as repeated addition, then they can begin to connect the concepts. Also, when students review their understanding of addition and subtraction as inverse operations, they can apply this knowledge to multiplication and division being inverse operations.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.4 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7 . This standard provides a foundation for work with understanding properties of multiplication and the relationship between multiplication and division because once students are able to understand and interpret products, they can begin to build the understanding of the properties of multiplication as well as the relationship of multiplication and division. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences include ways to recruit interest such as creating accepting and supportive classroom climate because students who feel free to take chances and make mistakes learn more than students who are afraid of failure or afraid to be wrong. By taking chances and trying new strategies as well as sharing their thinking students can find misconceptions and correct them as well as further build and cement their understanding of the concept.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because when students can personally relate to a concept they understand the concept at a deeper level.

Students connecting multiplication and the relationship of multiplication to real life situations based on their interest and background will provide them the opportunity to relate something new to them to their lives which is very familiar and comfortable to relate to.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because when students to connect what they already know it helps build a connection to what they are learning. If students can connect the relationships of the operations of addition and subtraction and addition and multiplication to the relationship of multiplication and division, then they build a deeper understanding of the relationship of multiplication and divisions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because when students are able to solve problems in multiple ways. The more ways students can manipulate numbers the more they will understand the relationship between multiplication and division.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as embedding new ideas in familiar ideas and contexts because when students begin to see the connections of familiar contexts and ideas and the new ideas they begin to develop an understanding based on concepts they are already confident and comfortable with. For example, understanding the connection between addition and subtraction, addition and multiplication, will help students understand the relationship between multiplication and division.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding properties of multiplication and the relationship between multiplication and division by revisiting student thinking through a short mini-lesson because through resisting their thinking students will need to think deeper about the ideals they have formed. Often through revisiting their thinking students will begin to find misconceptions or errors in their reasoning that they must re-examine in order to explain.

Re-teach (intensive) What assessment data will help identify content needing to be revisited for intensive interventions?

- For example, some students may benefit from intensive extra time during and after a unit understanding properties of multiplication and the relationship between multiplication and division by helping students move from specific answers to generalizations for certain types of problems because this allows students to develop patterns and understand what the product is and how it relates to division. Rather than just a memorization of the facts, students need an understanding of what the product represents in order to understand multiplication and its relationship to division. Through understanding the properties of multiplication students will better understand how it is related to division.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying understanding properties of multiplication and the relationship between multiplication and division because it allows students to extend their understanding of multiplication and division and relate it to the relationship of other operations. It also helps students begin to explore the relationship of larger numbers.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying understanding properties of multiplication and the relationship between multiplication and division the types of mathematical tasks are critical because if procedural knowledge is built without conceptual understanding students are only learning a process or steps, however they are not learning the WHY or the meaning of the math. Without conceptual knowledge students are basically doing what a calculator or computer could accomplish. Through building a conceptual understanding of the properties of multiplication and the relationship of multiplication and division students do not have to rely merely on a procedure to get an answer. Students can think through the math and use the procedure as a tool of calculating their answer. Students should also then be able to assess the reasonableness of their procedural answer when they have a conceptual understanding of the concept.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia Testlet for Grade 3 Operations and Algebraic Thinking

STANDARD: Apply properties of operations as strategies to multiply and divide (students need not use formal terms for these properties). Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) (03.OA.02.05)

LEARNING TARGET: I can identify different ways to multiply three numbers. DOK: 2

1. Look at this problem.

$$2 \times 4 \times 3$$

Maggie and Jon worked this problem using different methods.

- Maggie multiplied 2 and 4, then she multiplied the answer by 3.
- Jon multiplied 2 and 3, then he multiplied the answer by 4.

Whose method is correct?

- Ⓐ Only Maggie's method is correct.
- Ⓑ Only Jon's method is correct.
- Ⓒ Both Maggie's and Jon's methods are correct.
- Ⓓ Neither Maggie's nor Jon's method is correct.

Relevance to families and communities:

During a unit focused on multiplication, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, when making a quilt or blanket of a certain size, with a certain size of squares, how many squares would you need to create or make a quilt/blanket.

Cross-Curricular Connections:

Science: Students could learn about Science by implementing information from plants, earth and space, cycles of life, animals, electricity and magnetism, and motion and sound to solve word problems.