New Mexico Mathematics Instructional Scope for Third Grade

June 2020

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands
[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

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Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
Full text of the standard	The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	The cognitive skills students perform to demonstrate to comprehension of a standard. **Depth Of Knowledge:** Correlation of standard to Webb's Depth of Knowledge **Bloom's Taxonomy:** Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities:	**Cross Curricular Connections:**
Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

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<tr><td colspan="3">Cluster Statement: A: Represent and solve problems involving multiplication and division.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

3.OA.A.1
Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.</td><td rowspan="3">Standard for Mathematical Practices

SMP2: Students can reason abstractly and quantitatively by using reasoning to determine what is happening when they multiply (given the number of groups and the number of items in a group, they find the total number of items).

SMP3: Students can construct viable arguments and critique the reasoning of others by comparing their strategies with those of classmates. Students understand and express connections among ideas and between concrete models and numerical notations.</td><td>Students who demonstrate understanding can:
Demonstrate understanding by representing multiplication with equal groups.
Represent multiplication with arrays. Use repeated addition to represent multiplication.
Utilize the number line to represent multiplication with equal jumps
Analysis of operational patterns, grouping patterns, grouping of numbers, arrays, and area-based strategies.
Apply the standard algorithms and their conceptual basis utilizing drawings and equations.
Demonstrate algorithms to provide computational efficiency utilizing problem solving and solving problems.</td></tr>
<tr><td>Depth of Knowledge: 1</td></tr>
<tr><td>Bloom’s Taxonomy: Remember, Understand</td></tr>
<tr><td>Standard Text

3.OA.A.2
Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in</td><td>Standard for Mathematical Practices

SMP2: Students can reason abstractly and quantitatively by understanding what is happening when they divide (given the total number of items and the number of groups, they find the number of items in a group or given the total number of items and the number of items in a group, they find the number of groups) .</td><td>Students who demonstrate understanding can:
Explain the meaning of division and what the numbers in a division problem represent.
Explain that a dividend is the number of objects to be shared equally. The divisor is the number of equal shares or the number in each equal share. The quotient is an answer to a division problem.</td></tr>
</table>

which a number of shares or a number of groups can be expressed as 56 ÷ 8.	SMP3: Students can construct viable arguments and critique the reasoning of others by creating mathematical arguments to explain their reasoning and compare their strategies with those of classmates. Students will make connections among ideas and between concrete models and numerical notations.	• Describe the meaning of division and quotients of whole numbers • Interpret whole-number quotients of whole numbers • Utilize partition division in which you divide an amount into a given number of groups. • Utilize measurement division which is repeated subtraction division in which you divide an amount into groups of a given size. • Demonstrate division with manipulatives and other visuals.
		Depth of Knowledge: 1
		Bloom's Taxonomy: Remember, Understand
Standard Text **3.OA.A.3** **Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.**	**Standard for Mathematical Practices** SMP1: Students make sense of problems and persevere in solving them by deciphering word problems to distinguish relevant information and appropriate strategies and apply them to find solutions. SMP2: Students can reason abstractly and quantitatively by identifying and expressing what is happening when they multiply (given the number of groups and the number of items in a group, they find the total number of items) and divide (given the total number of items and the number of groups, they find the number of items in a group or given the total number of items and the number of items in a group, they find the number of groups).	**Students who demonstrate understanding can:** • Utilize visuals to represent, interpret, and solve one-step problems involving multiplication and division. • Solve multiplication word problems by utilizing models, drawings, and equations. • Represent the problem using arrays, pictures, and/or equations with a symbol for the unknown number to represent the problem. • Solve division word problems with a divisor and quotient utilizing models, drawings, and equations. • Represent the problem using arrays, pictures, repeated subtraction and/or equations with a symbol for the unknown number to represent the problem. • Explain connections of equations solved with their models or drawings to reinforce multiplication and division within 100

		• Develop strategies using models, drawings, and equations to demonstrate student understanding.
		Depth of Knowledge: 2-3
		Bloom's Taxonomy: Apply, Analyze
Standard Text **3.OA.A.4** **Determine the unknown whole number in a multiplication or division equation relating three whole numbers.** ***For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?***	**Standard for Mathematical Practices** SMP2: Students can reason abstractly and quantitatively by deciphering the process of multiplication (given the number of groups and the number of items in a group, they find the total number of items) and division (given the total number of items and the number of groups, they find the number of items in a group or given the total number of items and the number of items in a group, they find the number of groups). SMP7: Students can look for and make use of structure by expressing their knowledge of the algorithmic structure of multiplication and division to determine the unknown number.	**Students who demonstrate understanding can:** • Explain connections between an equation to a problem. • Multiply and divide within 100. • Determine which operation (multiplication or division) is needed to determine the unknown whole number and solve to find the unknown whole number in a multiplication or division equation. • Apply their understanding to demonstrate their knowledge of the relationship between multiplication and division • Examine patterns and use manipulatives as well as drawings to demonstrate their understanding of determining the unknown whole number in a multiplication or division equation.
		Depth of Knowledge: 1-2

		Bloom's Taxonomy: Remember, Understand
<u>Previous Learning Connections</u> • Connect to understanding of equal groups, skip counting by 2, 5, 10, 100's, work with arrays up to 5 rows and 5 columns. • Connect to whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (**2.OA.3**) • Connect to counting within 1000; skip-counted by 5s, 10s, and 100s. (**2.NBT.2**) • Connect to using addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (**2.OA.4**)	**<u>Current Learning Connections</u>** • Connect to division as an unknown-factor problem. (**3.OA.6**) • Connect to multiply and divide within 100, using strategies such as the relationship between multiplication and division. (**3.OA.7**) • Connect to apply properties of operations as strategies to multiply and divide. (**3.OA.5**) • Connect to relate area to the operations of multiplication and addition. (**3.MD.7**)	**<u>Future Learning Connections</u>** • Connect to understanding of multiplication and division, using various strategies to help with larger numbers. Learners will interpret a multiplication equation as a comparison, (**4.OA.1**) • Connect to apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (**4.NF.4**) • Connect to multiply or divide to solve word problems involving multiplicative comparison. (**4.OA.2**) • Connect to apply the area and perimeter formulas for rectangles in real world and mathematical problems. (**4.MC.3**)

Clarification Statement:
•Students need to explore and understand the relationship between multiplication and division
•They will need to determine the unknown number in multiplication and division problems such as the following examples: 8 x 9 = ?, 8 x ? = 48, ? x 3 = 27, 28 ÷ 7 = ?, ? ÷ 6 = 3, 35 ÷ ? = 7
• Students will apply their understanding of multiplication and division to identify an unknown in an equation. This means they will need to apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. The standard requires them to see the solution to an equation on both sides of the equal sign.
• Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Common Misconceptions:
•Students may think that 3 ÷ 15 = 5 and 15 ÷ 3 = 5 are the same equations. The use of models is essential in helping students eliminate this misunderstanding.
•Students may think a symbol used to represent a number once cannot be used to represent another number in a different problem/situation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying representing and solving problems involving multiplication and division because understanding the visual representations will help students understand the concept. Understanding the visual representations will also help provide students with a strategy to record their understanding.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. This standard provides a foundation for work with representing and solving problems involving multiplication and division because it provides students with the foundation and understanding of the visual representation of arrays. It also provides the basis of the understanding of multiplication being repeated addition. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with representing and solving problems involving multiplication and division benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because the learning must be relevant to student learning. It is important to provide options that will optimize what is relevant, important, meaningful and valuable to a learner. When students can relate the learning to their lives it will become more permanent learning.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and solving problems involving multiplication and division benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because when students discuss and engage with one another they not only build a deeper understanding of a concept but they also share ideas, teach, and support one another. This allows students the opportunity to talk out their ideas, share their thinking, and build a common understanding of a concept.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and solving problems involving multiplication and division benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.) because this allows all students an access point to understanding the information. It also helps remove language barriers that may exist and cause students to be confused by a concept.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representing and solving problems involving multiplication and division benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because students can focus on their strengths and weaknesses or their understandings and misconceptions. This allows students to get targeted feedback and adjust their thinking or understanding and develop a deeper understanding of the concept.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with representing and solving problems involving multiplication and division benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because this will allow students to reflect and correct if necessary. It forces students to think about their learning and ideas and therefore develop a deeper understanding of their learning, misconceptions, and understanding of the concept.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representing and solving problems involving multiplication and division by critiquing student approaches/solutions to make connections through a short mini-lesson because if students are able to critique student approaches and solutions they can then examine their own thinking and approach to determine if it is reasonable and accurate.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit representing and solving problems involving multiplication and division by offering opportunities to understand and explore different strategies because the more opportunities a student has to explore different strategies the more likely they will be able to find a strategy that makes sense to them and develop an understanding of a concept.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying representing and solving problems involving multiplication and division because this will allow students to dive deeper into the concept and begin to explore the next steps of the standard. It also allows students to begin thinking about application of the concept.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their “mathematical, social, and cultural competence”. By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying representing and solving problems involving multiplication and division the use of mathematical representations within the classroom is critical because it aids students in understanding the reasoning or the WHY behind the mathematics. Although procedural knowledge of multiplication and division is quicker, they are just steps without the understanding of the concepts and the procedure. Visual representations how students better visual and quantify the numbers and concepts.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/3/OA/A/3/tasks/262
Gifts from Grandma Variation 1 Task
Addresses standard 3. OA.A.3 - Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
The first of these is a multiplication problem involving equal-sized groups. The next two reflect the two related division problems, namely, "How many groups?" and "How many in each group?"

Sometimes the second type of problem is referred to as a measurement division or repeated subtraction problem. The third type of problem is sometimes called a partitive division or sharing problem. It asks how large is each share when a whole is divided equally into a specified number of pieces. It specifies the size of each share and asks how many of that size are in the whole. The language used in the solution reflects the language in the common core, which also refers to them "Number of Groups Unknown" or "Group Size Unknown," respectively.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on representing and solving problems involving multiplication and division, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, when cooking a meal or dessert you child could figure out how much each family member should get which will being an introduction to fractions.	Language Arts**:** Essential vocabulary: multiplication, factor, array, equal groups, and repeated addition. Writing math word problems utilizing punctuation and spelling. Also include a five square graphic organizer that includes 1. The question, 2. Important information from the problem, 3. visual representation of the important information to solve the problem, 4. Solve the problem, 5. Students explain in writing what they did to solve the problem and why they used their method to solve the problem.

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<table>
<tr><th colspan="3">3.OA: OPERATIONS & ALGEBRAIC THINKING</th></tr>
<tr><td colspan="3">Cluster Statement: B: Understand properties of multiplication and the relationship between multiplication and division.
Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

3.OA.B.5
Apply properties of operations as strategies to multiply and divide.2 Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)</td>
<td rowspan="3">Standard for Mathematical Practices

SMP2 Students can reason abstractly and quantitatively by expressing their understanding and applying "if, then" logic.

SMP8 Students can look for and express regularity in repeated reasoning by identifying and using repeated reasoning to apply commutative, associative, and distributive properties, i.e., realizing that a multidigit number can always be broken into a group of tens and ones. Students will use and explain their use of properties to multiply and divide.</td>
<td>Students who demonstrate understanding can:
Explain the commutative property of multiplication where two factors can be multiplied in either order and have the same product.
Explain the associative property of multiplication states that the position of three or more factors are grouped before multiplying does not affect the product
Explain that the distributive property allows you to separate numbers into parts so that the numbers are easier to work with, and apply properties to use with basic facts or multiply with multiplies.
Multiply and divide within 100 and explain how the properties of operations work. Apply properties of operations as strategies to multiply or divide
Utilize visuals that support their understanding of the application of properties as strategies to multiply and divide.</td></tr>
<tr><td>Depth of Knowledge: 2-3</td></tr>
<tr><td>Bloom's Taxonomy: Apply</td></tr>
</table>

1

<table>
<tr>
<td rowspan="3">Standard Text

3.OA.B.6
Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP5: Students can use appropriate tools strategically by using concrete models, pictures, words, and numbers to explain their thought process and justify their solutions, identifying the patterns they find in multiplication and division.

SMP6: Students can attend to precision by using the language of multiplication and division to communicate their thinking about the individual operations and how they are related to each other.</td>
<td>Students who demonstrate understanding can:
• Describe how multiplication and division are related.
• Identify the unknown factor in the related multiplication problem.
• Identify the multiplication problem related to the division problem.
• Use multiplication and division to solve division problems.
• Recognize multiplication and division as related operations and explain how they are related.
• Solve an unknown-factor problem, by using division strategies and/or changing it to a multiplication problem.
• Use visuals such as an array with related multiplication problems to demonstrate their understanding of division as an unknown factor problem.</td>
</tr>
<tr>
<td>Depth of Knowledge: 1-2</td>
</tr>
<tr>
<td>Bloom's Taxonomy: Remember, Understand</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>

Connect to previous work understanding equal groups, skip counting by 2, 5, 10, 100's, work with arrays up to 5 rows and 5 columns. (2.OA.3, 2.OA. 4 and 2.NBT.2)</td>
<td><u>Current Learning Connections</u>

Connections to multiplication and division exist across the standards in third grade. (3.OA.1) (3.OA.2)(3.OA.3) (3.MD.7) (3.OA.5) (3.OA.6) (3.NBT.3) (3.OA.4) (3.OA.7) (3.OA.8) (3.OA.9)</td>
<td><u>Future Learning Connections</u>

Connect to 4th grade, where students use multiplication and division with larger numbers. (4.NBT.5) (4.NBT.6)</td>
</tr>
<tr>
<td colspan="3">Clarification Statement:
• Requires students to apply properties of operations as strategies to multiply and divide
• Requires students to explain and represent the commutative property (two factors can be multiplied in either order and still have the same product
• Students need to explain and represent the associative property (the way in which three or more factors are grouped before multiplying does not affect the product)
• Students need to explain and represent the distributive property (breaking numbers into parts so that the numbers are easier to work with)
• Students need to apply properties to recall basic facts or multiply with multiples of 10
•These standards require students to explain the relationship between multiplication and division
•Students need to use multiplication to find an unknown in a division equation, as well as use division to find an</td>
</tr>
</table>

unknown in a multiplication equation.
• Students need to understand the relationship between multiplication and division and explain their processes of solving multiplication and division problems using an inverse operation.

Common Misconceptions
•Students may think that division is commutative. 5 ÷ 3 = 3 ÷ 5

•Students may see multiplication and division as different and unrelated operations.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying understanding properties of multiplication and the relationship between multiplication and division because when students review their understanding of arrays and multiplication as repeated addition, then they can begin to connect the concepts. Also, when students review their understanding of addition and subtraction as inverse operations, they can apply this knowledge to multiplication and division being inverse operations.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.4 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7. This standard provides a foundation for work with understanding properties of multiplication and the relationship between multiplication and division because once students are able to understand and interpret products, they can begin to build the understanding of the properties of multiplication as well as the relationship of multiplication and division. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences include ways to recruit interest such as creating accepting and supportive classroom climate because students who feel free to take chances and make mistakes learn more than students who are afraid of failure or afraid to be wrong. By taking chances and trying new strategies as well as sharing their thinking students can find misconceptions and correct them as well as further build and cement their understanding of the concept.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because when students can personally relate to a concept they understand the concept at a deeper level.

Students connecting multiplication and the relationship of multiplication to real life situations based on their interest and background will provide them the opportunity to relate something new to them to their lives which is very familiar and comfortable to relate to.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because when students to connect what they already know it helps build a connection to what they are learning. If students can connect the relationships of the operations of addition and subtraction and addition and multiplication to the relationship of multiplication and division, then they build a deeper understanding of the relationship of multiplication and divisions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because when students are able to solve problems in multiple ways. The more ways students can manipulate numbers the more they will understand the relationship between multiplication and division.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding properties of multiplication and the relationship between multiplication and division benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as embedding new ideas in familiar ideas and contexts because when students begin to see the connections of familiar contexts and ideas and the new ideas they begin to develop an understanding based on concepts they are already confident and comfortable with. For example, understanding the connection between addition and subtraction, addition and multiplication, will help students understand the relationship between multiplication and division.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding properties of multiplication and the relationship between multiplication and division by revisiting student thinking through a short mini-lesson because through resisting their thinking students will need to think deeper about the ideals they have formed. Often through revisiting their thinking students will begin to find misconceptions or errors in their reasoning that they must re-examine in order to explain.

Re-teach (intensive) What assessment data will help identify content needing to be revisited for intensive interventions?

- For example, some students may benefit from intensive extra time during and after a unit understanding properties of multiplication and the relationship between multiplication and division by helping students move from specific answers to generalizations for certain types of problems because this allows students to develop patterns and understand what the product is and how it relates to division. Rather than just a memorization of the facts, students need an understanding of what the product represents in order to understand multiplication and its relationship to division. Through understanding the properties of multiplication students will better understand how it is related to division.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying understanding properties of multiplication and the relationship between multiplication and division because it allows students to extend their understanding of multiplication and division and relate it to the relationship of other operations. It also helps students begin to explore the relationship of larger numbers.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying understanding properties of multiplication and the relationship between multiplication and division the types of mathematical tasks are critical because if procedural knowledge is built without conceptual understanding students are only learning a process or steps, however they are not learning the WHY or the meaning of the math. Without conceptual knowledge students are basically doing what a calculator or computer could accomplish. Through building a conceptual understanding of the properties of multiplication and the relationship of multiplication and division students do not have to rely merely on a procedure to get an answer. Students can think through the math and use the procedure as a tool of calculating their answer. Students should also then be able to assess the reasonableness of their procedural answer when they have a conceptual understanding of the concept.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 3 Operations and Algebraic Thinking
STANDARD: Apply properties of operations as strategies to multiply and divide (students need not use formal terms for these properties). Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) (03.OA.02.05)
LEARNING TARGET: I can identify different ways to multiply three numbers. DOK: 2

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1. Look at this problem.

$$2 \times 4 \times 3$$

Maggie and Jon worked this problem using different methods.

- Maggie multiplied 2 and 4, then she multiplied the answer by 3.
- Jon multiplied 2 and 3, then he multiplied the answer by 4.

Whose method is correct?

Ⓐ Only Maggie's method is correct.

Ⓑ Only Jon's method is correct.

Ⓒ Both Maggie's and Jon's methods are correct.

Ⓓ Neither Maggie's nor Jon's method is correct.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on multiplication, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, when making a quilt or blanket of a certain size, with a certain size of squares, how many squares would you need to create or make a quilt/blanket.	Science: Students could learn about Science by implementing information from plants, earth and space, cycles of life, animals, electricity and magnetism, and motion and sound to solve word problems.

3.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: C: Multiply and divide within 100.
Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.OA.C.7 **Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.**	SMP 5: Students can use appropriate tools strategically: by selection various tools and to show equal sets to determine the product or the missing factor. SMP6: Students can attend to precision by use appropriate vocabulary (factor, product, quotient) to associate meaning to their work and describe their thinking accurately to others. SMP8 Look for and express regularity in repeated reasoning by understanding how to apply the inverse properties of multiplying & dividing to build fluency.	• Solve problems using models and drawings as visuals as examples that represent multiplication and division facts. • Relate models to written equations. • Utilize strategies based on properties and patterns of multiplication to learn multiplication facts. • Use multiplication facts in terms of missing factors to learn division facts **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand
<u>Previous Learning Connections</u> Connect to previous work understanding equal groups, skip counting by 2, 5, 10, 100's, work with arrays up to 5 rows and 5 columns. (**2.OA.3, 2.OA. 4 and 2.NBT.2)**	**<u>Current Learning Connections</u>** Connections to multiplication and division exist across the standards in third grade. (**3.OA.1) (3.OA.2)(3.OA.3) (3.MD.7) (3.OA.5) (3.OA.6) (3.NBT.3) (3.OA.4) (3.OA.7) (3.OA.8) (3.OA.9**)	**<u>Future Learning Connections</u>** Connect to 4th grade, where students use multiplication and division with larger numbers. (**4.NBT.5) (4.NBT.6**)

Clarification Statement:
•This standard requires students to be fluent with multiplication and division facts
•They need to use strategies, knowledge of relationships between multiplication and division, and the properties of operations, to recall basic facts quickly and accurately.
•It is not enough for students to recall facts from memory from timed tests alone, but from experiences with

manipulatives, pictures, arrays, and word problems to internalize the basic facts.
•There are many strategies students may use to attain fluency:

- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
- Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
- Five facts (half of tens)
- Skip counting (counting groups of __ and knowing how many groups have been counted)
- Square numbers (ex: 3 x 3)
- Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3)
- Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
- Commutative Property of Multiplication
- Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24)
- Missing factors

•Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.
•Students should have exposure to equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Common Misconceptions
•Students may struggle with fully comprehending the strategies that will help them achieve fluency. It is critical for each of these strategies to be taught explicitly.
•Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as 15 ÷ 3 =? or 15 = • x 3.
•Students may think that 3 ÷ 15 = 5 and 15 ÷ 3 = 5 are the same equations.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying multiplying and dividing within 100 fluently because students will need to use many different strategies and properties for understanding multiplication and division in order to find products & quotients for both smaller and larger numbers.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.A.1 & 2.OA.A.2 These standards provide a foundation for work with representing and solving problems involving multiplication and division by interpreting products of whole numbers & interpreting whole-number quotients of whole numbers because students need to have a clear understanding of what multiplication & division means (using arrays, equal groups, area models, and repeated addition) before they would be able to find products, quotients, and become fluent . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction
Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with fluently multiplying and dividing within 100 benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to <requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard> because students need to use a variety of strategies at multiple levels and speeds to understand, practice and learn all the different multiplication & division facts with accuracy so they can become fluent with all the facts while meeting their individual learning needs.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with fluently multiplying and dividing within 100 benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because with feedback, students can keep data tracking their progress towards mastery of the multiplication & division facts so they can celebrate growth, and know which facts need maintenance versus which facts still need more practice for learning and memorizing.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with fluently multiplying and dividing within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because students need to rely on previously taught strategies such as drawing arrays or skip counting and properties such as decomposing numbers into known facts and fact families to understand and find products & quotients accurately before they can memorize them.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with fluently multiplying and dividing within 100 benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because students will need extensive practice (and motivation) with facts using different modalities and materials such as computer programs & games, timed tests, making visual support flash cards, buddy practice, board games, making arrays, oral recitation, and problem solving to both learn & move the fact knowledge to memory with accuracy and fluency.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with fluently multiplying and dividing within 100 benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because the student can set smaller achievable goals (learning their 3's facts) and get enough feedback to monitor progress and see growth to be aware of where they are in the process of meeting each goal and how that relates to final goal of learning all their facts fluently.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on 3.OA.B Understanding properties of multiplication and the relationship between multiplication and division by clarifying mathematical ideas and/or concepts through a short mini-lesson because applying properties like the commutative property, associative property, distributive property, and fact family knowledge of multiplication & division and their relationship is crucial for students to use to access multiple strategies for finding products & quotients before becoming fluent with facts.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit 3.OA.A Represent and solve problems involving multiplication and division by addressing conceptual understanding because students need to concretely understand the meaning of multiplication concretely using arrays, equal groups, area models, tape diagrams, and repeated addition before they can become proficient at fact knowledge and retrieval.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying multiplying and dividing within 100 fluently because students could see the application of multiplying & dividing as well as be provided with a realistic opportunity to extend their ability to multiply by a double digit number based on prior strategies when learning to multiply (such as decomposing numbers).

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying multiplying and dividing within 100 fluently goal setting is critical because students will need short term goals for learning facts clusters from memory because there are too many facts which would be overwhelming for students to memorize without having them broken down into smaller achievable pieces.

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying multiplying and dividing within 100 fluently the types of mathematical tasks are critical because students need to rely on a multitude of strategies to be able to understand and find products and quotients before they can commit the facts to memory.
Standards Aligned Instructionally Embedded Formative Assessment Resources: *Source*: http://tasks.illustrativemathematics.org/content-standards/3/OA/C/7 Kiri's Multiplication Matching Game Task There were no Cognia Testlets for this cluster of standards. Addresses standard 3. OA.C.7 - Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8×5=40, one knows 40÷5=8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. This game is a fun way for the student to practice arithmetic skills to the point where the single-digit facts are committed to memory. It reinforces the relationship between multiplication and division, and depending on the target cards can also connect these recall skills with other skills such as estimation and understanding of properties. The only necessary materials, the cards, can be produced easily and can be re-used. After playing regularly, students could be engaged in making new target cards.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on multiplying and dividing within 100 fluently, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways multiplying is used in the home and community for planning, shopping and cooking can a be a great way to connect schools tasks with home tasks.	Social Studies: Calculations related to populations, supply, goods, costs.

<table>
<tr><th colspan="3">3.OA: OPERATIONS & ALGEBRAIC THINKING</th></tr>
<tr><td colspan="3">Cluster Statement: D: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

3.OA.D.8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td><td rowspan="3">Standard for Mathematical Practices

SMP1: Students can make sense of problems and persevere in solving them by understanding single and multi-step word problems to distinguish relevant information and appropriate strategies, and apply them to find solutions.

SMP3: Students can construct viable arguments and critique the reasoning of others by constructing mathematical arguments to justify the reasonableness of their answer using rounding, mental computation, or other estimation strategies, and compare their strategies with those of classmates. Students make connections among ideas and between concrete models and numerical notations.</td><td>Students who demonstrate understanding can:
• Solve two-step word problems using addition, subtraction, and multiplication.
• Determine the first step in a two-step word problem. Students then are able to determine the second step in a two-step word problem.
• Utilize models, drawings, and equations to represent the equation.
• Represent problems using equations with a symbol for the unknown number.
• Develop their skills and assess the answer that it makes sense and correlates with visual equations</td></tr>
<tr><td>Depth of Knowledge: 2-3</td></tr>
<tr><td>Bloom's Taxonomy: Apply, Analyze</td></tr>
<tr><td>Standard Text

3.OA.D.9
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</td><td>Standard for Mathematical Practices

SMP5: Students can use appropriate tools strategically choosing a variety of representations to identify patterns.

SMP7: Students can look for and make use of structure by extending mathematical patterns in a variety of situations, including tables and problems, and connect those patterns to the properties. These patterns help students to understand the structure of the four operations.</td><td>Students who demonstrate understanding can:
• Identify patterns in addition and multiplication charts.
• Explain patterns when adjusting addends (any number that is added together in addition problem).
• Explain doubling a factor doubles the product.
• Explain a factor can be decomposed and the partial products can be put back together.
• Interpret patterns of multiplication on a hundreds board and/or multiplication table.</td></tr>
</table>

		• Use visuals that represent their thinking when identifying arithmetic patterns.
		Depth Of Knowledge: 1-2
		Bloom's Taxonomy: Remember, Understand
Previous Learning Connections Connect to addition and subtraction problems, skip counting and adding equal groups. Learners used addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (**2.MD.5) (2.OA.1) (2.NBT.5) (2.OA.3) (2.NBT.2)**	**Current Learning Connections** Connect to the work throughout third grade with multiplication and division problems. (**3.OA.3) (3.OA.6) (3.MD.8) (3.OA.3) (3.OA.7) (3.MD.7) (3.OA.5) (3.OA.4) (3.OA.8) (3.OA.9**)	**Future Learning Connections** Connect to future work with solving multi-step word problems using the four operations and generating patterns which follow a given rule. **(4.OA.3) (4.MD.2) (4.OA.5)**

Clarification Statement:
•This standard requires students to use their knowledge of the four **operations** to solve two-step word problems. They need to be able to determine the first and second step in a two-step word problem.
•They need to be able to represent a two-step word problem with models, pictures, and equations (two equations can be used in place of an equation with two operations). They also need to write an **equation** using a letter for the unknown.
•Students will determine if a solution to a two-step problem is **reasonable** using **mental computation** and **estimation** strategies including **rounding**.
•When adding and subtracting numbers, problems should include numbers within 1,000
•When multiplying numbers, problems should include single-digit factors and products less than 100.
•This standard requires students to examine patterns of multiplication. The ability to recognize and explain **patterns** in mathematics leads students to developing the ability to make generalizations, a foundational concept in algebraic thinking.
•Some of the patterns students in third grade are expected to describe and explain are:
- •Patterns in addition and multiplication charts
- •Patterns when adjusting addends (56 + 98 is the same as 54 + 100)
- •Doubling a factor doubles the product
- •A factor can be decomposed and the partial products can be put back together
- •Patterns in addition (even + even = odd, odd + odd = even, odd + even = odd, two addends less than 50 have a sum less than 100, a difference of numbers is unchanged when both numbers are adjusted by the same amount)
- •Patterns in multiplication (even x even = even, odd x odd = odd, and odd x even = even)

Common Misconceptions
•Many students may think a patter occurs if it only happens twice.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM.*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying Solving problems involving the four operations, and identifying and explaining patterns in arithmetic because students will need to apply the prior knowledge of addition and subtraction in conjunction with the newer knowledge of multiplying and dividing when solving multi-step problems involving a combination of the four operations and identifying patterns.

Pre-teach (intensive) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- 2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions: This standard provides a foundation for work with representing and solving problems involving addition and subtraction because using and understanding addition and subtraction to solve one & two steps problems is foundational to solving more complex problems involving any combination of the 4 operations. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with solving problems involving the four operations, and identify and explain patterns in arithmetic benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students will be more motivated to learn and solve problems if the problems are related to their lives or if the problems are real to the student for solving actual problems.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving problems involving the four operations, and identify and explain patterns in arithmetic benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because students can increase their effort and persistence with support of peers by fostering community and collaboration.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving problems involving the four operations, and identify and explain patterns in arithmetic benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because students will have better understanding of the problem when there are visual supports to help scaffold and clarify key ideas.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving problems involving the four operations, and identify and explain patterns in arithmetic benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students have a variety of experiences and knowledge and sharing multiple ways gives all students a potential clear method to understand how to solve a problem in a way that makes sense to them at their current ability as well as extend and expand their comprehension of the problem.

Internalize
Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with solving problems involving the four operations, and identify and explain patterns in arithmetic benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as provide tasks with multiple entry points and optional pathways because students approach problems with varying sets of abilities and the multiple entry points and pathways will demonstrate a student's level of comprehension by showing if the student relies on and understands how to use manipulatives, patterns, drawings, and formulas to solve the problems. This clarifies what supports the student may need and at what level of comprehension (concrete, visual, or abstract).

Re-teach
Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representing and solving problems involving multiplication and division by revisiting student thinking through a short mini-lesson because understanding and representing multiplication and division appropriately to solve one & two steps problems is foundational to solving more complex problems dependent on a mix of the 4 operations.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on understanding properties of multiplication and the relationship between multiplication and division by offering opportunities to understand and explore different strategies because students may need to see, draw, and use manipulatives to better clarify their understanding of properties of multiplication (distributive, commutative, associative) with arrays, equal groups and number lines to concretely solidify their understanding of the properties.

Extension
What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying Solving problems involving the four operations, and identify and explain patterns in arithmetic because students can be provided the opportunity of solving more complex questions, problems, and patterns without scaffolding assistance.

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Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying Solving problems involving the four operations, and identifying and explaining patterns in arithmetic. supporting productive struggle is critical because students need to be able to access (low entrance) to the problem so they feel they are capable of solving the problem using their repertoire of skills whether it is using manipulatives, drawing pictures, or jumping to formulas. They need to receive support at whatever level they begin to continue through and have expectations that they can solve the problem with their current skills whether or not it is the most efficient. Their thinking and processes need to be validated for correctness and shared as are more efficient methods of peers. Perseverance and comprehension should be celebrated.

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Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/3/OA/D/8/tasks/1301

The Class Trip Task

Addresses standard 3. OA.D.8 - Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The purpose of this instructional task is for students to solve a two-step word problem and represent the unknown quantity with a variable.

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Relevance to families and communities:

During a unit focused on Solving problems involving the four operations, and identifying and explaining patterns in arithmetic, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different examples of problem solving that occurs in the home and community can be a great connection to school. The problem solving can involve shopping, project costs, gardening resources, home repairs, building, and other home maintenance. Local jobs can also provide an opportunity to see the types of problem solving and pattern recognition that community workers are experiencing.

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Cross-Curricular Connections:

Language Arts: Students can write down a step by step instruction guides on arithmetic patterns using the addition table or multiplication table and explain how to use them using the properties of operation. Students can publish their guides and keep them in a resource writing center.

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3.NBT: NUMBERS & OPERATIONS IN BASE TEN

Cluster Statement: A: Use place value understanding and properties of operations to perform multi-digit arithmetic.
Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	SMP6 Students can attend to precision by using appropriate vocabulary including that of base ten place value and rounding to communicate their reasoning and justify their answer to rounding problems.	• Identifying the place value of digits in the ones, tens, hundred, and thousands place. • Round up a two-digit number in the tens place by looking at the place value of the ones. • Round up a three-digit number in the hundreds place by looking at the place value of the tens place. • Explain why and how they rounded with accuracy. • Demonstrate their understanding through visuals that correlate to place value understanding to round whole numbers
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	SMP 2 Students can reason abstractly and quantitatively by reasoning and express their knowledge of place value and of addition and subtraction algorithms when finding the unknown number. SMP 4 Students can model with mathematics by using representations, including number lines, bundling into groups of tens and groups of one hundred, to	• Demonstrate their understanding by fluently using place value to add and subtract. • Add and subtract whole numbers up to and including 1,000. • Use estimation strategies to assess reasonableness of answers. • Model and explain how the relationship between addition and subtraction can be applied to solve addition and subtraction problems.

	demonstrate and explain their thinking. SMP 6 Students can attend to precision by incorporating appropriate vocabulary including that of place value and properties of operations in their explanations. SMP 7 Students can look for and make use of structure by using the structure of place value (composing and decomposing tens and hundreds) and addition and subtraction algorithms to develop efficient strategies to add and subtract to within 1000.	• Use expanded form to decompose numbers and then find sums and differences • Utilize visuals that represent their understanding of place value to round whole numbers to the nearest 10 or 100. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text 3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**Standard for Mathematical Practices** SMP 2 Students can reason abstractly and quantitatively by using their knowledge of place value, properties of operations, and multiplication algorithms to find the unknown number. SMP 4 Students can model with mathematics by using concrete and pictorial models or manipulatives to represent base ten quantities for multiplying. SMP 6 Students can attend to precision by using appropriate vocabulary including that of place value and properties of operations in their explanations. SMP 7 Students can look for and make use of structure by utilizing the structure of numbers (i.e. place value, composing and decomposing tens and hundreds) and multiplication algorithms to develop efficient strategies to perform multiplication and explain their thought process.	**Students who demonstrate understanding can:** • Using strategies based on place value knowledge and the utilization of the properties of operations. • Use concrete and pictorial models, based on place value and the properties of operations to find the product of a one-digit whole number by a multiple of 10 in the range 10–90 **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• In 2nd grade, learners used place value understanding and properties of operations to add and subtract. • Connect to understanding that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases. (**2.NBT.1**) • Connect to numbers within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. **(2.NBT.7)** • Connect to why addition and subtraction strategies work, using place value and the properties of operations. (**2.NBT.9**) • Connect to mentally added 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. **(2.NBT.8)**	• Connect to apply properties of operations as strategies to multiply and divide.2 (**3.OA.5**) • Connect to fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (**3.NBT.2**) • Connect to relate area to the operations of multiplication and addition. (**3.MD.7**) • Connect to multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (**3.NBT.3**)	• Connect to use place value understanding and properties of operations to perform multi-digit arithmetic. Learners use place value understanding to round multi-digit whole numbers to any place. (**4.NBT.3**) • Connect to fluently add and subtract multi-digit whole numbers using the standard algorithm. (**4.NBT.4**) • Connect to multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (**4.NBT.5**) • Connect to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (**4.NBT.6**)

Clarification Statement:
• The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round.
• Students build on work in previous grades regarding strategies based on place value, the properties of operations, and relating addition to subtraction.
• Students continue adding and subtracting within 1,000, extending their understanding of place value by composing and decomposing tens and hundreds.
• Students explain their thinking and show their work and verify that their answer is reasonable. Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties.
• Estimation strategies include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations. Estimation strategies include, but are not limited to: front-end estimation with

adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts), rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), using friendly or compatible numbers such as factors

•Students extend on their work in multiplication by applying understanding of place value.

• The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10.

• Using the properties of operations (commutative, associative, and distributive) and place value, students are able to explain their reasoning.

Common Misconceptions

- Students may misunderstand "rounding down" and actually lower the value of the digit in the designated place.
- Students may misunderstand "rounding up" and change the digit in the designated place while leaving digits in smaller places as they are.
- Students who learn to add and subtract procedurally without a deep understanding of place value and regrouping will struggle to determine whether their answers are reasonable.
- Students may not understand the concept that 10 in any position (place) makes one (group) in the next position and vice versa.
- Students may not understand that multiplying 3 X 40 means you have 3 groups of 4 tens and that is 12 tens or 120 (rather than multiply 4 X 3 and "add a zero at the end").

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying using place value understanding and properties of operations to perform multi-digit arithmetic because understanding place value is the foundation of understanding numbers. Through understanding place value students will be able to easily manipulate numbers to help with mental math strategies. Understanding of place value also helps students to understand addition and subtraction.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.NBT.A.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. This standard provides a foundation for work with using place value understanding and properties of operations to perform multi-digit arithmetic because it helps students understand the process of regrouping and the reasoning of why we regroup when adding/subtracting. It also provides the basis of the place value system so students can expand their understanding to larger numbers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as layout of the visual or other elements lining the numbers up with larger graph paper, putting the numbers in a table, etc.. because this will allow students to access the numbers and the concept more effectively. The ability to manipulate sensory and perceptual features provides options for increasing the clarity and importance of the information for a wide range of learners. It also allows for adjustments and preferences of others when manipulating and working with the concept.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because all learners must be able to collaborate and communicate effectively. Groups with defined roles allows all students to be active participants in the group and helps keep students engaged in the group. It also allows for differentiation and guided peer support.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because when students understand the vocabulary they are better prepared to understand the concept. The key vocabulary is the gateway into discussing, making connections, and applying the concept. Understanding key vocabulary such as ones, tens, hundreds, thousands, addition, subtraction, etc. will help students when manipulating numbers.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because using a variety of tools and strategies allows students to express and show their learning in a way that is meaningful to them. It allows more points of access to the content. It also provides a more flexible and accessible toolkit in which learners can successfully take part in their learning and show or share what they understand and know.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing options for organizing and possible approaches (tables and representations for processing mathematical operations) because place value understanding including addition and subtraction of numbers is a key concept. All learners need to be able to generalize and transfer their learning into different contexts. Students need multiple representations for this to occur. In order to understand place value, addition, and subtraction students need to understand more than just an algorithm or step. Students need multiple representations to help understand the meaning and other ways to manipulate the numbers in order to build understanding of place value.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using place value understanding and properties of operations to perform multi-digit arithmetic by clarifying mathematical ideas and/or concepts through a short mini-lesson because it will provide students the opportunity to rethink about the value of numbers and digits in numbers. Understanding the value of digits and numbers provides students the foundation to understand addition and subtraction of numbers.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit using place value understanding and properties of operations to perform multi-digit arithmetic by clarifying mathematical ideas and/or concepts by addressing conceptual understanding because students need to understand and internalize the value of numbers. The more students understand the value of numbers the more they will be able to manipulate numbers and begin to understand addition and subtraction of numbers.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying using place value understanding and properties of operations to perform multi-digit arithmetic because when students are allowed to apply their understanding to real world tasks, they develop a deeper understanding of the concept. Through applying their understanding of place value as well as adding and subtracting numbers to a real world, multi-disciplinary tasks students must take their possibly isolated understanding of a concept and integrate and apply it to their own interests and lives.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying using place value understanding and

properties of operations to perform multi-digit arithmetic goal setting is critical because it allows students the opportunity to think about and evaluate where they are at in their learning. It helps students focus on where they need to go and helps them develop a plan to get where they need to be.
Standards Aligned Instructionally Embedded Formative Assessment Resources: Source: Cognia Testlet for Grade 3 Numbers and Operations in Base Ten STANDARD: Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (03.NBT.01.03) LEARNING TARGET: I can add a three-digit and two-digit number and find the missing number in a multiplication expression that equals the sum of the addition expression. DOK: 3 **1.** An equation is shown. $316 + 44 = 6 \times ?$ **a.** What is the sum of the left side of the equation? Show your work or explain how you know. **b.** What number can be written in place of the question mark that makes the equation true? Show your work or explain how you know.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on rounding, adding, and subtraction consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about how families determine how much and what type of cloth to make various textiles for their family and the community.	Language Arts: Expository writing to describe scientific or statistical data. Social Studies: Understanding statistical data in current events

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3.NF: NUMBER & OPERATIONS-FRACTIONS		
Cluster Statement: A: Develop understanding of fractions as numbers. **Major Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text **3.NF.A.1** **Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$.**	**Standard for Mathematical Practices** SMP4: Model with mathematics: Students will use pictures or tiles to help create & cut equal parts of a fraction and label each part correctly. SMP6: Attend to precision: Students will use fraction vocabulary and relate to models with precision (numerator, denominator, unit fraction, whole, non-unit fraction) SMP7: Look for and make use of structure: Students need to recognize the structure and visual pattern when numerators change to determine how a fraction would be named when more than one-unit part is shaded/considered by adding parts or counting on from the unit fraction.	**Students who demonstrate understanding can:** • Recognize a unit fraction such as ¼ as the quality formed when the whole is portioned into 4 equal parts. • Identify a fraction such as 2/3 and explain that quantity formed is 2 equal parts of the whole portioned into 3 equal parts (1/3 and 1/3 of the whole 3/3). • Express a fraction as the number of unit fractions • Use accumulated unit fractions to represent numbers equal to, less than, and greater than one (1/3 and 1/3 is 2/3; 1/3,1/3 1/3, 1/3 is 4/3). • Explain and represent a unit fraction. • Explain and represent a non-unit fraction. • Identify the numerator and denominator and understand the meaning of each in the fraction • Explain how fraction representations are related (1/b relates to a/b) • Identify a unit fraction and build other fractions from the unit fraction.
		Depth of Knowledge: 1
		Bloom's Taxonomy: Remember, Understand

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.NF.A.2 **Understand a fraction as a number on the number line; represent fractions on a number line diagram.** • **3.NF.A.2.A: Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.** • **3.NF.A.2.B: Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.**	SMP4: Model with mathematics: Students will use pictures, tiles, and number lines to represent fractions visually. SMP2: Reason abstractly and quantitatively: Students will reason about what it means when numerators & denominators get larger/smaller and determine how to break up a number line (using the denominator) & where the fraction would be located. SMP5: Use appropriate tools strategically: Students can use fraction tiles, pictures, and number line to help create and represent fractions on a number line diagram.	• Understand fractions as numbers • Interpret fractions with denominators of 2,3,4,6, and 8 using area and length models • Using an area model, explain that the numerator of a fraction represents the number of equal parts of the unit fraction • Using a number line, explain that the numerator of a fraction represents the number of lengths of the unit fraction from 0 • Recognize a unit fraction such as ¼ as the quantity formed when the whole is partitioned into 4 equal parts • Express a fraction as the number of unit fractions • Define the interval from 0 to 1 on a number line as the whole • Divide a whole on a number line into equal parts • Recognize that the equal parts between 0 and 1 have a fractional representation • Represent each equal part on a number line with a fraction • Explain that the endpoint of each equal part represents the total number of equal parts
		Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
3.NF.A.3 **Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.** • **3.NF.A.3.A: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.** • **3.NF.A.3.B: Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.** • **3.NF.A.3.C: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.** ***Examples: Express 3 in the form 3 = $\frac{3}{1}$; recognize that $\frac{6}{1}$ = 6; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.*** • **3.NF.A.3.D: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.**	SMP2: Students can reason abstractly and quantitatively by applying reasoning skills to analyze what it means when numerators & denominators get larger/smaller and place fractions on a number line that could go larger than 1 whole in order to compare the fractions to benchmark numbers, whole numbers and to each other when the numerators or denominators are the same. SMP3: Students can construct viable arguments and critique the reasoning of others by justifying their answers while comparing fractions with a clear explanation using words, pictures, or numbers.	• Compare fractions by reasoning about their size to determine equivalence • Use number lines, size, visual fraction models, etc. to find equivalent fractions • Recognize whole numbers written in fractional parts on a number line • Recognize the difference in a whole number and a fraction • Explain how a fraction is equivalent to a whole number • Explain what the numerator in a fraction represents and its location • Explain what the denominator in a fraction represents and its location • Explain that a fraction with the same numerator and denominator equals one whole • Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers • Recognize whether fractions refer to the same whole • Determine if comparisons of fractions can be made (if they refer to the same whole). • Compare two fractions with the same numerator by reasoning about their size. • Compare two fractions with the same denominator by reasoning about their size. • Compose and decompose fractions into equivalent fractions using fractions: halves, fourths and eighths; thirds and sixths • Record the results of comparisons using symbols <, =, or >. • Justify conclusions about the equivalence of fractions

		Depth of Knowledge: 2-3
		Bloom's Taxonomy: Apply, Analyze
Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• Connect to partition shapes, measure length and solve problems using addition and subtraction. • Connect to equally partitioned circles and rectangles into halves, thirds and fourths, and recognized that equal shares of identical wholes need not have the same shape. (**2.G.3**) • Connect to measure the length of an object twice, using length units of different lengths for the two measurements; described how the two measurements relate to the size of the unit chosen (**2.MD.2**). • Connect to use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (**2.MD.5**) • Connect to represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1, 2..., and represent whole-number sums and differences within 100 on a number line diagram (**2.MD.6**). • Connect to generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a	• Connect to partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (**3.G.2**). • Connect to generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters (**3.MD.4**)	• Connect to understanding of fraction equivalence, build fractions from unit fractions and understand and compare decimal fractions. Learners will understand a fraction a/b with a>1 as a sum of fractions 1/b (**4.NF.3**). • Connect to make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction of fractions by using information presented in line plots (**4.MD.4**). • Connect to apply and extend previous understanding of multiplication to multiply a fraction by a whole number (**4.NF.4**) • Connect to explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (**4.NF.1**)

line plot, where the horizontal scale is marked off in whole-number units (**2.MD.9**)		

Clarification Statement:
• Students use area and length models to compose and decompose fractions into **equivalent fractions** using related fractions: **halves, fourths, eighths, thirds,** and **sixths**.
• **Related fractions** are fractions in which one denominator is a multiple of the others; thirds and sixths are related fractions, while fourths and sixths are not related fractions.
• Students should be able to explain that fractions with the same **numerator** and **denominator** equal one whole.
• Renaming fractions with the same numerator and denominator as one whole without a model is not sufficient for this standard.
• The standard also expects students to express **whole numbers** as **fractions**. This work is limited to whole numbers less than 4.
• Expressing whole numbers as fractions lay the groundwork for seeing a fraction as a division problem, e.g., the fraction 4/2 represents 4 pieces that are a half each that equal 2 wholes.
• This standard is the building block for later work in Grade 5 where students divide a set of objects into a specific number of groups.

Common Misconceptions
• Students may not use benchmark numbers like 0, 1/2, and 1 to compare fractions because they have restricted their understanding of fractions to part-whole situations and do not think of the fractions as numbers.
• Students may overgeneralize and think that "all 1/4 s (for example) are equal".
• Students may not understand that the size of the whole determines the size of the fractional part.
• Students may struggle with the idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying develop understanding of fractions as numbers because while using different representations, such as pictures, students are introduced to appropriate labels to communicate the meaning of their representation.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.G.A.3: This standard provides a foundation for work with develop understanding of fractions as numbers because student need the foundational understanding of equal share partitioning, how to accurately describe the shares, and the understanding that decomposing into more equal shares creates smaller shares in order to make the connection between concrete (e.g., pictures) and abstract (e.g., fractions as numbers). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with develop an understanding of fractions as numbers benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because students may recognize the underlying mathematical relationships in representations quickly but may need support perceiving them in a different representation. By showing several representations students can connect and build from one representation to another allowing for greater long-term access to all representations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because the external environment must provide options that can equalize accessibility by supporting learners who differ in initial motivation, self-regulation skills, etc.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because text is a particularly weak form of presentation for learners who have text- or language-related disabilities. Providing alternatives—especially illustrations, simulations, images or interactive graphics—can make the information more comprehensible for any learner and accessible for some who would find it completely inaccessible in text.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-10 blocks, number lines, fraction bars) because virtual and concrete manipulatives provide a more flexible and accessible toolkit with which learners can more successfully take part in their learning and articulate what they know. Unless a lesson is focused on learning to use a specific tool (e.g., learning to draw with a compass), curricula should allow many alternatives. Learners should learn to use tools that are an optimal match between their abilities and the demands of the task.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with develop understanding of fractions as numbers benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections (e.g., teaching literacy strategies in the social studies classroom) because all learners can benefit from assistance in how to transfer the information they have to other situations, as learning is not about individual facts in isolation, and students need multiple representations for this to occur.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on developing understanding of fractions as numbers by revisiting student thinking through a short mini-lesson because when students can explain their thought process, they also understand the possibility of different interpretations and therefore the necessity for precision in their work.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on developing understanding of fractions as numbers by confronting student misconceptions because it takes time and multiple passes to develop understanding, so students need regular opportunities to think about, talk through, and refine ideas

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying develop understanding of fractions as numbers because in order to develop and solidify ideas, students need to be able to connect what they are learning to multiple disciplines and real-world connections through the productive struggle of open-ended tasks.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying understanding fractions as numbers supporting productive struggle is critical because building and internalizing fractions as parts of a whole that can be represented by numbers is foundational to all future comprehension of fraction math problems. It is through productive struggle that students develop their own understanding of math concepts in a mental context that they own, that has meaning to them, and that they can easily access and manipulate for future procedural tasks and problem solving.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 3 Number and Operations Fractions
STANDARD: Develop understanding of fractions as numbers: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (03.NF.01.03)
LEARNING TARGET: I can find two equivalent fractions and compare them to a third fraction. DOK: 2

1. Look at these fractions.

$$\frac{2}{3}, \frac{1}{2}, \frac{4}{6}$$

a. Which two of the fractions are equivalent to each other? Show or explain how you know.

b. Is the remaining fraction greater than or less than the two equivalent fractions? Show or explain how you know.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on understanding fractions as numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example: increasing or decreasing ingredients in a recipe; using statistics from current events, relevant to a family or community, to determine their impact (converting a whole number statistic into a fraction to show its impact as part of a greater whole population); using a given amount of time, determining how it is used, and comparing the portions of that time for time management goals, using a given amount of money, determining how it is used, and comparing the portions of that money to set a budget; dividing a plot of land into equal portions to plan and plant a garden, etc.	Social Studies: Based on current events and topics, create a survey, collect data, and represent results in each category as fractions of the whole survey population. Music: Reading the value of musical notes 1/4, 1/2, 3/4, 1 and the relation of note count to measure.

3.MD: MEASUREMENT & DATA

Cluster Statement: A: Solve problems involving measurement and estimation.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<table>
<tr>
<td rowspan="3">Standard Text

3.MD.A.1: Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by interpreting, analyzing, and solving word problems involving elapsed time using number line diagrams or other strategies.

SMP 6: Students can attend to precision by accurately reading and writing time to the nearest minute.

SMP 7: Students can look for and make use of structure by applying the patterns and structures of analog and digital clocks, such as there are 5 minutes between each marked number, to write and tell time.</td>
<td>Students who demonstrate understanding can:
• Recognize minute marks on an analog clock and minute position on a digital clock f
• Write time to the nearest minute
• Tell time to the nearest minute
• Find elapsed time in minutes using a number line diagram
• Solve word problems involving elapsed time in minutes by using a number line diagram</td>
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<tr>
<td>Depth of Knowledge: 1-2</td>
</tr>
<tr>
<td>Bloom's Taxonomy:
Remember, Understand and Apply</td>
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<tr>
<td>Standard Text

3.MD.A.2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</td>
<td>Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by interpreting, analyzing, and solving word problems involving liquid volume and mass.

SMP 5: Students can use tools to estimate and measure to solve liquid volume and mass word problems.

SMP 6: Students can attend to precision by using appropriate mathematical language and</td>
<td>Students who demonstrate understanding can:
• Estimate and measure liquid volumes using standard units of liters (l)
• Solve one-step word problems involving liquid volume given in the same units
• Estimate and measure masses of objects using standard units of grams(g) and kilograms (kg)
• Solve one-step word problems involving masses given in the same units
• Represent a word problem involving liquid volume or mass using various strategies</td>
</tr>
</table>

	abbreviations as they name units of measurement (grams, kilograms, & liters).	**Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Previous Learning Connections • Connect to telling and writing time from analog and digital clocks to the nearest 5 minutes using AM and PM. (**2.MD.7**)	**Current Learning Connections** • Connect to understanding a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; and understanding a fraction a/b as the quantity formed by a parts of size 1/b. **(3.NF.1)** • Connect to solving two-step word problems using the four operations; and **r**epresenting these problems using equations with a letter standing for the unknown quantity. Connect to assessing the reasonableness of answers using mental computation and estimation strategies including rounding. **(3.OA.8)**	**Future Learning Connections** • Connect to recording measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36). (**4.MD.1)** • Connect to multiplying or dividing to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **(4.OA.2)** • Connect to using the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Connect to representing measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **(4.MD.2)**
Clarification Statement: In this standard, students reason about the units of length, capacity and weight using customary units. Students need to develop a basic understanding of the size and weight of customary units and apply this understanding when estimating and measuring. Students are not expected to convert between units. The focus is on measuring and also reasoning as they estimate, using benchmarks to measure length, weight, and capacity.		

Word problems should only be one-step and include the same unit. The number range for these tasks should match the number size described in the OA and NBT standards

Common Misconceptions

● Students might overgeneralize the base-10 structure and apply it to time, such as changing 1 hour 15 minutes to minutes as 115 minutes or 25 minutes

● Students may believe that a larger object automatically has more mass.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects because students are building on previous knowledge from earlier grades where they described , measured, estimated, and compared amounts, using standard units of measurement as well as non-standard units such as scoops or cups to measure liquids. Students are also building on previous knowledge from earlier grades in telling and writing time to the nearest hour and half hour.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.MD.a.1: This standard provides a foundation for work with solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects because students will be using measurement and estimation to solve problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects benefit creating an accepting and supportive classroom climate because students need to feel safe and supported in their environment so they are willing to take risks and make mistakes so they can do their best learning. If students feel supported and safe then they know they can approach math in a way that makes sense to them and are supported by their teacher and peers. Activities or tasks are chosen to foster the use of imagination, to solve novel and relevant problems, or to make sense of complex ideas in creative ways.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because using flexible grouping gives students the opportunity to ask questions, ask for clarification, or encourage one another through coaching and the use of questioning when students get stuck. Students working in groups are encouraged to persevere and are celebrated for their thinking and effort. Students may use different strategies to solve problems involving time, liquid volumes, and mass. Students may use different manipulatives but are still exploring finding the solution to the problem. Talking with peers provides opportunities for students to sustain their effort and perseverance.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because it's important for students to have an understanding of important mathematical vocabulary so it doesn't impede their learning during the lesson. Before the lesson begins, the teacher may break down the objectives and use pictures, examples, or different words to pre teach vocabulary and symbols. This gives the students background knowledge and helps students make connections to their own experiences and prior knowledge.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because it allows for students to solve the problem in a way that makes sense to them and allows them to be able to talk about their thinking and their strategy when solving real world problems involving time, liquid volume, and mass.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as asking questions to guide self-monitoring and reflection because it is important for students to ask themselves questions as they learn and reflect on their learning. If students are consistently doing this then they know whether they understand or are still struggling with the concept.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects by providing specific feedback to students on their work through a short mini-lesson because students may need more time exploring measurement and working with what units they are using to measure objects and why. Students also may need more time exploring reading time. As the students explore, the teacher gives them specific feedback to help clear up misconceptions.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited?*

- For example, students may benefit from re-engaging with content during a unit on solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects by confronting student misconceptions in a small group setting or pull out if needed. In a smaller group setting, the teacher will be able to monitor students' progress on a biweekly basis to assess student growth and their responses to intensive interventions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects because students can make connections between different types of measurement and how they are used in the real world and across different topics.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying solving problems involving measurement and estimation of intervals of time, liquid volumes, and mass of objects the pattern of questions within the classroom is critical because these purposeful questions are used to guide students and advance students' reasoning and make sense about important mathematical ideas and relationships. Students having access to funneling and focusing questions gives them the support they need to be successful. When students have access to purposeful questions in the classroom, students understand that the teacher or their peers are seeking to understand their thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Sources: Cognia Testlet for Grade 3 Measurement and Data
3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
Learning Target: I can show how to solve telling time word problems by reading a clock and adding and subtracting minutes. DOK: 2

1. Look at this clock.

a. What time does the clock show?

Carter is meeting Zachary in 23 minutes.

b. At what time is Carter meeting Zachary? Show your work or explain how you know.

The clock shows the time that Carter finished reading an article. He spent 30 minutes reading the article.

c. At what time did Carter start reading the article? Show your work or explain how you know.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on measurement, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different structures for the different units of measurement across the languages in your classroom can lead to a more robust understanding of units used for measurement for all students by making connections to the different structures of units of measurement used in other countries. Relate the metric system to the countries of origin of your students.	Science: In third grade the NGSS recommends students work with data related to weather conditions. Consider providing a connection for students to collect and measure the amount of rain during different seasons. Language Arts**:** When students have independent reading time during school or at home, consider having them track when they started and ended and using this as the context for elapsed time problems.

3.MD: MEASUREMENT & DATA

Cluster Statement: B: Represent and interpret data.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.MD.B.3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	SMP 1: Students can make sense of problems and persevere in solving them by solving one- and two-step problems by analyzing and deciphering information presented in the scaled graphs. SMP 4: Students can model with mathematics by using strategies to determine the appropriate scale to draw scaled graphs. SMP 6: Students can attend to precision by accurately reading the key or scale and using appropriate vocabulary to describe bar and picture graphs.	• Interpret data in a scaled picture and bar graphs (e.g., one box equals 4 students). • Collect data by asking a question that yields data in several categories • Draw a scaled picture graph and a scaled bar graph (with axes provided) to represent a data set with several categories. • Solve one and two-step "how many more" and "how many less" problems using information from these graphs. **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Understand, Apply and Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.MD.B.4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	SMP 5: Students can use tools by using rulers to measure items and creating line plots to represent the data. SMP 6: Students can attend to precision by measuring with a ruler to the nearest quarter- or half-inch and using appropriate vocabulary to label and describe line plots. SMP 7: Students can look for and make use of structure by to noting that on a line plot the largest	• Generate measurement data by measuring lengths using rules marked with whole inches and halves and fourths of an inch. • Create a line plot where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters. • Analyze data from a line plot.

	quantity of a particular length is found by the most X's, the longest length is found with the measurement of the X furthest to the right, and the total amount of measurements is equal to the number of X's.	**Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply and analyze
Previous Learning Connections • Connect to practice measuring objects and inputting data on picture and bar graphs to represent data and well as answering questions relating and comparing the data represented on the graphs. **(2.MD.1-4, 10)**	**Current Learning Connections** • Connect to solving two-step word problems using the four operations and representing these problems using equations with a letter standing for the unknown quantity. **(3.OA.8)** • Connect to understanding a fraction as a number on the number line and representing fractions on a number line diagram. **(3.NF.2)**	**Future Learning Connections** • Connect to making a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and solving problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. **(4.MD.4)** • Connect to understanding a fraction a/b with a > 1 as a sum of fractions 1/b. **(4.NF.3)**

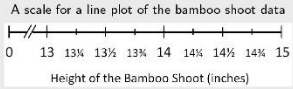
Clarification Statement:
3.MD.B.3: In Grade 3, the most important development in **data** representation for **categorical data** is that students now draw **picture graphs** in which each picture represents more than one object, and they draw **bar graphs** in which the **height** of a given bar in **tick marks** must be multiplied by the **scale factor** in order to yield the number of objects in the given category. These developments connect with the emphasis on **multiplication** in this grade.

3.MD.B.4:

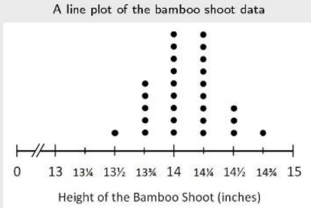
Students' measurements of a statue and of a bamboo shoot

Statue measurements		Bamboo shoot measurements	
Student's initials	Student's measured value (inches)	Student's initials	Height value (inches)
W.B.	64	W.B.	13 3/4
D.W.	65	D.W.	14 1/2
H.D.	65	H.D.	14 1/4
G.W.	65	G.W.	14 3/4
V.Y.	67	V.Y.	14 1/4
T.T.	66	T.T.	14 1/2
D.F.	67	D.F.	14
B.H.	65	B.H.	13 1/2
H.H.	63	H.H.	14 1/4
V.H.	64	V.H.	14 1/4
I.O.	64	I.O.	14 1/4
W.N.	65	W.N.	14
B.P.	69	B.P.	14 1/2
V.A.	65	V.A.	13 3/4
H.L.	66	H.L.	14
O.M.	64	O.M.	13 3/4
L.E.	65	L.E.	14 1/4
M.J.	66	M.J.	13 3/4
T.D.	66	T.D.	14 1/4
K.P.	64	K.P.	14
H.N.	65	H.N.	14
W.M.	67	W.M.	14
C.Z.	64	C.Z.	13 3/4
J.I.	66	J.I.	14
M.S.	66	M.S.	14 1/4
T.C.	65	T.C.	14
G.V.	67	G.V.	14
O.F.	65	O.F.	14 1/4

A scale for a line plot of the bamboo shoot data



A line plot of the bamboo shoot data



In Grade 3, students are beginning to learn fraction concepts (3.NF). They understand **fraction equivalence** in simple cases, and they use visual fraction models to represent and order fractions. Grade 3 students also **measure lengths** using **rulers** marked with **halves** and **fourths** of an **inch**. They use their developing knowledge of **fractions** and **number lines** to extend their work from the previous grade by working with measurement data involving fractional measurement values.

Common Misconceptions

● Students many not count each square or tick mark on a scaled graph as one (rather than one unit)

● Students may confuse the axes on a graph

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying representing and interpreting data because this will allow students time to practice new concepts without fear and to experiment with ideas that they may not otherwise explore.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 1.MD.C.4: This standard provides a foundation for work with representing and interpreting data because this standard introduces the idea of collecting data and representing as well as analyzing "one more" and "one less", thus connecting to the comparative aspect of interpreting data in 3rd grade . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and interpreting data benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students need to feel empowered to self-regulate by creating motivation to reach their goals through support and guidance. Students are encouraged to use information learned and then incorporate their learning into representations and interpretations of data.

Build

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and interpreting data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students are able to benefit from learning experiences that create a safe environment to learn and understand that attends to students as individual learners.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with representing and interpreting data benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students make meaningful and relevant connections when supported through scaffolding and opportunities to practice.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representing and interpreting data by revisiting student thinking through a short mini-lesson because this will allow both the student and the teacher to investigate where the student's thinking went off track and perform some error analysis on the path the student took.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit representing and interpreting data by confronting student misconceptions because interpreting data can involve many student misconceptions as to how to go about interpreting the information; therefore confronting those misconceptions is vital in helping students to understand the process and purpose for interpretation.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying representing and interpreting data because this would allow students to explore how they can apply this learning as individuals.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying representing and interpret data the pattern of questions within the classroom is critical because in data interpretation there is always various perspectives that play into what that data set reflects and asking deep questions allows students to explore those perspectives together as a class.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/3/MD/B/3/tasks/1315
Classroom Supplies Task
Addresses standard 3. MD.B.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.
The purpose of this task is for students to "Solve problems involving the four operations" (3.OA. A) and "Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories" (3. MD.3).

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on representing and interpret data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, discussing data that is relevant to the lives of students in the	Science: In third grade the NGSS recommends students work with data related to weather conditions. Consider providing opportunities to make a picture graph or bar graph to display and analyze weather data. Social Studies: In third grade one of ideas the New Mexico Social Studies Standards state focuses on is the rights of citizens. Consider providing opportunities to use

class such as exploring data trends regarding human migration, population growth, and language usage.	and interpret data within this context, such as the growth of the civil rights movement in the 1950s-60s.

3.MD: MEASUREMENT & DATA

Cluster Statement: C: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.MD.C.5: Recognize area as an attribute of plane figures and understand concepts of area measurement. • 3.MD.C.5.A: A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. • 3.MD.C.5.B: A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	SMP 6: Students can attend to precision by using specific vocabulary to describe the dimensions when measuring area. SMP 7: Students can look for and make use of structure by using their knowledge of the mathematical structure of multiplication and/or arrays and applying that knowledge to area.	• Recognize that area is the measurement as the space occupied by a flat shape or the surface of an object. • Recognize a unit square has 1 square unit of area and is used to measure area of two-dimensional shapes. • Recognize any plane figure covered without gaps or overlaps and filled with *n* unit squares indicates the total square units or area. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
3.MD.C.6: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	SMP 4: Students can model with mathematics by using tiles or unit squares to solve area problems by filling in the area and counting squares or by coloring and counting squares on a graph paper. SMP 5: Students can use tools by using manipulatives to tile or cover areas without gaps or overlaps and discover the formula for area of a rectangle.	• Measure areas by counting unit squares of cm, m, in, ft, ad other sizes.

<table>
<tr><td rowspan="2"></td><td rowspan="2">SMP 6: Students can attend to precision by using consistent unit squares for measuring area for a given shape.</td><td>Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:

Understand and Apply</td></tr>
<tr><td rowspan="3">Standard Text

3.MD.C.7: Relate area to the operations of multiplication and addition.

• 3.MD.C.7.A: Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
• 3.MD.C.7.B: Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
• 3.MD.C.7.C: Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
• 3.MD.C.7.D: Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</td><td rowspan="3">Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by solving area problems involving decomposing larger rectangles (i.e. a double-digit length) or rectilinear figures.

SMP 4: Students can model with mathematics by illustrating the distributive property with a drawing, graph paper, or tiles and explaining how multiplication and addition are used to find the area of a rectangle or rectilinear figure.

SMP 8: Students look for and express regularity in repeated reasoning by recognizing the shortcut of multiplying length by width for rectangles gives the same area as using repeated addition.</td><td>Students who demonstrate understanding can:
• Relate area to the operations of multiplication and addition.
• Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
• Apply their knowledge of area of rectangles with whole-number side lengths in the context of problem solving.
• Illustrate and explain that the area of a rectangle can be found by partitioning it into two smaller rectangles using tiles and/or arrays to and that the area of the larger rectangle is the sum of the two smaller rectangles.</td></tr>
<tr><td>Depth of Knowledge: 2-3</td></tr>
<tr><td>Bloom's Taxonomy:

Understand, Apply and Analyze</td></tr>
</table>

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to measuring the length of an object by selecting and using appropriate tools such as rulers, yardsticks meter sticks, and measuring tapes. **(2.MD.1)** • Connect to partitioning a rectangle into rows and columns of same-size squares and counting to find the total number of them. **(2.G.2)** • Connect to using addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and writing an equation to express the total as a sum of equal addends. **(2.OA.4)**	• Connect to understanding division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. **(3.OA.6)** • Connect to solving two-step word problems using the four operations and representing these problems using equations with a letter standing for the unknown quantity. **(3.OA.8)** • Connect to applying properties of operations as strategies to multiply and divide. **(3.OA.5)**	• Connect to applying the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. **(4.MD.3)**

Clarification Statement:
3.MD.C.5: Students need to learn to conceptualize **area** as the amount of **two-dimensional space** in a bounded region and to measure it by choosing a **unit of area**, often a **square**. A **two-dimensional geometric figure** that is covered by a certain number of squares without **gaps** or **overlaps** can be said to have an area of that number of **square units**.

3.MD.C.6: To begin an explicit focus on area, teachers might then ask students which of three rectangles covers the most area. Students may first solve the problem with **decomposition** (cutting and/or folding) and r**e-composition**, and eventually analyses with **area-units**, by covering each with unit squares (tiles).

3.MD.C.7: Students can be taught to **multiply length measurements** to find the area of a **rectangular** region. But, in order that they make sense of these quantities, they first learn to interpret measurement of rectangular regions as a **multiplicative relationship** of the number of square units in a **row** and the number of rows. Students learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5 or by adding two products, e.g., 10 x 5 and 2 x 5, illustrating the **distributive property**.

Common Misconceptions
● Students may not understand how to find the number of square units for non-rectangular shapes, such as combining two half-square unites to make a whole square unit

● Students may confuse perimeter and area

● Students may believe that all shapes with a given perimeter have the same area

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying understanding concepts of area and relating area to multiplication and addition because students are building on prior understanding of linear measurement, tiling rectangles, and partitioning a rectangle into rows and columns of same-size squares and count to find the total number of them.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.G.A.2: This standard provides a foundation for work with understanding concepts of area and relating area to multiplication and addition because this standard focuses on reasoning with shapes and their attributes and partitioning a rectangle into rows and columns of same-size squares and counting to find the total number of them. This prior knowledge and understanding is important because students can make connections from tiling rectangles and decomposing them into rows and columns of same-sized squares to using square units to measure the amount of space covered by a rectangle. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with understanding concepts of area and relating area to multiplication and addition benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because this helps students make sense and understand important math vocabulary and will be able to focus more on the mathematics and not get frustrated or consumed by things that don't make sense to them or are confusing. Students use manipulatives and work with a partner.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding concepts of area and relating area to multiplication and to addition benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because using flexible grouping gives students the opportunity to ask questions, ask for clarification, or encourage one another through coaching and the use of questioning when students get stuck. Students working in groups are encouraged to persevere and are celebrated for their thinking and effort. Students may use different strategies to figure out the area of a two-dimensional figure. Students may use different manipulatives but are still exploring finding the area. Talking with peers provides opportunities for students to sustain their effort and perseverance.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding concepts of area and relating area to multiplication and to addition benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because it's important for students to have an understanding of important mathematical vocabulary so it doesn't impede their learning during the lesson. Before the lesson begins, the teacher may break down the objectives and use pictures, examples, or different words to pre teach vocabulary and symbols. This gives the students background knowledge and helps students make connections to their own experiences and prior knowledge.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding concepts of area and relating area to multiplication and to addition benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because it allows for students to solve the problem in a way that makes sense to them and allows them to be able to talk about their thinking and their strategy when finding the area of two-dimensional figures.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

For example, learners engaging with understanding concepts of area and relating area to multiplication and to addition benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because giving students opportunities for review and practice when finding the area of two-dimensional figures helps them to build confidence and work through misconceptions and to build on their conceptual knowledge and fluency.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding concepts of area and relating area to multiplication and addition by critiquing student approaches/solutions to make connections through a short mini-lesson because being able to see different strategies and solutions and make connections between those strategies and solutions will promote discourse between students and helps to clear up misconceptions and give students the opportunity to reflect on where they are in their learning and what they might still be struggling with.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited?*

- For example, students may benefit from re-engaging with content during a unit on understanding concepts of area and relating area to multiplication and addition by offering students opportunities to understand and explore different strategies in a small group setting or pull out if needed. In a smaller group setting, the teacher will be able to monitor students' progress on a biweekly basis to assess student growth and their responses to intensive interventions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying understanding concepts of area and relating area to multiplication and addition because it gives students the opportunity to explore with measuring efficiently and effectively using standard units, their learning experiences can be directed to situations that encourage them to "discover" measurement formula. Students can also make connections to what types of jobs might utilize area and why it is important to be accurate and efficient.

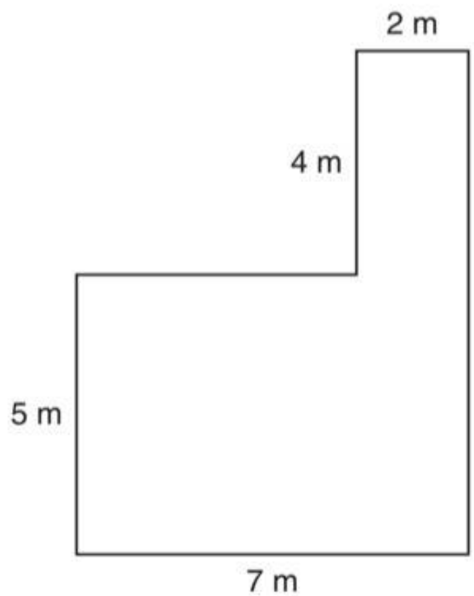
Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying understanding concepts of area and relate area to multiplication and addition goal setting is critical because students understand and know the learning goals and expectations for the lesson. When using both content and language objectives, the students are able to clearly understand the expectations and learning goals for the lesson, feel comfortable in the classroom setting, and are immersed in a classroom culture where ALL students can engage with interesting and rigorous mathematical content and achieve in mathematics to high levels.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source Cognia Testlet for Grade 3 Measurement and Data
STANDARD: Relate area to the operations of multiplication and addition: Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (03. MD.03.07. d)
LEARNING TARGET: I can find the area of a shape by breaking it down into smaller rectangles and then adding those areas to find the total area. DOK: 2

5. A patio is in the shape of two rectangles, as shown below.



What is the area of the patio?

Ⓐ 18 square meters
Ⓑ 35 square meters
Ⓒ 41 square meters
Ⓓ 43 square meters

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on area, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways area is used in the home and community can a be a great way to connect schools tasks with home tasks. Students can talk with their family members about the concept of area. They can make connections to buying and laying flooring or carpet, planning and building a garden, or planning and laying bricks to build a rectangular patio floor.	Science: In third grade the NGSS states students should be able to "make a claim about the merit of a design solution that reduces the impacts of a weather-related hazard." Consider providing a connection where students look at how area can relate to the impact of weather-related hazards, such as tsunamis and stilt houses. Art: Painting a wall requires knowing the area of the wall and also how much area each paint can will cover. Consider providing an opportunity for students to apply their knowledge of area to a real-life application within the school, such as painting an outside wall used during recess.

NEW MEXICO
Public Education Department

3.MD: MEASUREMENT & DATA		
Cluster Statement: D: Geometric measurement: recognize perimeter. **Additional Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text 3.MD.D.8: Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**Standard for Mathematical Practices** SMP1: Students can make sense of problems and persevere in solving them by solving one- and two-step problems involving perimeter of polygons including finding rectangles with the same perimeter & different areas as well as rectangles with the same area and different perimeters. SMP 4: Students can model with mathematics by using models to show some rectangles can have the same perimeter with different areas and that others can have the same area, but different perimeters. SMP 8: Students look for and express regularity in repeated reasoning by noticing short cuts for finding the perimeter of a rectangles (i.e. twice the length plus twice the width, or multiplying the length by the number of sides for any regular shape such as a square).	**Students who demonstrate understanding can:** • Solve problems involving perimeters of polygons • Determine the perimeter given the side lengths • Determine an unknown side length given the perimeter • Illustrate how multiple rectangles can have same perimeters and different areas or vice-versa **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
<u>Previous Learning Connections</u> • Connect to measuring the length of an object by selecting and using appropriate tools such as rulers, yardsticks meter sticks, and measuring tapes. **(2.MD.1)** • Connect to using addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of	**<u>Current Learning Connections</u>** • Connect to solving two-step word problems using the four operations and representing these problems using equations with a letter standing for the unknown quantity. **(3.OA.8)**	**<u>Future Learning Connections</u>** • Connect to applying the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. **(4.MD.3)**

rulers) and equations with a symbol for the unknown number to represent the problem. **(2.MD.5**)		

Clarification Statement:
3.MD.D.8: **Perimeter** problems for **rectangles** and **parallelograms** often give only the **lengths** of two **adjacent sides** or only show numbers for these sides in a drawing of the **shape**. The common error is to add just those two numbers. Having students first **label** the lengths of the other two sides as a reminder is helpful. Students then find unknown side lengths in more difficult "missing **measurements**" problems and other types of perimeter problems.

Common Misconceptions
● Students may confuse area and perimeter.

● Students may not recognize that all rectangles have four dimensions, especially when only two side lengths are shown or provided.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying recognizing perimeter as an attribute of plane figures and distinguishes between linear and area measures because students build on prior knowledge of recognizing polygons and knowing their attributes and measuring and estimating lengths in standard units.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.MD.A.1: This standard provides a foundation for work with recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures because students will be measuring the distance around polygons using standard units and using these measurements to find the perimeter. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners recognize perimeter as an attribute of plane figures and distinguish between linear and area measures benefit creating an accepting and supportive classroom climate because students need to feel safe and supported in their environment so they are willing to take risks and make mistakes so they can do their best learning. If students feel supported and safe then they know they can approach math in a way that makes sense to them and are supported by their teacher and peers. Activities or tasks are chosen to foster the use of imagination, to solve novel and relevant problems, or to make sense of complex ideas in creative ways.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because using flexible grouping gives students the opportunity to ask questions, ask for clarification, or encourage one another through coaching and the use of questioning when students get stuck. Students working in groups are encouraged to persevere and are celebrated for their thinking and effort. Students may use different strategies to figure out the perimeter of a two-dimensional figure. Students may use different manipulatives but are still exploring finding the perimeter. Talking with peers provides opportunities for students to sustain their effort and perseverance.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because it's important for students to have an understanding of important mathematical vocabulary so it doesn't impede their learning during the lesson. Before the lesson begins, the teacher may break down the objectives and use pictures, examples, or different words to pre teach vocabulary and symbols. This gives the students background knowledge and helps students make connections to their own experiences and prior knowledge.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because it allows for students to solve the problem in a way that makes sense to them and allows them to be able to talk about their thinking and their strategy when finding the perimeter of two-dimensional figures.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with u recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as asking questions to guide self-monitoring and reflection because it is important for students to ask themselves questions as they learn and reflect on their learning. If students are consistently doing this then they know whether they understand or are still struggling with the concept.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures by clarifying mathematical ideas and/or concepts through a short mini-lesson because this gives students time to explore more with perimeter and clear up their misconceptions. Students may need time to explore that perimeter is the distance around the shape, which is different from area.

Re-teach (intensive) What assessment data will help identify content needing to be revisited?

- For example, students may benefit from re-engaging with content during a unit on recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures by addressing conceptual understanding in a small group setting or pull out if needed. In a smaller group setting, the teacher will be able to monitor students' progress on a biweekly basis to assess student growth and their responses to intensive interventions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures because students can explore tasks that involve "Big Ideas" and spend time reasoning and making connections which the students can visualize, play, and investigate. This leads to a deeper understanding of the mathematical content.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures the types of mathematical tasks are critical because they should be accessible to ALL students. Choosing floor to ceiling tasks gives each student an opportunity to access the task at their level and build on and deepen their understanding of the mathematics being explored. Tasks should promote reasoning and problem solving, promote student discourse, and support students in productive struggle.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Illustrative Mathematics for Grade 3 Measurement and Data; Geometric Measurement
Shapes and Their Insides Task
http://tasks.illustrativemathematics.org/content-standards/3/MD/D/tasks/1514
3.MD.D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
There were no Cognia Testlets for this cluster of standards.

The purpose of this task is to help students differentiate between a polygon and the region inside of a polygon so that they understand what is being measured when the perimeter and area are being found.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. Example 1: During a unit focused on perimeter, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different structures for the different units of measurement across the languages in your classroom can lead to a more robust understanding of units used for measurement for all students by making connections to the different structures of units of measurement used in other countries. Relate the metric system to the countries of origin of your students. Example 2: During a unit focused on perimeter, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways perimeter is used in the home and community can a be a great way to connect schools tasks with home tasks. Students can talk with their families about perimeter. What are some items in your home that have a perimeter? What units of measure would you use? Example 3: During a unit focused on perimeter, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the mathematics used within the different careers of your family and community can provide a strong connections between school and careers. What careers in their family or community would utilize perimeter? Maybe landscapers, interior designers, what about sports fields? Have you ever noticed the lines on the field or the court?	Science: In third grade the NGSS states students should be able to "make a claim about the merit of a design solution that reduces the impacts of a weather-related hazard." Consider providing a connection where students look at how perimeter can relate to the impact of weather-related hazards, such as wildfires and leaving containers of water around the perimeter of a property. Social Studies: In third grade the New Mexico Social Studies Standards recommend students "identify the components of the Earth's biosystems and their makeup (e.g., air, land, water, plants, and animals)." Consider providing opportunities for students to think about farmland and animals and how to best construct a fence for them within the context of perimeter.

3.G: GEOMETRY

Cluster Statement: A: Reason with shapes and their attributes.
Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	SMP 3: Students can construct viable arguments and critiques the reasoning of others by recognizing and applying attributes to justify classification of shapes and support their claim that some shapes may fit in many categories & a category may be subset of a larger category (i.e. a rhombus is a subset of parallelograms which is a subset of quadrilaterals), as well as support their argument that some shapes do not belong to any of the subcategories. SMP 6: Students can attend to precision by accurately using attributes to help classify shapes and see that some shapes fit in many categories & a category may be a subset of a larger category (i.e. a rhombus is a subset of parallelograms which is a subset of quadrilaterals). SMP7: Students can look for and make use of structure by identifying a quadrilateral as any closed figure with four straight sides. Furthermore, the student will apply attributes of angles (right or not) and the relationship between opposite sides being parallel or equal in length to more specifically classify and draw shapes as trapezoids, parallelograms, rectangles, rhombuses, and squares. The experience of sorting, discussing, and describing attributes of shapes will help	• Investigate characteristics of and compose triangles and quadrilaterals. • Decompose quadrilaterals. • Recognize and draw both examples and non-examples of a variety of quadrilaterals including rhombuses, rectangles, squares, parallelograms, and trapezoids. • Communicate their reasoning by explaining their thinking and sharing their solutions • Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). **Depth of Knowledge: 2** **Bloom's Taxonomy: Understand, apply**

	students understand geometric structure and categories	
Standard Text 3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.	**Standard for Mathematical Practices** SMP4: Students can model with mathematics: Students analyze shapes to identify and draw cut lines that demonstrate equal pieces and name the parts and whole. SMP6: Students can attend to precision by how they draw and cut different shapes (circles, rectangles, graph paper shapes) into pieces of equal area, how they would label each of those pieces or a set of those pieces as a unit fraction of the whole. Students can also use accurate vocabulary to explain their models as a representation of fractional parts and wholes.	**Students who demonstrate understanding can:** • Partition shapes into equal parts understanding that the parts have equal areas • Write a unit-fraction or a non-unit fraction for partitioned shapes • Know that shapes can be partitioned into equal areas • Describe the area of each part as a fractional part of the whole • Relate fractions to geometry by expressing the area of part of a shape as a unit fraction of the whole **Depth of Knowledge: 1-2** **Bloom's Taxonomy:** Remember, Understand
<u>Previous Learning Connections</u> • Connect to the many experiences with specific attributes of shapes such as triangles, hexagons and cubes. They have learned the difference between defining attributes and non-defining attributes. • Connect to drawing shapes having specified attributes, such as a given number of angles or a given number of equal faces. They identified triangles, quadrilaterals, pentagons, hexagons, and cubes. (**2.G.1**)	**<u>Current Learning Connections</u>** • Connect to understanding a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. (**3.NF.1**) • Connect to understanding a fraction as a number on the number line; represent fractions on a number line diagram. (**3.NF.2**) • Connect to partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape. (**3.G.2**)	**<u>Future Learning Connections</u>** • Connect to understanding angles of geometric shapes and angle measurements. • Connect to drawing points, lines, line segments, rays, angles (right, obtuse, acute) and perpendicular and parallel lines in 2-D figures. **(4.G.1)** • Connect to angles as geometric shapes which are formed when two rays share a common endpoint and understand concepts of angle measurement. (**4.MD.5**)

Clarification Statement:
Students should **categorize** shapes by **attribute**. They can name **rhombuses, squares,** and **rectangles** as types of **quadrilaterals**. They can also draw an example of a quadrilateral that is not a rhombus, rectangle, or square. Students should break shapes into **equal parts** to illustrate **fractions**. They can tell you that one part of that shape is a part of a whole. For example: A student may have a circle. They will draw lines in that circle to break it into 3 equal parts. The student can tell you that one of the four pieces is 1/3 of the circle.

Common Misconceptions
Students may have difficulty recognizing the subtle differences between shapes such as the size of angle where two sides meet.

Students might mistakenly mislabel types of quadrilaterals due to vocabulary difficulty.

Students may be able to tell that squares and rectangles are related shapes but they may mistakenly label a rectangle as a kind of square rather than the other way around.

Students might be confused with the concept that equal shares of identical wholes may not have the same shape.

Students may also not understand an area model represents one out of two or three or four fractional parts without the understanding the parts are equal shares.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted) *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time when studying reasoning with shapes and their attributes because this will engage visual learners and help students to relate vocabulary of shapes and their attributes to concrete examples.

Pre-teach (intensive) *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.G.A.1: This standard provides a foundation for work with reasoning with shapes and their attributes because this standard asks students to begin reasoning with shapes as well as laying the groundwork for key vocabulary that will be repeated in the current work of the grade. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access
Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with reason with shapes and their attributes benefit when learning experiences include ways to recruit interest such as supporting culturally relevant connections (i.e home culture) because it helps create support for student understanding by building connections through background knowledge and language to support mathematical concepts.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with reason with shapes and their attributes benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students need to feel empowered to self-regulate by creating motivation to reach their goals through support and guidance.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with reason with shapes and their attributes benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students are able to benefit from learning experiences that create a safe environment to learn and understand that attends to students as individual learners.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with reason with shapes and their attributes benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video because it's important to provide multiple modalities for expression, so students are able to express knowledge, ideas, and concepts in the learning environment.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with reason with shapes and their attributes benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections (e.g., teaching literacy strategies in the social studies classroom) because the more connections that students make the more meaningful and relevant connections can be made and retained to the subjects and skills they are learning.

Re-teach

Re-teach (targeted) *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on reasoning with shapes and their attributes by clarifying mathematical ideas and/or concepts through a short mini-lesson because this standard is heavy with mathematical vocabulary and allowing students time to clarify their understanding of these ideas will be key in helping them to meet the standard.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit reasoning with shapes and their attributes by offering opportunities to understand and explore different strategies because this will allow students to find methods for reasoning with shapes that fit their mental schema as individual learners.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying reasoning with shapes and their attributes because this allows them to explore higher level applications of reasoning with shapes and to move at a pace that is more appropriate for them rather than working with the rest of the class and then receiving "extra" work.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM, consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying reasoning with shapes and their attributes the types of mathematical tasks are critical because there are a great many concrete ways in which this area of study can be applied to students' lives and experiences therefore there is a rich groundwork on which to build understanding of the content through tasks that are aimed at student experiences.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/3/G/A/2/tasks/1502
Halves, Thirds, and Sixths Task
Addresses Standard 3.G.A.2 - Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.
The purpose of this task is for students to use their understanding of area as the number of square units that covers a region (3.MD.6), to recognize different ways of representing fractions with area (3.G.2), and to understand why fractions are equivalent in special cases (3.NF.3.b).

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on reasoning with shapes and their attributes, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, studying the architecture of different cultures to investigate how shapes and their attributes are utilized in different ways.	Language Arts: Read *Keeping Quilt* by Patricia Polacco. Explore geometry of quilt designs. Student create a quilt design using plane shapes. Art: Architecture/design by finding shapes in buildings.

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ⍰ How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ⍰ What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ⍰ How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ⍰ What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ⍰ How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ⍰ What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding	
ENGAGEMENT ⍰ How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ⍰ What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols**: ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication**: ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: ? What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension**: ? What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ? How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ? What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

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[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5- 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* x 1/*a* = 1/*a* x *a* = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.