4.MD: MEASUREMENT & DATA

Cluster Statement: C: Geometric measurement: understand concepts of angle and measure angles.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
4.MD.C.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: • 4.MD.C.5.A: An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. • 4.MD.C.5.B: An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	SMP 4: Students can model with mathematics by creating examples of acute, obtuse and right angles. SMP 6: Students can attend to precision by using specific vocabulary to describe the measurements of angles, including measure, point, end point, geometric shapes, ray, angle, circle, degree and protractor.	• Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. • Relate angle measurement to circles. • Demonstrate acute, obtuse, and right angles. • Use precise vocabulary when describing angles. **Depth Of Knowledge:** 1 **Bloom's Taxonomy:** Remember and Understand
Standard Text 4.MD.C.6: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by drawing and creating angles of a given measurement. SMP 5: Students can use tools by measuring angles with different protractors, including traditional and circular protractors.	**Students who demonstrate understanding can:** • Recognize that angles are measured in degrees. • Measure angles in whole-number degrees using a protractor. • Make sketches of specified angle measures. • Observe that the orientation of an angle does not affect its measure.

		Depth Of Knowledge: 2
		Bloom's Taxonomy: Understand and Apply
Standard Text 4.MD.C.7: Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	**Standard for Mathematical Practices** SMP 1: Students can make sense of problems and persevere in solving them by interpreting, analyzing, and solving word problems involving measurement of angles. SMP 2: Students can reason abstractly and quantitatively by recognizing angle measurement as additive in relation to the reference to a circle.	**Students who demonstrate understanding can:** • Recognize angles as additive. • Solve addition and subtraction problems to involving unknown angles • Write an equation for a word problem involving angle measurement and represent the unknown number with a symbol.
		Depth Of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply and Analyze
Previous Learning Connections • Connect to understanding that shapes in different categories may share attributes. **(3.G.1)** • Connect to recognizing and drawing shapes having specified attributes, such as a given number of angles. **(2.G.1)** • Connect with using addition and subtraction within 100 to solve one-and two-step word problems. **(2.OA.1)** • Connect to finding the unknown whole number using addition and subtraction. **(1.OA.8)**	**Current Learning Connections** • Connect to drawing and identifying lines and angles. **(4.G.1)** • Connect to classifying two-dimensional figures based on lines and angles. **(4.G.2)**	**Future Learning Connections** • Connect to constructing triangles from three measures of angles. **(7.G.2)** • Connect to using facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to writing and solving simple equations for an unknown angle in a figure. **(7.G.5)**

Clarification Statement:
4.MD.C.5: As with **length**, **area**, and **volume**, children need to understand **equal partitioning** and **unit iteration** to understand **angle and turn measure**. Whether defined as more statically as the measure of the figure formed by the **intersection** of two **rays** or as turning, having a given angle measure involves a relationship between components of **plane figures** and therefore is a **property**.

4.MD.C.6: If examples and tasks are not varied, students can develop incomplete and inaccurate notions. For example, some come to associate all **slanted lines** with 45 **degree** measures and **horizontal and vertical lines** with measures of 90 degrees. Others believe angles can be "read off" a **protractor** in **"standard" position**, that is, a **base** is horizontal, even if neither **arm of the angle** is horizontal. Measuring and then sketching many angles with no horizontal or vertical arms, perhaps initially using **circular 360 protractors**, can help students avoid such limited conceptions.

4.MD.C.7: Students with an accurate conception of angle can recognize that angle measure is **additive**. As with length, area, and volume, when an angle is **decomposed** into **non-overlapping parts**, the angle measure of the **whole** is the **sum** of the angle measures of the parts. Students can then solve interesting and challenging **addition** and **subtraction** problems to find the measurements of unknown angles on a **diagram** in real world and mathematical problems.

Common Misconceptions
● Students may think size of an angle's arms or rays affects its measure

● Students may think you can only use a protractor when there is a horizontal line

● Students may think the direction of an angle matters.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach
Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*
- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying geometric measurement: understand concepts of angle and measure angles because students will need to know how to draw and label line segments.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*
- 4.G.A.1: This standard provides a foundation for work with drawing and identifying lines and angles, and classifies shapes by properties of their lines and angles because students need to be able to draw and recognize angles. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction
Access
Interest: *How will the learning for students provide multiple options for recruiting student interest?*
- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because students have had prior work with angles and angle measurements. Students can associate angles with topics that are relevant in their lives to make connections.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*
- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for

visualizing desired outcomes because students will need scaffolds to help with work with angles. They may need step by steps or prompts for different angles and measuring angles.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*
- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students have prior experience with working with angles. This cluster has very domain specific vocabulary that students need to know, use, and be able to identify.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*
- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) because students will need conceptual understanding about angles. Manipulatives that help them move angles and change them will help with building understanding. Practice with using different protractors including digital is beneficial.

Internalize
Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*
- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using multiple examples and non-examples to emphasize critical features because students will need work with different examples and non examples. This way students have a variety of practice with different angles and can identify if a figure is not angle or does not have angle.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*
- For example, students may benefit from re-engaging with content during a unit on geometric measurement: understand concepts of angle and measure angles by clarifying mathematical ideas and/or concepts through a short mini-lesson because students need to be able to recognize angles and recognize the degrees each angle represents.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*
- For example, some students may benefit from intensive extra time during and after a unit understand concepts of angle and measure angles by addressing conceptual

understanding because students who continue to struggle will need support with building conceptual understanding. This might include work with manipulatives or real-world examples.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying understand concepts of angle and measure angles because this cluster can be cross curricular and have ties to real world examples. Students can explore these links.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Goal Setting): Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying geometric measurement: understand concepts of angle and measure angles, goal setting is critical because it is recognizing important concepts that allows for the teacher to create a clear plan. Teachers are encouraged to think critically about students' culture background, strengths and weaknesses. Teacher is then looking at the students individual needs to plan accordingly so that students reach their end goal.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia Testlet for Grade 4- Measurement and Data

Standard: 4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. 4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

Learning Target: I can add or subtract to find the unknown angle on a diagram.

1. Ray *RS* divides angle *QRT* into two smaller angles, as shown.
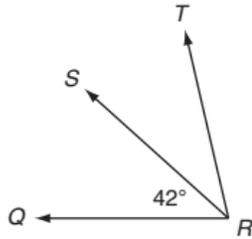
The measure of angle *QRT* is 77 degrees.

a. What is the measure of angle *SRT*? Show your work or explain your reasoning.

Ray *RW* is drawn to create angle *TRW*. *QRW* is a straight line.

b. What is the measure of angle *TRW*? Show your work or explain how you know.

This type of assessment question requires students to use the information given to them. They need to know the measurement of the whole angle before beginning work. This task encompasses SMP 6 due to the vocabulary content within the problem. Teacher determine different reteach aspects from this task, for example a student might be adding when they are supposed to be subtracting or ask you what a "ray" is. These such aspects are good to note for reteaching.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on geometric measurement: understand concepts of area and relate area to multiplication and to addition, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example utilizing vocabulary cards with visuals to present the geometric shape to explain the concept of area. Students can then understand the area as how much two-dimensional space a shape takes up and then practice by using addition to counting individual square units or by multiplying the length and width of the shape. Students can use repeated addition to support their multiplication skills.	Social Studies: In fourth grade the New Mexico Social Studies Standards state students should "explain how the Earth-Sun relationships produce day and night". Consider providing a connection to the angle of Earth's axis in relation to the Sun. Language Arts: Literature can offer connections about angles such as: *What's Your Angle, Pythagoras?* by Julie Ellis or *The Adventures of the Angles* by Kristie Carpenter.