

4.NF: NUMBER & OPERATIONS-FRACTIONS

Cluster Statement: B: Build fractions from unit fractions.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text

4.NF.B.3

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

- 4.NF.B.3.A: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- 4.NF.B.3.B: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.
- 4.NF.B.3.C: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- 4.NF.B.3.D: Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively by understanding that adding and subtracting fractions involves joining and separating parts that refer to the same whole.

SMP 4: Students can model with mathematics by decomposing fractions into a sum of fractions and justifying with a visual model.

Students who demonstrate understanding can:

- Explain why a fraction is the sum of multiple fractions
- Explain why addition and subtraction of fractions with the same denominator is joining or separating parts referring to the same whole.
- Explain why a fraction can be a sum of different like denominator fractions.
- Explain why mixed numbers can be added or subtracted
- Add and subtract fractions and mixed numbers with like denominators.
- Write an equation when decomposing fractions.
- Solve addition word problems involving fractions with like denominators using models and equations
- Solve subtraction word problems involving fractions with like denominators using models and equations

Depth Of Knowledge: 1,2

Bloom's Taxonomy: Apply

<p>Standard Text</p> <p>4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <ul style="list-style-type: none"> 4.NF.B.4.A: Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. <i>For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (1/4)$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (1/4)$.</i> 4.NF.B.4.B: Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. <i>For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $\frac{6}{5}$. (In general, $n \times (\frac{a}{b}) = (n \times \frac{a}{b})$)</i> 4.NF.B.4.C: Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i> 	<p>Standard for Mathematical Practices</p> <p>SMP6: Students can attend to precision by building on previous understandings of the meaning of numerator and denominator.</p> <p>SMP 7: Students can look for and make use of structure by using previous understanding of numerators and denominators to see the structure multiplication of fractions.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Extend the understanding of multiplication to problems that have fractions. Multiply a unit fraction (numerator of 1) by a whole number. Multiply a fraction with a numerator greater than 1 by a whole number. Use a number line to represent fraction multiplication. Explain why a fraction is a multiple of a unit fraction Explain why multiplying a whole number times a fraction can be changed to a whole number times a unit fraction. Solve multiplication word problems involving whole numbers and fractions using models and equations. Restate word problems involving multiplication of a whole number and a fraction. Draw a diagram and write an equation represent and solve a word problem involving multiplication of a whole number and a fraction.
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to understanding a $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. (3.NF.A1) Connect fraction and decimal notation to measuring the 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect converting fractions and decomposing fractions to converting measurements from a larger unit to a smaller unit (4.MD A2) 	<p>Depth Of Knowledge: 1,2</p> <p>Bloom's Taxonomy: Apply</p> <p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to extending and applying multiplication to multiply a fraction or whole number by a fraction (5.NF.B4) Connect to interpreting multiplication as scaling/ resizing (5.NF.B5)

<p>length of an object to $\frac{1}{2}$ and $\frac{1}{4}$ inches (3.MD.b4)</p> <ul style="list-style-type: none"> Connect to multiplication and division within 100 involving arrays, equal groups, and measurement quantities (3.OA.A.3) 		
<p>Clarification Statement:</p> <p>4.NF.B3: This standard builds on prior work with unit fractions where students will now investigate fractions other than unit fractions such as $\frac{2}{3}$, they should then be able to join (compose) AND separate (decompose) the fraction of the same whole. In order to gain conceptual understanding of this standard, students must be able to visualize the composition and decomposition into unit fractions. This skill will aid in the development needed to then move into adding and subtracting fractions. For students to visualize they must have multiple opportunities to model this concept by using hands on manipulatives and other appropriate tools such as creating an original drawing to develop the skill. The models should not be limited to area models only (vary the type of area model), and should include length models such as number lines, folded paper, rulers, fraction strips, and set models as well.</p> <p>4.NF.B4: aStudents will be able to model multiplication of whole numbers by unit fraction. Building on ideas of decomposing fraction into unit fractions, students will apply knowledge and work with multiplying of whole numbers to this work. Using similar language, such as “groups of” or “jumps” on a number line. Students will be able to explain this using models, words and numbers. Students will also be able to look at a given picture and describe multiplication that is present, in essence working backwards. b. Students will be able to apply patterns from multiplying whole numbers by a unit fraction to multiplying whole numbers by any fractions. Students are still using models and manipulatives for this work. Students will be able to explain this using models, words and numbers. This work includes fractions that are greater than 1 or mixed numbers. Students can apply previous work with whole number multiplication to multiplication with fractions, such as area model, distributive property, etc. c. Students will be able to solve multiplication word problems that include whole number by a fraction or mixed number. Students are still using models and manipulatives for this work. Students will be able to explain this using models, words and numbers.</p>		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> Students may misunderstand and believe when you multiply a fraction by a whole number, first you multiply the numerator by the whole number and then you multiply the denominator by the whole number. Students that are taught to put a 1 under the whole number and multiply straight across may lack an understanding of why this algorithm works. 		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach</p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers because it allows the teacher to plan in order to meet individual student needs, small group needs, and whole class needs. <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p>		

- 3.NF.A: These standards provide a foundation for work with building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers because it helps students understand what the parts of a fraction represent and see fractions as numbers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because breaking down a whole into equal parts are more visible when related to their own lives. Students have experience with breaking whole numbers into "groups of" but this will be new for them to view one whole as unit fractions.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because using real world, relevant examples will help students see parts of one whole in terms of unit fractions in a way that makes sense to them will build math confidence.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because the term "unit fraction" is new and essential to learning within this cluster. Breaking down, or decomposing, a fraction into unit fractions helps students relate to size of the units in relation to the whole.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-

10 blocks, algebra blocks) because physically working with materials to decompose and compose fractions will help students move toward explaining their thinking with justifications.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because keeping a graphic organizer, such as a chart, which includes the fraction, decomposition using unit fractions, and a visual, will help students see patterns when working on future similar problems.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers by providing specific feedback to students on their work through a short mini-lesson because specific feedback during a one-on-one or small group mini-lesson helps students see the mistake. Working through a few more problems where the student can immediately apply the feedback strengthens their understanding of building from unit fractions and/or decomposing a fraction into unit fractions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers by addressing conceptual understanding because students need work with concrete manipulatives when struggling with fractions. This can include area models, fraction strips, number lines, and other visual models. This will build conceptual understanding and mental pictures for student work with fractions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers because students need extended work with fractions. They need to reason about fractions and part of a whole. Students can use extended work to make generalizations and reasonings about work with fractions. For example, the denominator stays the same because the fractional pieces are

equivalent. This also includes connections between addition, multiplying, and properties of operations that apply here as well.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Building Procedural Fluency from Conceptual Understanding): Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers the types of mathematical tasks are critical because we are connecting to previous work with decomposing whole numbers and to work with "groups of" used in multiplication.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/NF/B/3/tasks/837>

Standard: 4.NF.3b

Task: Making 22 Seventeenths in Different Ways

Making 22 Seventeenths in Different Ways

Which of the following sums are equal to $\frac{22}{17}$?

a. $\frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17}$

b. $\frac{3}{17} + \frac{8}{17} + \frac{3}{17} + \frac{10}{17}$

c. $\frac{6}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17}$

d. $\frac{12}{17} + \frac{10}{17}$

e. $\frac{1}{17} + \frac{1}{17} + \frac{9}{17} + \frac{3}{17}$

Find another way to write $\frac{22}{17}$ as a sum of fractions.

This type of assessment question requires students to decompose fractions in more than one way. Students reason about different equations listed and have to produce one different way from those listed. This can be a lead task into mixed numbers. This standard is a vital part of fraction work in 4th grade, students that struggle with this type of task need more manipulative or visual models. Some might need manipulatives to complete task.

Relevance to families and communities:

During a unit focused on building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, examining fractions of students in the classroom and fractions of members in a family to help students work within decomposing one whole. The one whole is the classroom, family, team, or other group a student is connected to outside of school. This will help students understand a whole can vary in size.

Cross-Curricular Connections:

STEM Connection: Students can create a measuring cup for a science experiment. As they do, have students pay attention to precision and remind them the parts must be equal. The act of partitioning reinforces an understanding of the relationship between the unit fraction and the whole. Reinforce the relationship between mixed numbers and their fraction equivalent.

Music: Students can partition a unit fraction (or beat) into smaller unit fractions (e.g., subdividing each fourth note to create 2 eighth notes). This helps students see mathematical and musical relationships among:

- the denominator (the type of note)
- the number of parts in the whole quantity (how many fit into a bar)
- the size of the part (the duration of that note)

<https://thelearningexchange.ca/wp-content/uploads/2018/04/cbs-fraction-across-curriculum-en.pdf>