

New Mexico Mathematics Instructional Scope for Fourth Grade

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. [Section 1](#) describes how to use this document to support equitable and excellent mathematics instruction. [Section 2](#) contains planning support for each cluster of mathematics standards within the grade level or course. [Section 3](#) provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside [High Quality Instructional Materials \(HQIM\)](#). A [sample template](#) is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our [references](#). This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the [references](#) in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*¹ on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

¹ Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from <https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics>

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.²

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb’s Depth of Knowledge and Bloom’s Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.³ This planning support considers which aspect(s) of rigor are within each standard and then identifies academic skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb’s Depth of Knowledge and Bloom’s Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

² Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

³Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from <https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics>

formative assessment, or it can be used to plan/design appropriate formative assessment.⁴ The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.⁵ The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire⁶ over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports (MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

⁴ English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from <https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands>

⁵ Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from <https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics>

⁶ Cardone, T. (n.d.). Nix the Tricks. Retrieved from <https://nixthetricks.com/>

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see [Standards Aligned Instructionally Embedded Formative Assessment Resources](#).

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the [Universal Design Learning \(UDL\) Framework](#), additional planning supports based on this framework can be found in Section 3 of this document in the part titled, [Planning Guidance for Multi-Layered Systems of Support: Core Instruction](#).

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.⁷

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.⁸ Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

⁷ Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

⁸ Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>		
<p>Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.</p> <p>Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.) Identifies if the cluster is major, additional or supporting work of the grade.</p>		
<p>Standard Text Full text of the standard</p>	<p>Standard for Mathematical Practices The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.</p>	<p>Students who demonstrate understanding can: The cognitive skills students perform to demonstrate to comprehension of a standard.</p>
		<p>Depth Of Knowledge: Correlation of standard to Webb's Depth of Knowledge</p>
		<p>Bloom's Taxonomy: Correlation of standard to Bloom's Taxonomy</p>
<p>Connections to Previous Learning: Supports student connections to learning from previous grade levels.</p>	<p>Connections to Current Learning Supports student connections to learning within the grade level.</p>	<p>Connections to Future Learning Supports student connections to learning in a future grade.</p>
<p>Clarification Statement: Clarifies the language of the standard.</p>		
<p>Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.</p>		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning. Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.</p> <p>Core Instruction Access: Guidance for optimizing universal access to learning experiences. Build: Guidance for supporting students build their understanding of the cluster. Internalize: Guidance for ensuring student internalization of the learning goal.</p> <p>Re-teach Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data. Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching. Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.</p>		
<p>Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards</p>		
<p>Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.</p>		
<p>Relevance to Families and Communities: Connecting with families and communities to create relevant connections between mathematics inside and outside of school.</p>	<p>Cross Curricular Connections: Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.</p>	

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

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Operations & Algebraic Thinking

[4.OA.A](#)

[4.OA.B](#)

[4.OA.C](#)

Number & Operations in Base Ten

[4.NBT.A](#)

[4.NBT.B](#)

Number & Operations – Fractions

[4.NF.A](#)

[4.NF.B](#)

[4.NF.C](#)

Measurement & Data

[4.MD.A](#)

[4.MD.B](#)

[4.MD.C](#)

Geometry

[4.G.A](#)

4.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: A: Use the four operations with whole numbers to solve problems.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 3: Students can construct viable arguments and critique the reasoning of others by explaining their thinking and listening to the reasoning of others and look for similarities and differences in strategies</p> <p>SMP 7: Students can look for and make use of structure by using properties of operations to explain calculations.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Explain multiplication equations as multiplicative comparisons (28 is 7 times as many 4 and 4 times as many as 7) • Explain how multiplication can compare quantities - Interpret multiplicative comparison language within a word problem • Represent multiplicative comparisons • Identify a multiplication equation as showing two ways to describe a product • Write equations to represent multiplicative comparisons • Write word problems using multiplicative comparisons to describe a multiplication equation <p>Depth Of Knowledge: 1-2</p> <p>Bloom's Taxonomy: Apply</p>
<p>Standard Text</p> <p>4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 4: Students can model with mathematics by solving single and multistep problems that include all four operations using models, pictures, words, and numbers.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Use drawings and equations (with symbols to represent an unknown) to solve multiplication word problems • Use drawings and equations (with symbols to represent an unknown) to solve division word problems • Contrast a multiplicative comparison from an additive comparison

		Depth Of Knowledge: 1,2
		Bloom's Taxonomy: Apply
<p>Standard Text</p> <p>4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 2: Students can reason abstractly and quantitatively by interpreting the remainder in a multi-step problem involving the four operations.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Use drawings and equations (with symbols to represent an unknown) to solve multiplication word problems Use drawings and equations (with symbols to represent an unknown) to solve division word problems Use mental computation and estimation to check for reasonable solutions
		Depth Of Knowledge: 1-2
		Bloom's Taxonomy: Apply
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to interpreting products of whole numbers as the total number of objects in a set of groups (3.OA.A1) Connect to using addition to find the total number of objects arranged in a rectangular array (2.OA.C4) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to process of generating a number or shape pattern that follows a given rule (4.OA.C5) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect multiplying and dividing whole numbers to future work of multiplying and dividing fractions (5.NF.B3)
<p>Clarification Statement:</p> <p>4.OA.A1: This standard requires students to use multiplication equations to represent verbal multiplicative comparisons. This standard also calls for students to conceptually represent multiplicative comparisons. The focus lies in understanding comparisons NOT simply identifying each factor or product without understanding the meaning. Students should relate multiplicative reasoning to iterating-that is, to making multiple copies- and partitioning sets of objects as well as to the length, area, and volume of physical space.</p> <p>4.OA.A2: This standard requires students to use drawings and equations to solve word problems with multiplication and division. Symbols are used to represent the unknown. Students must distinguish between additive and multiplicative comparison. (more than vs. times as) In an additive comparison, the underlying question is 'what amount would be added to one quantity in order to result in the other?' In a multiplicative comparison, the underlying question is 'what factor would multiply one quantity in order to result in the other?'</p>		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> Students may confuse addition and multiplication. For example, when asked to write an equation for 7 times as many as 5, a student may write $7 + 5$ instead of 7×5. Students may have trouble with the language in the word problems. For example, "3 times fewer" may be interpreted as the same as "3 less than" when solving word problems. 		

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach Targeted: *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying using the four operations with whole numbers to solve problems because students need to represent verbal statements of multiplicative comparisons as multiplication equations.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.D.8 This standard provides a foundation for work with using the four operations with whole numbers to solve problems because this standard works on two step problems using the four operations. It also asks students to create an equation using a letter for unknown. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with using the four operations with whole numbers to solve problems benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because students develop interest in things they know such as using their names or topics that are relevant to students (i.e. money, video game points).

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using the four operations with whole numbers to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because students need conceptual support and a place to refer for reference when working with word problems or different operations.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using the four operations with whole numbers to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because students need direct instruction on drawing models or using manipulatives to determine what word problems are asking. This is also true to the four operations. See CCSS Math Glossary Table 1 (<http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/>) and

CCSS Math Glossary Table 2

(<http://www.corestandards.org/Math/Content/mathematics-glossary/Table-2/>)

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using the four operations with whole numbers to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks because manipulatives will help with this cluster. Students can create diagrams for word problems or create arrays to answer multiplication problems.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using the four operations with whole numbers to solve problems benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need repeated, ongoing practice with word problems. They need access to problems examples in CCSS Math Glossary Tables 1 & 2. This practice and review will help with this cluster.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using the four operations with whole numbers to solve problems by examining tasks from a different perspective through a short mini-lesson because students need support in discovering different ways to solve problems. This can be through group work or small group.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit using the four operations with whole numbers to solve problems by confronting student misconceptions because students will need clear understanding to solve multi-step problems using the four operations. Students need practice and support when they don't understand word problems. Students need direct instruction using manipulatives.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying using the four operations with whole numbers to solve problems because word problems are seen through the students' academic careers and will be found in a variety of places. Working on different real world problems or creating problems help students understand different situations and help with understanding of complex problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Posing Purposeful Questions): CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying, four operations with whole numbers to solve problems, the pattern of questions within the classroom is critical because it is important to include every student in no particular order. When grouped appropriately, students can share prior knowledge and support each other's strengths and weaknesses. Students set group norms in respect of their cultures. This enables the development of a culture of productive discourse/discussion. Encourages questioning and validation among students' groups, the use of sentence frames and positive reinforcement. The teacher can facilitate conversations through strategic questioning.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia Testlet Grade 4- Operations and Algebraic Thinking

Standard: 4.OA.A.3

Learning Target: I can use the four operations to solve multistep word problems.

1. A garden store sells two types of flowers. The store sells 40 daisies. The store sells 8 times as many tulips as daisies.
 - a. How many tulips does the store sell? Show your work or explain how you know.
The garden store sells each daisy for \$4 and each tulip for \$6.
 - b. How much more money does the store make from the sale of tulips than from the sale of daisies? Show your work or explain how you know.

This type of assessment question requires students to interpret multiplication equations as a comparison, solve multiplication or division word problems, and solve multistep word problems using the four operations. These are all standards that make up this cluster. This task might be used towards the end of unit with this cluster because it asks students to do all operations stated in standards of this cluster. Teacher will be able to see misconceptions and where students are having errors for reteach.

Relevance to families and communities:

During a unit focused on using the four operations with whole numbers to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example teachers can encourage students to write a word problem about something at home. This could be written in conjunction with family members. This gets the family talking about the math and different examples brought back into the classroom.

Cross-Curricular Connections:

Science: In fourth grade the NGSS recommends that students will study energy. Teachers should give students opportunities to use the four operations with whole numbers to solve problems. Students will also study Earth and human activity. Teachers should give students opportunities to be quantitative in descriptions. Consider providing a connection for students to be quantitative when discussing environmental effects.

4.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: B: Gain familiarity with factors and multiples.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 3: Students can construct viable arguments about whether a number is prime or composite using their understanding of factors.</p> <p>SMP 7: Students can look for and make use of structure by using basic multiplication facts to find factors for a number.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Identify factor pairs for a number using basic multiplication facts. • Determine whether a number is a multiple of another number using basic multiplication facts. • Identify prime or composite numbers. • Find all factors pairs for whole numbers 1-100 • Determine if a number in the range of 1-100 is a multiple of a given one-digit number • Understand that a whole number is a multiple of its factors • Determine if a number in the range 1-100 is prime or composite number
		<p>Depth Of Knowledge: 1-2</p>
		<p>Bloom’s Taxonomy: Remember</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> • Connect to determining the unknown value in a multiplication or division equation (3. O.A.A.4) • Connect to fluently multiplying and dividing numbers within 100 (3.OA.C7) • Connect to learning about representing unknown quantities with a letter (3.OA.D8) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> • Connect factor pairs to multiplicative comparisons (4.OA. A1) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> • Connect to representing expressions with whole number exponents (6.EE.A1) • Connect to determining the greatest common factor and least common multiple of two whole numbers (6.NS.B4)

Clarification Statement:

4.OA.B4: This standard requires students to find all **factor pairs** for **whole number** in the range of **1-100**. It also requires students to determine whether a whole number in the range of 1-100 is **prime** or **composite**.

Common Misconceptions

- A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite.
- Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying gaining familiarity with factors and multiples because it helps students with conceptual understanding and being able to visually see patterns within factors and multiples. This can be connected to different standards in previous grades such as skip counting.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.C.7 This standard provides a foundation for work with gaining familiarity with factors and multiples because students will need to fluently multiply and divide within 100 and know from memory all products of two one-digit numbers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with gaining familiarity with factors and multiples benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because factors are about different patterns. Teachers can create interest by using topics that students are familiar with or can create interest as students work on them.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with gaining familiarity with factors and multiples benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students need a way to conceptually see patterns. Using mathematics representations and scaffolds allows the teacher to give support without having to prompt students at every step.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with gaining familiarity with factors and multiples benefit when learning experiences attend to the linguistic and nonlinguistic

representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because using highlighting or color coding to help students see patterns will help with conceptual understanding.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with gaining familiarity with factors and multiples benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because students need a way to make sense of the patterns of factors and multiples by using manipulatives. As students can see patterns quicker, the manipulatives can be removed. Those who cannot see patterns will need more work with manipulatives.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with gaining familiarity with factors and multiples benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because giving students a way to organize factors and/or multiples will help them. This also will reduce students repeating factors (for example: 1×8 and 8×1).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on gaining familiarity finding factor pairs of whole numbers by revisiting student thinking through a short mini-lesson because students that are struggling need support where they are having difficulty. Can they multiply or divide within 100? Do they need supports for determining factors? Are they confused with the vocabulary? These are student thinking that the teacher can work on in small groups

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit gaining familiarity finding factor pairs of whole numbers by addressing conceptual understanding because students who have trouble with coming up with factors quickly will need additional support with conceptual understanding. This includes manipulatives or visual models. It also might include the use of mathematical tools such as hundreds charts.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying gaining familiarity finding factor pairs of whole numbers because students can look for factor relations through

different curriculum data or real-world examples. This will help with work into 5th (fractions) and 6th (ratios).

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the way students talk about who is and isn't capable of mathematics, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying gaining familiarity with factors and multiples facilitating meaningful mathematical discourse is critical because factors and multiples can be interpreted in many different ways. These are types of patterns that are seen in different areas of academic and life. Supporting mathematical discourse around tasks or problems with factors and multiples allows the teacher to determine misconceptions and helps the classroom develop different strategies for determining answers. A teacher can use questioning to guide and further students thinking through discourse. In the same way, the teacher can ask guiding questions about misconceptions and lead students to understanding

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/OA/B/4/tasks/938>

Standard: 4.OA.B.4

Task: The Locker Game

The Locker Game ✕

The 20 students in Mr. Wolf's 4th grade class are playing a game in a hallway that is lined with 20 lockers in a row.



- The first student starts with the first locker and goes down the hallway and opens all the lockers.
- The second student starts with the second locker and goes down the hallway and shuts every other locker.
- The third student stops at every third locker and opens the locker if it is closed or closes the locker if it is open.
- The fourth student stops at every fourth locker and opens the locker if it is closed or closes the locker if it is open.

This process continues until all 20 students in the class have passed through the hallway.

This type of assessment question requires students to extend work on understanding of multiplication. Students will look for patterns within problem and connect those to factors. Students might have this task towards the end of unit work. This task can also be differentiated to meet needs of students, but still give teacher the reteach information they need.

Relevance to families and communities:

During a unit focused on gaining familiarity with factors and multiples, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can discover different patterns that lend to factors and multiples in their home. They can further mathematical discourse with family about patterns and different ways factors and multiples are used in everyday life.

Cross-Curricular Connections:

Social Studies: Connect students to the history behind the Sieve of Eratosthenes, the ancient algorithm that helps us to determine factors, multiples, primes, and composites for all numbers. Study the life and accomplishments of the Greek astronomer Eratosthenes of Cyrene and teach students how to use the Sieve.

4.OA: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: C: Generate and analyze patterns.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP 8: Students look for and express regularity in repeated reasoning by using rules to generate or extend a number or shape pattern.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Describe rules in number and shape patterns. Identify features of a pattern when given a rule. Make observations about a resulting sequence given a rule, such as noticing that the terms alternate between even and odd numbers. Model and solve multi-step word problems using equations.
		<p>Depth Of Knowledge: 2-3</p>
		<p>Bloom's Taxonomy: Evaluate, Create</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to arithmetic patterns and properties of operations (3.OA.D.9) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to solving problems using a letter to stand in for an unknown quantity (4.OA.A2) Connect patterns to multiples and the multiplication table. 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to future learning of generating two numerical patterns given two rules (5.OA.B3)
<p>Clarification Statement: 4.OA.C5: Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features. After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.</p>		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> Students may not think a number is a multiple of itself. Students may think that numbers with a greater value have more factors than numbers with a lesser value. 		

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying generating and analyzing patterns because students will be building on arithmetic that was built in previous grades. This standard in 4th grade will build on working with patterns, which began in previous grades.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.D.9: This standard provides a foundation for work with generating and analyzing patterns because students are reasoning about operations and begin identifying patterns in addition and multiplication. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with generating and analyzing patterns benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as concrete manipulatives and models or other mathematical tools such as tables because this allows students to gain concrete understanding of different patterns and visually see what is happening between patterns. As students become more efficient with concrete models, they can move towards tables to organize and make connections through numbers.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with generating and analyzing patterns benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students can rise to high expectations using flexible tools and supports.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with generating and analyzing patterns benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because students will be able to construct meaning from visuals, symbols, and numbers using different representations.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with generating and analyzing patterns benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because students will be able to express their learning in flexible ways and communicate their thoughts and ideas through using these physical manipulatives. For example, student can see different patterns by seeing them laid out with counters.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with generating and analyzing patterns benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because students might look at patterns and become frustrated, however when given feedback and encouragement students can progress and develop independent understanding. For example, questions and comments will be based on group or individual work and progressing them through understanding.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on generating and analyzing patterns by revisiting student thinking through a short mini-lesson because students can usually identify patterns, but may not know how to put them into a rule or statement. Reviewing student thinking will help students identify patterns and put them into mathematical representation or tables.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit generating and analyzing patterns by addressing conceptual understanding because students will be able to see patterns better with conceptual understanding (examples: manipulatives or concrete models). Looking at a list of numbers is an abstract skill that struggling students need to work towards. The teacher will need to support conceptual work and move students to more abstract work (example: manipulatives to data table).

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying generating and analyzing patterns because data is accessible in many different subjects. Students can explore data in social studies, science, or independent investigations. In this way, students learn more about practical or real-world applications.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying generating and analyzing patterns the use of mathematical representations within the classroom is critical because students need to understand patterns, create tables that accurately represent the mathematics, and answer questions based on the data. These representations are seen in many different areas of academics, but in everyday life and can be related to home cultures.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/OA/C/5/tasks/487>

Standard: 4.OA.C.5

Task: Double Plus One

Double Plus One



a.

The table below shows a list of numbers. For every number listed in the table, multiply it by 2 and add 1. Record the result on the right.

number	double the number plus one
0	
1	
2	
3	
4	
5	
10	
23	
57	
100	
309	

b. What do you notice about the numbers you entered into the table?

C.

Sherri noticed that all the numbers she entered are odd.

i.

Does an even number multiplied by 2 result in an even or odd number? Why do you think this is?

ii.

Does an odd number multiplied by 2 result in an even or odd number? Why do you think this is?

iii.

Does an even number plus 1 result in an even or odd number? Why do you think this is?

iv.

Does an odd number plus 1 result in an even or odd number? Why do you think this is?

v.

Explain why the numbers you entered in the table are all odd.

This type of assessment question requires students to find, extend, and describe patterns. Students will use SMP 4, SMP 5, and SMP 7. These math practices use manipulatives or visual models for determining patterns within problems. Students are also asked to determine patterns (which can be found in multiple areas in this task). This task might be used within instruction of this standard. Students can work with groups to determine and discuss different patterns seen. This task can also be broken down into multiple days or parts as students work through it. Differentiation can be built in also for this task (for example, having students finish one part before moving to the next).

Relevance to families and communities:

During a unit focused on generating and analyzing patterns, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning can be expanded to

Cross-Curricular Connections:

Art: Patterns are prevalent in artistic compositions. Have students generate a rule to create a mosaic pattern. For example: Given the rule "add three" and the starting point "one" create a mosaic or other artistic composition using two different colors that follows the pattern. Increase the complexity of the pattern to result in a more complex

home. Students can ask questions or gather data that pertains to their home unit or culture.	color scheme. This could also be used with notes in music.
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4.NBT: NUMBER & OPERATIONS IN BASE TEN

Cluster Statement: A: Generalize place value understanding for multi-digit whole numbers.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
<p>4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</p>	<p>SMP 2: Students can reason abstractly and quantitatively by making sense of quantities and their relationships in problem situations.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Recognize that $700 \div 70 = 10$ by applying concepts of place value and division Understand that a quantitative relationship exists between the digits in place value positions of a multi-digit number. Explain that a digit in one place represents ten times what it would represent in the place to its right.
		Depth Of Knowledge: 1,2
		Bloom's Taxonomy: Understand
<p>4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<p>SMP 3: Students can construct viable arguments and critique the reasoning of others by explain their thinking when comparing numbers and critique the reasoning of their peers explanations.</p> <p>SMP 6: Students can attend to precision when comparing two numbers.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Explain the difference between standard, word, and expanded forms. Read multi-digit whole numbers using base-ten numerals, number names, and expanded form. Write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers and write the comparison using symbols.
		Depth Of Knowledge: 1,2

		Bloom's Taxonomy: Apply
<p>Standard Text</p> <p>4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place.</p>	<p>Standard for Mathematical Practices</p> <p>SMP3: Students can construct viable arguments and critique the reasoning of others by explaining how they rounded whole numbers to a given place using place value understanding.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Explain the role of place value when rounding whole numbers • Round multi-digit whole numbers to any place.
		<p>Depth Of Knowledge: 1,2</p>
		<p>Bloom's Taxonomy: Apply</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> • Connect to understanding that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (2.NBT.1) • Connect to reading and writing numbers to 1,000 using base-ten numerals, number names and expanded form (2.NBT.3) • Connect to comparing two three-digit numbers based on meanings of the hundreds, tens, and one's digits, using $>$, $=$, and $<$ symbols to record the results of comparisons. (2.NBT.4) • Connect to using place value understanding to round two-digit and three-digit numbers to the nearest 10 and 100 (3.NBT.1) • Connect to multiplying one-digit whole numbers by multiples of ten (3.NBT.3) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> • Connect to multiplying a whole number up to four digits by a one-digit whole number, and multiply two two-digit numbers using strategies based on place value (4.NBT.5) • Connect to finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value (4.NBT.6) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> • Connect to recognizing that in a multi-digit number, a digit in one place represent 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left. (5.NBT.1) • Connect to explaining patterns in the numbers of zeros of the product when multiplying a number by powers of 10 (5.NBT.2) • Connect to reading, writing, and comparing decimals to thousandths. (5.NBT.3) • Connect to using place value understanding to round decimals to hundredths (5.NBT.4)

Clarification Statement:

4.NBT.A1: This standard calls for students to extend their understanding of **place value** related to **multiplying** and **dividing** by **multiples of 10**. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and **analyze** the **relationships** of numbers that they are working with.

4.NBT.A2: This standard requires students to **read** and **write multi-digit whole numbers** using **numerals (standard form), word form, and expanded form**. It also requires students to **compare** 2 multi-digit whole numbers (based on place value meaning) using the symbols $<$, $>$, $=$. Be mindful when teaching not to teach a number is larger because it has more digits or a number is smaller because it has fewer digits. This will confuse students when they move into comparing decimal numbers.

4.NBT.A3: The standard requires students to use **place value** to **round** with any given **whole number** to any given place value. The standard focuses on using place value. Students need to use **visual model** or **manipulatives** when learning to round numbers so they understand the mathematical reasoning for rounding up or down. A **number line** may be a good visual when rounding.

Common Misconceptions

- Students may struggle with numbers such as one thousand two. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two).
- Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole.
- Students may get confused when rounding to specific place values. For example, when asked to round 712 to the nearest ten, they may round to the nearest hundred. Students need work with number line and other mathematical tools to help build this understanding of rounding.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying generalizing place value understanding for multi-digit whole numbers because students are working with multiples of ten moving from one place value to another. Students that are taught to "just add a zero" when multiplying by ten will not understand mathematically why this works. Similarly, with rules for rounding, students need to understand why we round up from 5 and up. Knowing this rule does not help them, but visually seeing which number is closer will help with rounding.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.NBT.A.1: This standard provides a foundation for work with generalizing place value understanding for multi-digit whole numbers because this standard build student understanding of place values and values of these numbers up to the hundreds place. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with generalizing place value understanding for multi-digit whole numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students are working with very large numbers, most of which are very abstract, and will need support and guidance through work with these large numbers.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with generalizing place value understanding for multi-digit whole numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students are working with very large numbers, most of which are very abstract, and will need support and guidance through work with these large numbers.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with generalizing place value understanding for multi-digit whole numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because students can begin to see patterns in the symbols thus constructing meaning.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with generalizing place value understanding for multi-digit whole numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students can express their learning in different ways and share these thoughts and ideas.

Internalize

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

- For example, learners engaging with generalizing place value understanding for multi-digit whole numbers benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students can become frustrated without beginning to work on a problem, but through encouragement and support students can change their thinking about math work and their thinking when beginning a math problem.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on generalizing place value understanding for multi-digit whole numbers by revisiting student thinking through a short mini-lesson because targeting revisiting of student thinking will allow the teacher to correct misconceptions and/or give more help with conceptual understanding.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit generalizing place value understanding for multi-digit whole numbers by revisiting student thinking by addressing conceptual understanding because conceptual understanding needs to be built with manipulatives or visual models.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying generalizing place value understanding for multi-digit whole numbers because students need extensive practice to become fluent with multi-digit numbers, including manipulating them and using them in different contexts.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Eliciting and Using Evidence of Student Thinking): Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time. For example, when studying generalizing place value understanding for multi-digit whole numbers eliciting and using student thinking is critical because students need to work with very abstract numbers. Students will need to manipulate these numbers by understanding movement between place values and rounding numbers. This work is best done with students working together on tasks to explain and expand thinking through discourse. The teacher can further discourse by asking prompting questions or extending thinking through questions.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia Testlet for Grade 4- Numbers and Operations in Base Ten

Standard: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Learning Targets: I can write numbers in number form given the expanded and word forms. I can compare multi-digit whole numbers. I can relate the value of the same digit in two numbers.

1. The number of people that live in Madison's town is shown in expanded form.

$$400,000 + 2,000 + 700 + 9$$

The number of people that live in Keisha's town is four hundred twenty-seven thousand nineteen.

- a. Write the number of people in Madison's town in number form.
- b. Write the number of people in Keisha's town in number form.
- c. Compare the numbers in parts (a) and (b) using $<$, $>$, or $=$.
- d. How many times greater is the value of the 2 in the number of people that live in Keisha's town than the value of the 2 in the number of people that live in Madison's town? Show your work or explain how you know.

This type of assessment question requires students to determine number form, comparing numbers, and comparing place values within two different numbers. This task uses SMP 1 because students have to determine which information is needed to answer each step in the problem. This task can be used as independent end of unit assessment. It could also be used in smaller chunks throughout a week of instruction to help drive instruction.

Relevance to families and communities:

During a unit focused on generalizing place value understanding for multi-digit whole numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can gather information from home about ways that parents would use rounding in everyday life. These examples can be shared and explored as tasks that directly relate back to home and culture.

Cross-Curricular Connections:

Science: Study of planets' distance from the sun may present an opportunity to connect to concepts of base-10 and place value.

Social Studies: Study of populations (state, country, and world) may present an opportunity to connect to concepts of base-10 and place value.

4.NBT: NUMBER & OPERATIONS IN BASE TEN

Cluster Statement: B: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p>	<p>Standard for Mathematical Practices</p> <p>SMP6: Students can attend to precision when they carefully regroup when adding and subtracting.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Fluently use standard algorithm to add multi-digit whole numbers Fluently use the standard algorithm to subtract multi-digit whole numbers <p>Depth Of Knowledge: 1,2</p> <p>Bloom’s Taxonomy: Apply</p>
<p>Standard Text</p> <p>4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p>Standard for Mathematical Practices</p> <p>SMP3 Students can construct viable arguments and critique the reasoning of others by explaining their strategy for multiplying multi-digit numbers.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Explain the role of place value and the properties of operations when multiplying multi-digit numbers Solve multi-digit multiplication problems <p>Depth Of Knowledge: 1,2</p> <p>Bloom’s Taxonomy: Apply, Understand</p>

<p>Standard Text</p> <p>4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p>Standard for Mathematical Practices</p> <p>SMP3: Students can construct viable arguments and critique the reasoning of others by explaining their calculations using equations, rectangular arrays, AND/OR area models.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Find whole number quotients with up to 4-digit dividends and 1-digit divisors using strategies based on place value, properties of the operations, AND/OR the relationship between multiplication and division. Students can successfully use one of the following: Illustrate division with equations Illustrate division with rectangular arrays Illustrate division with area models <p>Depth Of Knowledge: 1,2</p> <p>Bloom’s Taxonomy: Understand</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to fluently adding and subtracting within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (2.NBT.5) Connect to adding and subtracting within 1,000 using concrete models or drawings (2.NBT.7) Connect to fluently adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (3.NBT.2) Connect to multiplying one-digit whole numbers by multiples of 10 in the range 10–90, for example, 9×80 and 5×60 (3.NBT.3) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to recognizing that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (4.NBT.1) Connect to finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors (4.NBT.6) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to fluently multiplying multi-digit whole numbers using the standard algorithm (5.NBT.5) Connect to finding quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies (5.NBT.6) Connect to adding, subtracting, multiplying, and dividing decimals to hundredths, using concrete models or drawings and strategies (5.NBT.7) Connect to explaining patterns in the number of zeros of the product when multiplying a number by powers of 10 (5.NBT.2)
<p>Clarification Statement: 4.NBT.B.4: The standard requires students to add and subtract multi-digit whole numbers using the standard algorithm. Students who struggle with the algorithm needs more experience with hands-on materials. Scaffolding students with place value understanding will help students with regrouping misconceptions.</p>		

4.NBT.B.5: This standard requires students to **multiply** up to **4 digits by 1 digit** and **2 digits by 2-digit** numbers using place value AND the properties of operations. Students may calculate using equations, **rectangular arrays**, AND/OR **area models**. **Properties of operations: commutative, associative, distributive**. Previous grade level standards focused on using place value and properties of operations. Also, students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits. Specific strategies or students having difficulty with lining up similar place values in numbers as they are adding and subtracting.

4.NBT.B.6: General methods for computing **quotients of multi-digit numbers and one-digit numbers** rely on the same understandings as for multiplication, but cast in terms of division. One component is quotients of **multiples of 10, 100, or 1000** and one-digit numbers. Another component of understanding general methods for multi-digit division computation is the idea of **decomposing** the **dividend** into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This work can be done through methods such as **partial quotients** or **area model for division**.

Common Misconceptions

- Students may confuse the role of place-value when "regrouping". In addition, students may add $35 + 19$ and say the answer is 414. They may add 9 and 5 to get 14 then add 3 and 1 to get 4 and put them together. In subtraction, students may flip numbers in the subtrahend and minuend to make the numbers work to subtract. For example, for $51-27$, a student may think you cannot do $1-7$ and may flip it to be $7-1$, then subtract $5-2$ to get 3, then state the answer is 36.
- Students DO NOT use the standard algorithm to divide in 4th grade. Some students may struggle to recognize the place value inherent in multiplication. Do not rush to teach the algorithm, as a knowledge of where the numbers come from is necessary for a full understanding of the algorithm.
- Students DO NOT use the standard algorithm to divide in 4th grade. The standard algorithm is a 6th grade standard. 4th grade should focus on place value, properties, models, etc., to multiply multi-digit numbers. Students who have been taught the algorithm may not understand the importance of place value. They may misapply the algorithm and get a number that does not make sense. It is important to ensure a full understanding of the importance of place value before even considering the algorithm for multiplication or division.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying place value understanding and properties of operations to perform multi-digit arithmetic because reviewing models used prior to grade 4 will help students move to adding and subtracting using the standard algorithm. Students can use place value models they used for adding and subtracting as they create place value models when multiplying and dividing.

Pre-teach (intensive): What critical understandings will prepare students to access the mathematics for this cluster?

- 3.NBT.A.2: This standard provides a foundation for work with place value understanding and properties of operations to perform multi-digit arithmetic because being able to fluently add and subtract within 1000 using the properties of operations and/or the relationship between addition and subtraction is the foundation for accuracy and understanding of the algorithm. If students have unfinished learning within this standard, based on assessment data, consider ways to

provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences include ways to recruit interest such as providing choices in their learning such as choosing strategies to demonstrate understanding because students are applying and extending their knowledge of place value and properties of operations to reach fluency when performing multi-digit arithmetic.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as displaying the learning goals in multiple ways because using multiples strategies, based on place value and properties of operations demonstrates fluency with multi-digit arithmetic.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because it is important for students to make connections to previous work with using models to solve addition and subtraction problems as they may use the same models can be used to explain their thinking when multiplying and dividing based on place value and properties of operations.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because using multiple strategies to explain thinking develops fluency.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using place value understanding and properties of operations to perform multi-digit arithmetic benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as provide tasks with multiple entry points and optional pathways because multiple entry points will allow students at various readiness levels experience success and optional pathways to demonstrate understanding pushes students to a deeper understanding.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using place value understanding and properties of operations to perform multi-digit arithmetic by providing specific feedback to students on their work through a short mini-lesson because focusing on solidifying place value understanding will help students when adding and subtracting using the standard algorithm.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on using place value understanding and properties of operations to perform multi-digit arithmetic by confronting student misconceptions because identifying and correcting misconceptions with place value understanding is crucial to students being successful with adding and subtracting using the standard algorithm as well as using place value understanding to multiply and divide whole numbers.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying using place value understanding and properties of operations to perform multi-digit arithmetic because through working with adding, subtracting, multiplying, and dividing in various curriculum areas, students will not only make connections between math and the real world, but they will also see the value of math in life.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Goal Setting): Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying using place value understanding and properties of operations to perform multi-digit arithmetic goal setting is critical because when students reach goals with adding, subtracting, multiplying, and dividing they realize success which increases math confidence. Multi-digit arithmetic success relies on fluency, and many students may need to continue to work toward fluency.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/NBT/B/tasks/1189>

Standard: 4.NBT.B

Task: To Regroup or Not to Regroup

Task

Sometimes when we subtract one number from another number we "regroup," and sometimes we don't. For example, if we subtract 38 from 375, we can "regroup" by converting a ten to 10 ones:

$$\begin{array}{r}
 615 \\
 \cancel{3}7\cancel{5} \\
 - 38 \\
 \hline
 337
 \end{array}$$

Find a 3-digit number to subtract from 375 so that:

- a. You don't have to use regrouping.
- b. You would naturally use regrouping from the tens to the ones place.
- c. You would naturally use regrouping from the hundreds place to the tens place.
- d. You would naturally use regrouping in all places.

In each case, explain how you chose your numbers and complete the problem.

This type of assessment question requires students to think in different ways that are not procedural. Students can use different strategies to finding a number that meets each criterion. SMP 7 would apply with this task because students would find strategy and begin to see pattern emerge within work. This task could be used at

end of unit, however students that struggle can still determine answers using manipulatives or visual models. Those that struggle might have to go back to visual models or manipulatives to help with subtraction.

Relevance to families and communities:

During a unit focused on using place value understanding and properties of operations to perform multi-digit arithmetic, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, possible examples include: students look for ways they use arithmetic outside of school, students ask family members and/or friends how they use arithmetic in their everyday lives, students look for examples of arithmetic in their daily lives. Students can share their findings with class by: having conversations, creating written and/or visuals in print or non-print formats. As students share with each other, they will make connections.

Cross-Curricular Connections:

Social Studies: Consider giving students an opportunity to study and compare populations in various geographic areas within the state, country, or world. Students can use their understanding of place value to solve real-world problems that require them to compare populations using addition and subtraction.

4.NF: NUMBER & OPERATIONS-FRACTIONS

Cluster Statement: A: Extend understanding of fraction equivalence and ordering.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
<p>4.NF.A.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\left(\frac{n \times a}{n \times b}\right)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>	<p>SMP 4: Students can model mathematics by creating visual models of equivalent fractions to build understanding.</p> <p>SMP 8: Students can look for and express regularity in repeated reasoning by discussing patterns as they find equivalent fractions.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Use models to show the value of a fraction • Explain how a fraction model represents the quantity of a fraction • Use models to demonstrate that two fractions are equivalent • Represent equivalent fractions using models. • Multiply and divide to find equivalent fractions.
		Depth Of Knowledge: 1,2
		Bloom’s Taxonomy: Understand
<p>4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions,</p>	<p>Standard for Mathematical Practices SMP 3/4: Students can construct and justify viable arguments comparing the size of two fractions using benchmark fractions, number lines, and visual fraction models.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Explain how to convert two fractions to have common denominators. • Explain how to convert two fractions to have common numerators. • Convert fractions to have common denominators. • Convert fractions to have common numerators. • Compare two fractions with different numerators and denominators. • Use symbols ($<$, $>$, $=$) to compare two fractions

<p>e.g., by using a visual fraction model.</p>		<p>Depth Of Knowledge: 1,2</p>
		<p>Bloom’s Taxonomy: Apply</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> • Connect to partitioning shapes into halves, thirds, and fourths in 2nd grade. • Connect to finding equivalent fractions and using symbols to compare fractions in 3rd grade. 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> • Connect to learning about multiplicative comparison (4.OA. A2) • Connect to learning about using an understanding of relative size to convert between units of measurement (4.MD. A1) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> • Connect to adding and subtracting fractions and mixed numbers with unlike denominators (5.NF. A1) • Connect to solving fraction addition and subtraction word problems (5.NF. A2)
<p>Clarification Statement: Equivalent fractions are fractions that represent equal value. They are numerals that name the same fractional number. Equivalent fractions have wholes that are the same size, students need to understand this concept. Upon generating a rule for finding equivalent fractions, students should understand how that connects to the identity property of multiplication or division ($5/5 = 1$, therefore any fraction multiplied by $5/5$ would be the equivalent or equal). Students should generate and justify why their fractions are equivalent. This lends to the generation of the rule (or procedure) for equivalent fractions. Upon discovering the rule, students should be able to explain why the rule works.</p>		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> • Students may be confused by “reducing” since it implies that the number is getting smaller. 		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach</p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> ▪ For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying extend understanding of fraction equivalence and ordering because students have worked with simple equivalent fractions and comparing fractions with denominators of 2, 3, 4, 6, and 8. <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p> <ul style="list-style-type: none"> ▪ 3.NF.A.3: This standard provides a foundation for work with extend understanding of fraction equivalence and ordering because in this third grade standard students begin to work with simple equivalent fractions and comparing using similar numerator and denominator. This is foundational work with equivalent and ordering fractions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. <p>Core Instruction</p> <p>Access</p> <p>Physical Action: <i>How will the learning for students provide a variety of methods for navigation to support access?</i></p>		

- For example, learners engaging with extending understanding of fraction equivalence and ordering benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to pencil and paper while creating fractions and fraction equivalencies and range of motor action with instructional materials, physical manipulatives, and technologies; physically interacting with materials by hand or keyboard because constructing fractions and their equivalents using more than paper and pencil will help students develop a deeper understanding.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with extending understanding of fraction equivalence and ordering benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because fraction equivalents and ordering fractions can be applied to many different real-world situations that students can directly connect to their own lives thus making the work more meaningful.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with extending understanding of fraction equivalence and ordering benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because connecting the numeric fraction (a/b) to various models such as pictures, diagrams, 3D representations, etc.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with extending understanding of fraction equivalence and ordering benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives because students reach a deeper understanding of ordering fractions and fraction equivalents by creating hands-on representations.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with extending understanding of fraction equivalence and ordering benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important

information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as “chunking” information into smaller elements because focusing on working on finding equivalent fraction with regards to one unit fraction at a time or beginning with ordering two fractions and building to ordering more than two fractions builds capacity thereby increasing math confidence.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on extend understanding of fraction equivalence and ordering by revisiting student thinking through a short mini-lesson because teachers can assess what students already know and build on their thinking. Teachers can also see misconceptions in student thinking and correct them during a mini-lesson.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit extend understanding of fraction equivalence and ordering by addressing conceptual understanding because students need understanding in fractions. Students who struggle with fractions tend to look at numbers within fractions and try to generalize them. Students with these types of misconceptions will need concrete work with manipulatives to build conceptual understanding. They will need to physically see that $\frac{1}{2}$ bar is bigger than $\frac{1}{4}$ bar, even though in whole numbers a 4 is greater than 2.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying extend understanding of fraction equivalence and ordering because students need extensive work in equivalent fractions to become fluent with simple fractions ($\frac{2}{4}$ is equal to $\frac{1}{2}$, etc). This work will help them become flexible with equivalent fractions (reducing) and build foundational understanding for 5th grade work.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Using and Connecting Mathematical Representations): The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their “mathematical, social, and

cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying extending understanding of fraction equivalence and ordering fractions the use of mathematical representations within the classroom is critical because students need to work with fractions presentations in many ways, many times in order to develop a strong sense of benchmark fraction knowledge in order to help when rationalizing about fraction equivalents and ordering fractions.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/NF/A/1/tasks/881>

Standard: 4.NF.1

Task: Fractions and Rectangles

Fractions and Rectangles

a.

What fraction of the rectangle below is shaded?



b. Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct? Explain why or why not by using the picture.

This type of assessment question requires students to determine fraction, recognize equivalent fraction, and explain their thinking. This standard asks students to reason why fractions are equivalent using visual representations (SMP 4 & SMP 5). This task will help teacher see where students are having trouble with fractions and equivalence, which will help with reteaching or planning for future lessons. This task will also help determine misconceptions that can be corrected during reteach.

Relevance to families and communities:

During a unit focused on extending understanding of fraction equivalence and ordering fractions consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, students can look for ways they use fractions at home and share their findings with the classroom community.

Cross-Curricular Connections:

Science: Students may track precipitation levels in fractional amounts using a graduated cylinder or rain gage.

4.NF: NUMBER & OPERATIONS-FRACTIONS

Cluster Statement: B: Build fractions from unit fractions.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text

4.NF.B.3

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

- 4.NF.B.3.A: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- 4.NF.B.3.B: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.
- 4.NF.B.3.C: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- 4.NF.B.3.D: Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively by understanding that adding and subtracting fractions involves joining and separating parts that refer to the same whole.

SMP 4: Students can model with mathematics by decomposing fractions into a sum of fractions and justifying with a visual model.

Students who demonstrate understanding can:

- Explain why a fraction is the sum of multiple fractions
- Explain why addition and subtraction of fractions with the same denominator is joining or separating parts referring to the same whole.
- Explain why a fraction can be a sum of different like denominator fractions.
- Explain why mixed numbers can be added or subtracted
- Add and subtract fractions and mixed numbers with like denominators.
- Write an equation when decomposing fractions.
- Solve addition word problems involving fractions with like denominators using models and equations
- Solve subtraction word problems involving fractions with like denominators using models and equations

Depth Of Knowledge: 1,2

Bloom's Taxonomy: Apply

<p>Standard Text</p> <p>4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <ul style="list-style-type: none"> 4.NF.B.4.A: Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. <i>For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (1/4)$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (1/4)$.</i> 4.NF.B.4.B: Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. <i>For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $\frac{6}{5}$. (In general, $n \times (\frac{a}{b}) = (n \times \frac{a}{b})$)</i> 4.NF.B.4.C: Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i> 	<p>Standard for Mathematical Practices</p> <p>SMP6: Students can attend to precision by building on previous understandings of the meaning of numerator and denominator.</p> <p>SMP 7: Students can look for and make use of structure by using previous understanding of numerators and denominators to see the structure multiplication of fractions.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Extend the understanding of multiplication to problems that have fractions. Multiply a unit fraction (numerator of 1) by a whole number. Multiply a fraction with a numerator greater than 1 by a whole number. Use a number line to represent fraction multiplication. Explain why a fraction is a multiple of a unit fraction Explain why multiplying a whole number times a fraction can be changed to a whole number times a unit fraction. Solve multiplication word problems involving whole numbers and fractions using models and equations. Restate word problems involving multiplication of a whole number and a fraction. Draw a diagram and write an equation represent and solve a word problem involving multiplication of a whole number and a fraction.
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to understanding a $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. (3.NF.A1) Connect fraction and decimal notation to measuring the 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect converting fractions and decomposing fractions to converting measurements from a larger unit to a smaller unit (4.MD A2) 	<p>Depth Of Knowledge: 1,2</p> <p>Bloom's Taxonomy: Apply</p> <p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to extending and applying multiplication to multiply a fraction or whole number by a fraction (5.NF.B4) Connect to interpreting multiplication as scaling/ resizing (5.NF.B5)

<p>length of an object to $\frac{1}{2}$ and $\frac{1}{4}$ inches (3.MD.b4)</p> <ul style="list-style-type: none"> Connect to multiplication and division within 100 involving arrays, equal groups, and measurement quantities (3.OA.A.3) 		
<p>Clarification Statement:</p> <p>4.NF.B3: This standard builds on prior work with unit fractions where students will now investigate fractions other than unit fractions such as $\frac{2}{3}$, they should then be able to join (compose) AND separate (decompose) the fraction of the same whole. In order to gain conceptual understanding of this standard, students must be able to visualize the composition and decomposition into unit fractions. This skill will aid in the development needed to then move into adding and subtracting fractions. For students to visualize they must have multiple opportunities to model this concept by using hands on manipulatives and other appropriate tools such as creating an original drawing to develop the skill. The models should not be limited to area models only (vary the type of area model), and should include length models such as number lines, folded paper, rulers, fraction strips, and set models as well.</p> <p>4.NF.B4: a Students will be able to model multiplication of whole numbers by unit fraction. Building on ideas of decomposing fraction into unit fractions, students will apply knowledge and work with multiplying of whole numbers to this work. Using similar language, such as “groups of” or “jumps” on a number line. Students will be able to explain this using models, words and numbers. Students will also be able to look at a given picture and describe multiplication that is present, in essence working backwards. b. Students will be able to apply patterns from multiplying whole numbers by a unit fraction to multiplying whole numbers by any fractions. Students are still using models and manipulatives for this work. Students will be able to explain this using models, words and numbers. This work includes fractions that are greater than 1 or mixed numbers. Students can apply previous work with whole number multiplication to multiplication with fractions, such as area model, distributive property, etc. c. Students will be able to solve multiplication word problems that include whole number by a fraction or mixed number. Students are still using models and manipulatives for this work. Students will be able to explain this using models, words and numbers.</p>		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> Students may misunderstand and believe when you multiply a fraction by a whole number, first you multiply the numerator by the whole number and then you multiply the denominator by the whole number. Students that are taught to put a 1 under the whole number and multiply straight across may lack an understanding of why this algorithm works. 		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach</p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers because it allows the teacher to plan in order to meet individual student needs, small group needs, and whole class needs. <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p>		

- 3.NF.A: These standards provide a foundation for work with building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers because it helps students understand what the parts of a fraction represent and see fractions as numbers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because breaking down a whole into equal parts are more visible when related to their own lives. Students have experience with breaking whole numbers into "groups of" but this will be new for them to view one whole as unit fractions.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because using real world, relevant examples will help students see parts of one whole in terms of unit fractions in a way that makes sense to them will build math confidence.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because the term "unit fraction" is new and essential to learning within this cluster. Breaking down, or decomposing, a fraction into unit fractions helps students relate to size of the units in relation to the whole.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-

10 blocks, algebra blocks) because physically working with materials to decompose and compose fractions will help students move toward explaining their thinking with justifications.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because keeping a graphic organizer, such as a chart, which includes the fraction, decomposition using unit fractions, and a visual, will help students see patterns when working on future similar problems.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers by providing specific feedback to students on their work through a short mini-lesson because specific feedback during a one-on-one or small group mini-lesson helps students see the mistake. Working through a few more problems where the student can immediately apply the feedback strengthens their understanding of building from unit fractions and/or decomposing a fraction into unit fractions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers by addressing conceptual understanding because students need work with concrete manipulatives when struggling with fractions. This can include area models, fraction strips, number lines, and other visual models. This will build conceptual understanding and mental pictures for student work with fractions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers because students need extended work with fractions. They need to reason about fractions and part of a whole. Students can use extended work to make generalizations and reasonings about work with fractions. For example, the denominator stays the same because the fractional pieces are

equivalent. This also includes connections between addition, multiplying, and properties of operations that apply here as well.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Building Procedural Fluency from Conceptual Understanding): Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers the types of mathematical tasks are critical because we are connecting to previous work with decomposing whole numbers and to work with "groups of" used in multiplication.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/NF/B/3/tasks/837>

Standard: 4.NF.3b

Task: Making 22 Seventeenths in Different Ways

Making 22 Seventeenths in Different Ways

Which of the following sums are equal to $\frac{22}{17}$?

a. $\frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17}$

b. $\frac{3}{17} + \frac{8}{17} + \frac{3}{17} + \frac{10}{17}$

c. $\frac{6}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17}$

d. $\frac{12}{17} + \frac{10}{17}$

e. $\frac{1}{17} + \frac{1}{17} + \frac{9}{17} + \frac{3}{17}$

Find another way to write $\frac{22}{17}$ as a sum of fractions.

This type of assessment question requires students to decompose fractions in more than one way. Students reason about different equations listed and have to produce one different way from those listed. This can be a lead task into mixed numbers. This standard is a vital part of fraction work in 4th grade, students that struggle with this type of task need more manipulative or visual models. Some might need manipulatives to complete task.

Relevance to families and communities:

During a unit focused on building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, examining fractions of students in the classroom and fractions of members in a family to help students work within decomposing one whole. The one whole is the classroom, family, team, or other group a student is connected to outside of school. This will help students understand a whole can vary in size.

Cross-Curricular Connections:

STEM Connection: Students can create a measuring cup for a science experiment. As they do, have students pay attention to precision and remind them the parts must be equal. The act of partitioning reinforces an understanding of the relationship between the unit fraction and the whole. Reinforce the relationship between mixed numbers and their fraction equivalent.

Music: Students can partition a unit fraction (or beat) into smaller unit fractions (e.g., subdividing each fourth note to create 2 eighth notes). This helps students see mathematical and musical relationships among:

- the denominator (the type of note)
- the number of parts in the whole quantity (how many fit into a bar)
- the size of the part (the duration of that note)

<https://thelearningexchange.ca/wp-content/uploads/2018/04/cbs-fraction-across-curriculum-en.pdf>

4.NF: NUMBER & OPERATIONS-FRACTIONS

Cluster Statement: C: Understand decimal notation for fractions, and compare decimal fractions.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students can make sense of problems and persevere in solving them by first expressing two fractions with the same denominator and then adding them.</p> <p>SMP 8: Students look for and express regularity in repeated reasoning by generating fractions that can be added using concepts of base 10 and then adding them.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Explain how a fraction with a denominator of 10 is equal to a fraction with a denominator of 100. • Convert a fraction with a denominator of 10 to an equivalent fraction with a denominator of 100. • Add two fractions with denominators of 10 or 100.
<p>Standard Text</p> <p>4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 7: Students can look for and make use of structure by writing fractions as decimals using the previous knowledge of place value structure.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Write fractions with denominators of 10 or 100 as decimals. • Write decimals as fractions with denominators of 10 or 100. • Write a money amount given in words as a whole dollar and fraction amount. • Write a measurement using decimals. • Write two fractions and two decimals that represent the same amount. • Develop strategies to write decimals as equivalent fractions.
		<p>Depth Of Knowledge: 1,2</p>
		<p>Bloom's Taxonomy: Apply</p>
		<p>Depth Of Knowledge: 1,2</p>

		Bloom's Taxonomy: Understand, Apply
<p>Standard Text</p> <p>4.NF.C.7 <i>Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP3: Students can construct viable arguments and critique the reasoning of others by comparing the size of two decimals and justify the reason for the larger decimal.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Reason about the size of two decimals to the hundredths place. Use symbols (>, <, or =) when comparing decimals. <p>Depth Of Knowledge: 1,2</p> <p>Bloom's Taxonomy: Understand/ Apply</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to exploring fractions on a number line in third grade. Connect to explaining equivalence and generating equivalent fractions (3.NF.3) Connect to understanding that that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (2.NBT.1) Connect to comparing two fractions with the same numerator or the same denominator by reasoning about their size(3.NF.3d) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to adding and subtracting fractions with like denominators (4.NF.3) Connect to solving measurement word problems involving decimals (4.MD.2) Connect to reading, writing, and comparing multi-digit whole numbers (4.NBT.2) Connect to comparing fractions with different numerators and denominators(4.NF.2) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to recognizing that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left (5.NBT.1)
<p>Clarification Statement:</p> <p>4.NF.C5: This standard explores the relationship between fractions and decimals. Students use previous learning of equivalent fractions to apply to denominators of 10 and 100. This includes finding equivalence, adding and subtracting tenths and hundredths using models and explanations.</p> <p>4.NF.C6: Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal. Decimals and fractions both represent parts of a whole</p> <p>4.NF.C7: This standard requires students to compare decimals to the hundredths when those two decimals refer to the same whole. Students compare using the symbols <, >, = and justify their response by creating a visual model. When comparing decimals, students should use models (such as hundredths grids) and number lines. When locating decimals on a number line the smaller numbers are farther to the left and the greater number is farther to the right. Students need to understand that some decimals are equivalent. Sharing examples with models to show that .4 = .40 will help students see the equivalency. Decimal numbers are rational numbers and</p>		

so we can use them to indicate quantities that are less than one or between any two whole numbers. In between any two decimal numbers, there is always another decimal number.

Common Misconceptions

- Some students might think the longer the decimal, the greater the value, so 2.146 would be greater than 2.4. The shorter the decimal, the greater the value, so 6.31 would be greater than 6.482.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying understanding decimal notation for fractions, and comparing decimal fractions because teachers will need to review equivalent fractions and comparing fractions from 3rd and 4th grade standards. Students can also use base ten blocks to represent decimals. Previous work with base ten blocks will depend on student experience.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.NF.A.3: This standard provides a foundation for work with understanding decimal notation for fractions, and comparing decimal fractions because this third grade standard is the first traditional work with equivalent fractions for students. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding decimal notation for fractions, and compare decimal fractions benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because this is a difficult concept, but when relating it to their lives through money will help students connect to the concept .

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding decimal notation for fractions, and compare decimal fractions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because students need to understand the idea of decimals including the size. This can be done through using examples such as money, where pennies are hundredths and dimes are tenths.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)?*

- For example, learners engaging with understanding decimal notation for fractions, and compare decimal fractions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation because representation of decimals can be helpful for students to understand. This can be done through money or visuals models such as hundred grids. Multiple representations help students discover a strategy that helps them understand the mathematics of decimals.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding decimal notation for fractions, and compare decimal fractions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives because like fractions, students need to have a conceptual understanding of decimals. Using physical manipulatives (such as base ten blocks) helps students visualize decimals. Moving to a visual model (such as hundred grid) will help with building student understanding.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with understanding decimal notation for fractions, and compare decimal fractions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of “natural” aptitude (e.g., “how can I improve on the areas I am struggling in?” rather than “I am not good at math”) because expressing fractions as decimals and/or decimals as fractions and comparing them can pose a challenge for many students. These skills are essential. Stopping and reflecting on where the mistakes lie and working to understand them and correct them will strengthen their understanding.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding decimal notation for fractions, and comparing decimal fractions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students will need mathematical ideas clarified. Students tend to generalize numbers. For example, they might think $1/100$ is greater than $1/10$ because 100 is greater than 10. Revisiting these ideas and making clarification of these concepts will help students.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding decimal notation for fractions, and comparing decimal fractions by addressing conceptual understanding because students who struggle with fractions need support with concrete models. Using base ten blocks, place value

charts, or hundreds blocks that students can color in to show the fractions help students build visual pictures and strengthen conceptual understanding.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying understanding decimal notation for fractions, and comparing decimal fractions because students who have extensive practice with fractional decimals can quickly move from fraction to decimal form (and vice versa). This can extend to adding and subtracting fractional decimals within word or real world problems. This type of work would not extend students into different standards but build fluency for later work with decimals and fractions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Eliciting and Using Evidence of Student Thinking): Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying understanding decimal notations for fractions and compare decimal fractions eliciting and using student thinking is critical because <when working with expressing fractions with a denominator of 10 as a fraction with a denominator of 100 in order to add two fractions (or the opposite) turning decimals into fractions (or the opposite), and comparing decimals all require a solid understanding of part to whole relationships, renaming using equivalents based on place value, and overall place value understanding. When students habitually explain their thinking, they find successes and mistakes. These mistakes become learning opportunities as opposed to failures.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia Testlet for Grade 4 Numbers and Operations-Fractions

Standard: 4.NF.C.5

Learning Targets: I can add fractions with denominators 10 and 100. I can rewrite a fraction as an equivalent decimal.

Task:

1. Look at this addition problem.

$$\frac{5}{10} + \frac{6}{100}$$

- a. Write the sum of the two fractions.
- b. Rewrite the sum from part (a) as a decimal.

This type of assessment question requires students to add fractions with denominators of 10 and 100 finding equivalent denominator of 100. It also ask students to convert answer of fraction into decimal notation. This task will give the teacher different indicators for misconceptions that students have involving fraction, decimals, and the equivalence between the 10ths and 100ths.

Relevance to families and communities:

During a unit focused on understanding decimal notations for fractions and compare decimal fractions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students learn about the ways decimals are used in the real world, both at home and in the community. Students connect to the Imperial and Metric systems of measurement to make global connections. Students could examine ways decimals are used in various occupations which could connect to families and/or the world in general.

Cross-Curricular Connections:

STEM Connection: Students can create a measuring cup for a science experiment using the metric system. As they do, have students pay attention to precision and remind them the parts must be equal. The act of partitioning reinforces an understanding of the relationship between the decimal fraction and the whole.

4.MD: MEASUREMENT & DATA

Cluster Statement: A: Solve problems involving measurement and conversion of measurements.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.MD.A.1: Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students can make sense of problems and persevere in solving them by solving problems involving measurement and the conversion of measurements from a larger unit to a smaller unit.</p> <p>SMP 2: Students can reason abstractly and quantitatively by visually seeing and thinking about benchmark (or landmark) measurements and associating them with approximate lengths (things they already know).</p> <p>SMP 6: Students can attend to precision by using measurement vocabulary for both metric and standard measurements and their associated measurement abbreviations.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Recognize the relationship between kilometers, meters, and centimeters. • Recognize the relationship between yards, feet, and inches. • Recognize the relationship between pounds and ounces. • Recognize the relationship between hours, minutes, and seconds. • Recognize the relationship between liters and milliliters. • Express measurements in a large unit in terms of a smaller unit by recording measurements equivalent in a two-column table
<p>Standard Text</p> <p>4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students can make sense of problems and persevere in solving them by interpreting, analyzing, and solving word problems involving elapsed time, liquid volume, mass and money.</p> <p>SMP 4: Students can model with mathematics by representing the measurement quantities using diagrams that feature a measurement scale, such as number line diagrams.</p>	<p>Depth Of Knowledge: 1</p> <p>Bloom's Taxonomy: Remember and Understand</p> <p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Solve word problems involving elapsed time, liquid volume, mass and money involving the operations of addition, subtraction, multiplication and division and including whole numbers, fractions and decimals within the 4th Grade Standards • Represent a word problem involving elapsed time, liquid volume, mass or money using a diagram that features a measurement scale, such as number line diagrams. • Express measurements in a large unit in terms of a smaller unit by

	<p>SMP 6: Students can attend to precision by paying attention to the units given in the problem and including units in their answer.</p>	<p>recording measurements equivalent in a two-column table</p> <p>Depth Of Knowledge: 1-2</p> <p>Bloom’s Taxonomy: Understand, Apply and Analyze</p>
<p>Standard Text</p> <p>4.MD.A.3: Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students can make sense of problems and persevere in solving them by interpreting, analyzing, and solving word problems involving area and perimeter.</p> <p>SMP 7: Students can look for and make use of structure by noticing that perimeter is always measured in linear units and area is always measured in square units.</p> <p>SMP 8: Students look for and express regularity in repeated reasoning by seeing the area and perimeter formulas as summaries of all calculations to find the area or perimeter of a rectangle.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Apply the area formula to real-world and mathematics problems. • Apply the perimeter formula to real-world and mathematical problems. • Find an unknown length or width in an area problem by recognizing the area formula as a multiplication equation with an unknown factor • Find an unknown length or width in a perimeter problem by recognizing the perimeter formula as an addition equation with an unknown factor <p>Depth Of Knowledge: 1-2</p> <p>Bloom’s Taxonomy: Understand, Apply and Analyze</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> • Connect to measuring lengths with halve and fourths of an inch. (3.MD.4) • Connect to estimating, measuring, adding, and subtracting lengths using inches, feet, yards, centimeters, and meters. (2.MD.1-6) • Connect to measuring and estimating masses of objects using grams and kilograms and liquid volumes using milliliters and liters. (3.MD.2) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> • Connect to interpreting a multiplication equation as a comparison. (4.OA.1) • Connect to multiplying to solve word problems involving multiplicative comparisons. (4.OA.2) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> • Connect to using unit conversions in solving multi-step, real world problems. (5.MD.1) • Connect to using ratio reasoning to convert measurement units; connect to manipulating and transforming units appropriately when multiplying or dividing quantities. (6.RP.3.d)

<ul style="list-style-type: none"> • Connect to measuring and estimating; and to adding, subtracting, multiplying, or dividing to solve one-step word problems given the same units. (3.MD.2) • Connect to telling and writing time to the nearest minute. Connect to adding and subtracting time intervals in minutes using number line diagrams. (3.MD.1) 		
<p>Clarification Statement:</p> <ul style="list-style-type: none"> • 4.MD.A.1: Relating units within the metric system is another opportunity to think about place value. For example, students need to be able to create a table that shows measurements of the same lengths in centimeters and meters. • 4.MD.A.2: Students combine competencies from different domains as they solve measurement problems using all four arithmetic operations: addition, subtraction, multiplication, and division. For example, “How many liters of juice does the class need to have at least 35 cups if each cup takes 225 ml?” Students may use tape or number line diagrams for solving such problems (MP1). • 4.MD.A.3L Such abstraction and use of formulas underscores the importance of distinguishing between area and perimeter in Grade 3 (3.MD.8 3) and maintaining the distinction in Grade 4 and later grades, where rectangle perimeter and area problems may get more complex and problem solving can benefit from knowing or being able to rapidly remind oneself of how to find an area or perimeter. By repeatedly reasoning about how to calculate areas and perimeters of rectangles, students can come to see area and perimeter formulas as summaries of all such calculations (MP8). 		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> • Students may believe that larger units will give larger measures (as opposed to the larger the unit, the smaller the number you get when you measure). • Students may confuse area and perimeter. 		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach</p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> • For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying solving problems involving measurement and conversion of measurements because students need to understand the basic foundations of content being covered in this standard. This includes measurement names, abbreviations, and prior work in word problems. Eliciting prior learning will help in understanding what students already know and what needs to be covered more thoroughly. <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p> <ul style="list-style-type: none"> • 3.MD.A.2: This standard provides a foundation for work with solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit because it is essential for students to understand and be able to solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. If students have unfinished learning within this standard, based on assessment data, 		

consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with solving problems involving measurement and conversion of measurements will know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs. Also represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. Students will be able to find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. Students benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as visuals that will enhance the understanding of solving problems involving measurement because students can see and make connections and conversion of measurement. Students may also recognize the underlying mathematical relationships in representations quickly but may need support perceiving them in a different representation.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with measurement and conversions of measurement benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because analyzing proportional relationships is difficult and critical for all further development of mathematics.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with measurement in the specific areas of length, distance, intervals of time, volume, mass, money, area, and perimeter benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and applying learning to new contexts such as anchoring instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) because students will be able to apply knowledge to real world experiences to display understanding of content.

Re-teach

Re-teach (targeted): What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?

- For example, students may benefit from re-engaging with content during a unit on solving problems involving measurement and conversion of measurement from a larger unit to a smaller unit, by revisiting student thinking through a short mini-lesson because it is essential for students to be able to explain their thinking and reasoning behind it to portray understanding and learning of content. Revisiting student thinking will also allow the teacher to clear up misconceptions or student thinking that is not based in measurement.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit to solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit by addressing conceptual understanding because first students need to understand what they are doing and the why so sense making of the intended content is developed with an understanding and building on their knowledge. Students will need explicit work with concrete materials to build conceptual understand. For example, using centimeter blocks to see that 100 centimeters equals one meter.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit because this will allow students to make connections to prior learning and build on their understanding and knowledge of measurement and data.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Posing Purposeful Questions): CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying solving problems involving measurement and conversion of measurements, the pattern of questions within the classroom is critical because it is important to include every student in conversation to elicit their understanding and prior knowledge of content. Allowing students to collaborate will promote a culture of productive talk and allow students to express their thinking which is relevant to their life experiences. Students have the opportunity to take ownership of their learning and sense making of intended content. Through discussion, multiple strategies can evolve for students to utilize.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/MD/A/3/tasks/876>

Standards: 4.MD.A.3 4.OA.A.3

Task: Karl's Garden

Karl's rectangular garden is 20 feet by 45 feet, Makenna's is 25 feet by 40 feet. Whose garden is larger in area?

This type of assessment question requires students to determine area using a strategy that works for them. They will use SMP 6 to communicate who has a larger garden using vocabulary from cluster and problem. This task could be an end of unit check or can be used within unit to check for understanding. This task could vary depending on the classroom, for example using groups and having groups share out rather than individually.

Relevance to families and communities:

During a unit focused on solving problems involving measurement and conversion of measurements, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example finding measurement of family garden, backyard or any space that is accessible from home so that there is relevance in finding measurement (finding the area or perimeter of the garden).

Cross-Curricular Connections:

Science: In fourth grade the NGSS recommends students work with measurement related to erosion. Consider providing a connection for students to determine the area of vegetation in a certain place.

Language Arts: Literature can offer connections about area and perimeter such as: *Spaghetti and Meatballs for All* by Marilyn Burns.

4.MD: MEASUREMENT & DATA

Cluster Statement: B: Represent and interpret data.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.MD.B.4: Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students can make sense of problems and persevere in solving them by interpreting and making sense of word problems involving information presented in line plots.</p> <p>SMP 2: Students can reason abstractly and quantitatively by attending to the meaning of the measured objects and plots on the number line by using addition and subtraction involving fractions.</p> <p>SMP 5: Students can use tools by measuring objects to the nearest $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ inch using a ruler.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Identify benchmark fractions. Make a line plot to display a data set of measurements in fractions of a unit. Solve problems involving information presented in line plots which use fractions of a unit by adding and subtracting fractions.
		<p>Depth of Knowledge: 1-2</p>
		<p>Bloom's Taxonomy: Remember and Apply</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to generating measurement data and making line plots using whole number units. (2.MD.9) Connect to generating measurement data by measuring lengths using rulers marked with halves and fourths of an inch and showing the data by making a line plot where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters. (3.MD.4) Connect to understanding line plots represent measurement data, not categorical data. (3.MD.3-4) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to using the four operations to solve word problems, including simple fractions and representing measurement quantities using diagrams. (4.MD.2) Connect to explaining why fractions are equivalent and generating equivalent fractions. (4.NF.1) Connect to adding and subtracting mixed numbers with like denominators. (4.NF.3c) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to making line plots with measurements to the half, quarter, and eighth of a unit and solving problems involving operations of fractions. (5.MD.2) Connect to solving real world problems involving the addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. (5.NF.2)

<p>Clarification Statement: 4.MD.B.4: Grade 4 students learn elements of fraction equivalence and arithmetic, including multiplying a fraction by a whole number and adding and subtracting fractions with like denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. For example, with reference to the line plot above, students might find the difference between the greatest and least values in the data. (In solving such problems, students may need to label the measurement scale in eighths so as to produce like denominators. Decimal data can also be used in this grade.)</p>		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> • Students may not understand that it is possible to graph with fractions. 		
<p>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</p> <p>Pre-Teach</p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> • For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying, representing and interpreting data because students will have created line plots representing whole, halves, and fourths. <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p> <ul style="list-style-type: none"> • 3.MD.B.4: This standard provides a foundation for work with representing and interpreting data because students will have previous work with measuring to halves and fourths then creating line plot with this information. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments. <p>Core Instruction</p> <p><i>Access</i> Interest: <i>How will the learning for students provide multiple options for recruiting student interest?</i></p> <ul style="list-style-type: none"> • For example, learners engaging with representing and interpreting data benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because students are more engaged when topics are about them. For example, data can be gathered on height, shoe size, number of siblings, etc. Whole class data charts can be created to serve as anchor charts for other tasks. <p><i>Build</i> Effort and Persistence: <i>How will the learning for students provide options for sustaining effort and persistence?</i></p> <ul style="list-style-type: none"> • For example, learners engaging with representing and interpreting data benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students need to understand the data to be able to interpret it. Giving students opportunities to work with peers or allow them for support gives students encouragement to persist longer in math work. <p>Language and Symbols: <i>How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or</i></p>		

puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with representing and interpreting data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because students will have to interpret data based on a table or create a table based on data therefore students might need to highlight certain pieces for understanding. For example, highlighting x and y labels to help understanding.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with representing and interpreting data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips because students need a way to organize data into verbal or written form for the data they are creating or interpreting. For example, ____ represents _____. might be a sentence starter/strip for interpreting a data chart. This gives students a place to start when speaking with a partner or writing about a data table.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with representing and interpreting data benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) because students need to understand the purpose for representing and interpreting data. For example, students can gather data and create tables. Students can generate questions to represent in a data table. These types of activities will help students understand data and how to represent it.

Re-teach

Re-teach (targeted): What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?

- For example, students may benefit from re-engaging with content during a unit on representing and interpreting data by providing specific feedback to students on their work through a short mini-lesson because students will need feedback for mistakes they are making with creating or interpreting line plots. Attending to precision with setting up line plots.

Re-teach (intensive): What assessment data will help identify content needing to be revisited for intensive interventions?

- For example, some students may benefit from intensive extra time during and after a unit representing and interpreting data by confronting student misconceptions because if students can set up line plots, it is important to look at the misconceptions that still have students. This will allow the teacher to isolate issues and work with the student in that area.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying representing and interpreting data because students can explore where these types of data tables would be used or generate them from given information.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Building Procedural Fluency from Conceptual Understanding): Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying representing and interpreting data the types of mathematical tasks are critical because this cluster deals with collection of data, creating line plots based on fractions, and interpreting data from a data table. These mathematical concepts need procedural and conceptual understanding. Students need to understand how to set up a number line that will represent their data, this includes fractional measurements. Students need understanding in fractions and fraction measurements to be able to properly step up number lines in creating the line plot. Conceptual understanding comes through concrete work with these mathematical concepts.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <http://tasks.illustrativemathematics.org/content-standards/4/MD/B/4/tasks/1039>

Standard: 4.MD.B4

Task: Button Diameters

- a. With a partner or group, gather a handful of round buttons from a diverse collection, and use a rule to measure the diameter of each button to the nearest eighth-inch.
- b. Make a line plot of buttons diameters, marking your scale in eighth-inch increments.
- c. What is the most common diameter in your collections? How does that compare with the collection from another group?
- d. Now measure the diameters of these same buttons to the nearest quarter-inch.
- e. Make a line plot of button diameters, marking your scale in quarter-inch increments.
- f. Describe the difference between the two line plots you created. Which one gives you more information? Which one is easier to read?

This type of assessment question requires students to use SMP 1 to reason about the task they are asked to do. This task allows students to relate information they gather to a word problem they may experience later. Using SMP5, students will use measurement to gather data to create their line plot. Students will have experience with mathematical tools for measurement, but also for creating a line plot that represents their data with two different increments of measurements and the use of measurement tool. This task will help with identifying misconceptions or reteaching about fraction ordering, measurement, plotting data on line plot, and interpretation of data on the line plot.

<p>Relevance to families and communities:</p> <p>During a unit focused on representing and interpreting data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can gather data at home, discuss data tables with parents, or discuss other topics that can be used to gather information for data tables.</p>	<p>Cross-Curricular Connections:</p> <p>Science: In fourth grade the NGSS recommends students “develop a model of waves to describe patterns in terms of amplitude and wavelength”. Consider providing a connection for students to determine the length of various waves that measure in fractional units. Then have students graph and analyze that data.</p> <p>Social Studies: In fourth grade the New Mexico Social Studies Standards state students should “understand how visual data (e.g., maps, graphs, diagrams, tables, charts) organizes and presents geographic information.” Consider having students gather, graph and analyze geographic data that contains measurements in fractions of a unit and can be displayed using a line plot. Consider providing opportunities to consider what type of data suits a line plot best.</p>
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4.MD: MEASUREMENT & DATA

Cluster Statement: C: Geometric measurement: understand concepts of angle and measure angles.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>4.MD.C.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <ul style="list-style-type: none"> 4.MD.C.5.A: An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. 4.MD.C.5.B: An angle that turns through n one-degree angles is said to have an angle measure of n degrees. 	<p>Standard for Mathematical Practices</p> <p>SMP 4: Students can model with mathematics by creating examples of acute, obtuse and right angles.</p> <p>SMP 6: Students can attend to precision by using specific vocabulary to describe the measurements of angles, including measure, point, end point, geometric shapes, ray, angle, circle, degree and protractor.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. Relate angle measurement to circles. Demonstrate acute, obtuse, and right angles. Use precise vocabulary when describing angles.
<p>Depth Of Knowledge: 1</p>		
<p>Bloom's Taxonomy: Remember and Understand</p>		
<p>Standard Text</p> <p>4.MD.C.6: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 4: Students can model with mathematics by drawing and creating angles of a given measurement.</p> <p>SMP 5: Students can use tools by measuring angles with different protractors, including traditional and circular protractors.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Recognize that angles are measured in degrees. Measure angles in whole-number degrees using a protractor. Make sketches of specified angle measures. Observe that the orientation of an angle does not affect its measure.

		Depth Of Knowledge: 2
<p>Standard Text</p> <p>4.MD.C.7: Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students can make sense of problems and persevere in solving them by interpreting, analyzing, and solving word problems involving measurement of angles.</p> <p>SMP 2: Students can reason abstractly and quantitatively by recognizing angle measurement as additive in relation to the reference to a circle.</p>	<p>Bloom’s Taxonomy: Understand and Apply</p> <p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Recognize angles as additive. • Solve addition and subtraction problems to involving unknown angles • Write an equation for a word problem involving angle measurement and represent the unknown number with a symbol.
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> • Connect to understanding that shapes in different categories may share attributes. (3.G.1) • Connect to recognizing and drawing shapes having specified attributes, such as a given number of angles. (2.G.1) • Connect with using addition and subtraction within 100 to solve one-and two-step word problems. (2.OA.1) • Connect to finding the unknown whole number using addition and subtraction. (1.OA.8) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> • Connect to drawing and identifying lines and angles. (4.G.1) • Connect to classifying two-dimensional figures based on lines and angles. (4.G.2) 	<p>Depth Of Knowledge: 1-2</p> <p>Bloom’s Taxonomy: Understand, Apply and Analyze</p> <p>Future Learning Connections</p> <ul style="list-style-type: none"> • Connect to constructing triangles from three measures of angles. (7.G.2) • Connect to using facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to writing and solving simple equations for an unknown angle in a figure. (7.G.5)
<p>Clarification Statement:</p> <p>4.MD.C.5: As with length, area, and volume, children need to understand equal partitioning and unit iteration to understand angle and turn measure. Whether defined as more statically as the measure of the figure formed by the intersection of two rays or as turning, having a given angle measure involves a relationship between components of plane figures and therefore is a property.</p>		

4.MD.C.6: If examples and tasks are not varied, students can develop incomplete and inaccurate notions. For example, some come to associate all **slanted lines** with 45 **degree** measures and **horizontal and vertical lines** with measures of 90 degrees. Others believe angles can be “read off” a **protractor** in “**standard**” **position**, that is, a **base** is horizontal, even if neither **arm of the angle** is horizontal. Measuring and then sketching many angles with no horizontal or vertical arms, perhaps initially using **circular 360 protractors**, can help students avoid such limited conceptions.

4.MD.C.7: Students with an accurate conception of angle can recognize that angle measure is **additive**. As with length, area, and volume, when an angle is **decomposed** into **non-overlapping parts**, the angle measure of the **whole** is the **sum** of the angle measures of the parts. Students can then solve interesting and challenging **addition** and **subtraction** problems to find the measurements of unknown angles on a **diagram** in real world and mathematical problems.

Common Misconceptions

- Students may think size of an angle’s arms or rays affects its measure
- Students may think you can only use a protractor when there is a horizontal line
- Students may think the direction of an angle matters.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying geometric measurement: understand concepts of angle and measure angles because students will need to know how to draw and label line segments.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.G.A.1: This standard provides a foundation for work with drawing and identifying lines and angles, and classifies shapes by properties of their lines and angles because students need to be able to draw and recognize angles. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because students have had prior work with angles and angle measurements. Students can associate angles with topics that are relevant in their lives to make connections.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for

visualizing desired outcomes because students will need scaffolds to help with work with angles. They may need step by steps or prompts for different angles and measuring angles.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students have prior experience with working with angles. This cluster has very domain specific vocabulary that students need to know, use, and be able to identify.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) because students will need conceptual understanding about angles. Manipulatives that help them move angles and change them will help with building understanding. Practice with using different protractors including digital is beneficial.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding concepts of angles and measure angles benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using multiple examples and non-examples to emphasize critical features because students will need work with different examples and non examples. This way students have a variety of practice with different angles and can identify if a figure is not angle or does not have angle.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on geometric measurement: understand concepts of angle and measure angles by clarifying mathematical ideas and/or concepts through a short mini-lesson because students need to be able to recognize angles and recognize the degrees each angle represents.

Re-teach (intensive) *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understand concepts of angle and measure angles by addressing conceptual

understanding because students who continue to struggle will need support with building conceptual understanding. This might include work with manipulatives or real-world examples.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying understand concepts of angle and measure angles because this cluster can be cross curricular and have ties to real world examples. Students can explore these links.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Goal Setting): Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying geometric measurement: understand concepts of angle and measure angles, goal setting is critical because it is recognizing important concepts that allows for the teacher to create a clear plan. Teachers are encouraged to think critically about students' culture background, strengths and weaknesses. Teacher is then looking at the students individual needs to plan accordingly so that students reach their end goal.

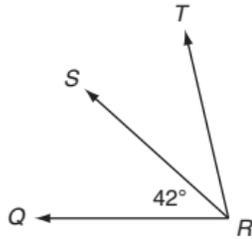
Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia Testlet for Grade 4- Measurement and Data

Standard: 4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. 4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

Learning Target: I can add or subtract to find the unknown angle on a diagram.

1. Ray RS divides angle QRT into two smaller angles, as shown.



The measure of angle QRT is 77 degrees.

- a. What is the measure of angle SRT ? Show your work or explain your reasoning.
Ray RW is drawn to create angle TRW . QRW is a straight line.
- b. What is the measure of angle TRW ? Show your work or explain how you know.

This type of assessment question requires students to use the information given to them. They need to know the measurement of the whole angle before beginning work. This task encompasses SMP 6 due to the vocabulary content within the problem. Teacher determine different reteach aspects from this task, for example a student might be adding when they are supposed to be subtracting or ask you what a "ray" is. These such aspects are good to note for reteaching.

Relevance to families and communities:

During a unit focused on geometric measurement: understand concepts of area and relate area to multiplication and to addition, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example utilizing vocabulary cards with visuals to present the geometric shape to explain the concept of area. Students can then understand the area as how much two-dimensional space a shape takes up and then practice by using addition to counting individual square units or by multiplying the length and width of the shape. Students can use repeated addition to support their multiplication skills.

Cross-Curricular Connections:

Social Studies: In fourth grade the New Mexico Social Studies Standards state students should "explain how the Earth-Sun relationships produce day and night". Consider providing a connection to the angle of Earth's axis in relation to the Sun.

Language Arts: Literature can offer connections about angles such as: *What's Your Angle, Pythagoras?* by Julie Ellis or *The Adventures of the Angles* by Kristie Carpenter.

4.G: GEOMETRY

Cluster Statement: A: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
<p>4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p>	<p>SMP3: Students can construct viable arguments and critique the reasoning of others by explaining the differences between points, lines, line segments, rays, angles, and perpendicular and parallel lines.</p> <p>SMP6: Students can use appropriate tools strategically when drawing points, lines, line segments, rays and angles.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Draw/identify points, lines, line segments • Draw/identify rays • Draw/identify angles (right, acute, obtuse) • Draw/identify perpendicular lines • Draw/identify parallel lines
		<p>Webb’s Depth Of Knowledge: 1,2</p>
		<p>Bloom’s Taxonomy: Understand and Apply</p>
<p>4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</p>	<p>SMP 3: Students can construct viable arguments and critique the reasoning of others by sorting and classifying 2D figures, including types of triangles, and explain their thinking using Tier 3 vocabulary</p> <p>SMP 5: Students can use appropriate tools strategically when they select tools to verify their thinking when sorting 2D figures.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Classify 2D figures with parallel lines • Classify 2D figures with perpendicular lines • Recognize 2D figures based on angle size • Recognize and identify right triangles
		<p>Webb’s Depth of Knowledge: 1, 2</p>
		<p>Bloom’s Taxonomy: Understand</p>

<p>Standard Text</p> <p>4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>	<p>Standard for Mathematical Practices</p> <p>SMP3: Students can construct viable arguments and critique the reasoning of others by explaining why or why not figures have line symmetry</p> <p>SMP 6: Students can attend to precision by ensuring figures are folded EXACTLY in half</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Recognize line symmetry in 2D figures as a folded line creating two matching parts Identify 2D figures with line symmetry Draw lines of symmetry in 2D figures Identify that 2D shapes can consist of more than one line of symmetry <p>Webb’s Depth of Knowledge: 1,2</p> <p>Bloom’s Taxonomy: Understand</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to recognizing attributes of quadrilaterals, including parallel lines and right angles. (3.G.1) Connect to identifying and distinguishing between attributes and non-attributes of trapezoids, squares, rectangles, circles, hexagons, rhombuses and parallelograms and had to build and draw shapes that possess these attributes. (2.G.1) Connect to partitioning shapes into halves. (2.G.3, 2.G.3) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to recognizing angles as geometric figures that form wherever two rays share a common endpoint, and understand concepts of angle measurement. (4.MD.5) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to understanding attributes belong to a category of two-dimensional figures belong to subcategories of that category. (5.G.3) Connect to classifying two-dimensional figures based on properties. (5.G.4) Connect to understanding reflection, rotation, and translation. (8.G.2)
<p>Clarification Statement:</p> <ul style="list-style-type: none"> 4.G.A.1: Points, lines, segments, rays, and angles are the building blocks of the geometry. Point and line are undefined terms because they do not have definitions. We can understand these terms by thinking of examples of what a point and line might look like. A point can be a tip of a pencil; it has position but no dimension. Euclid described a line by saying that through any two points there is always a line and every line contains at least two points. Line segment is part of a line and it contains two endpoints meaning it has a beginning and endpoints. A line contains an infinite number of points and has no endpoints and goes on and on forever. A ray is part of a line that has one endpoint and extends forever in only one direction. Parallel lines are lines that never cross and are the same distance apart. Perpendicular lines intersect to form right angles. Essential vocabulary for this standard includes: point, line, line segment, ray, parallel lines, perpendicular lines, intersecting lines, and endpoint. 4.G.A.2: This standard requires students to describe parallel and perpendicular lines. Students need to classify 2D figures based on parallel and/or perpendicular line segments as well as classify them by their angles. Students need to be able to classify triangles by their angles and by their side lengths. Essential vocabulary for this standard includes: parallel, perpendicular, acute, obtuse, right, right 		

triangle, isosceles, scalene, equilateral, and equiangular.

- 4.G.A.3: This standard requires students to recognize a **line of symmetry**. A line of symmetry divides a figure into two **congruent** mirrored parts. A figure may have multiple lines of symmetry. The folded line is called a line of symmetry. A figure is symmetrical if it has a line of symmetry. This standard also requires students to identify figures with line **symmetry**. Students are required to draw lines of symmetry within figures. Essential vocabulary for this standard includes **symmetrical**, symmetry, and line of symmetry.

Common Misconceptions

- Students may confuse lines, line segments, and rays.
- Students may confuse acute and obtuse angles.
- Students may confuse perpendicular and parallel lines.
- Students may confuse the types of triangles.
- Students may confuse matching parts that are created by halving then rotating a part of the 2D figure. Halves must fold over to match. No other movement is allowed to create a of line symmetry.
- Students may think figures can only have one line of symmetry; some figures have more than one line of symmetry. Some figures do not have any lines of symmetry.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles because this is the first time that students are exposed to rays, angles, and perpendicular and parallel lines, and these are the building blocks of geometry, so a strong foundational understanding is crucial. This cluster has vocabulary that is cluster specific. Many of these vocabulary words are new, but some are reviewed from previous grade levels. Students need practice in using and interacting with mathematical language and embedding vocabulary for this cluster.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.G.A.1: This standard provides a foundation for work with drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles because this is where students are introduced to recognizing that different figures have different attributes, including angles and lines. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with draw and identify lines and angles, and classify shapes by properties of their lines and angles benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as real words and pictorial representation that students can move

around and manipulate because this standard asks students to work with two dimensional representations that can seem very abstract to students. When they can move and manipulate two dimensional shapes, they can begin to think in a more abstract way.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students develop and practice explicit awareness of and vocabulary for many concepts they have been developing, including points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Vocabulary should be used/modeled by teacher as well.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as providing graphic symbols with alternative text descriptions because it is important to construct examples of these concepts, such as drawing angles and triangles that are acute, obtuse, and right to help students form richer concept images connected to verbal definitions and to have a more complete and accurate mental images and associated vocabulary for geometric ideas. It is also important to develop nonexamples to associate with this cluster vocabulary.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with drawing and identifying lines and angles, and classifying shapes by properties of their lines and angle benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) because students will learn to apply these concepts in varied contexts. For example, they learn to use a digital protractor to measure digital angles. Practice with different types of protractors is beneficial for students.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with drawing and identifying lines and angles, and classifying shapes by properties of their lines and angle benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as making explicit cross-curricular connections (e.g., teaching literacy strategies in the social studies classroom) because angles are an abstract concept for students, so linking it to something relevant and connecting the concept to real world

applications will help students apply their skills into usable knowledge (e.g., using angles to engineer/design something, athletes using angles to increase performance).

Re-teach

Re-teach (targeted): What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?

- For example, students may benefit from re-engaging with content during a unit on drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles by clarifying mathematical ideas and/or concepts through a short mini-lesson because students develop explicit awareness of and vocabulary for many concepts they have been developing, including points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines, and clarifying these terms will help students form richer concept images connected to verbal definitions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles by confronting student misconceptions because general misunderstandings regarding specific attributes exist when student are not exposed to multiple opportunities to see shapes visually.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles because it is important for students to link such an abstract concept into concrete situations. Students can use their knowledge of shapes and attributes to design a room/building/city, use their knowledge of lines and engineer a road system for a town, etc.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Using and Connecting Mathematical Representations): The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence"¹. By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles the use of mathematical representations within the classroom is critical because students can draw on their own knowledge based on cognates, and can

¹ Boston et al., 2017, pp. 122

express their knowledge, questions, and reasoning using multiple representations. For example, a student might make the connection between parallel and paralela, and be able to represent parallel lines with the symbol \parallel .

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Illustrative Mathematics

Standard: 4.G.A.1

Task: The Geometry of Letters

The Geometry of Letters



Letters can be thought of as geometric figures.

A B C D E F G
H I J K L M N
O P Q R S T U
V W X Y Z

a.

How many line segments are needed to make the letter A? How many angles are there? Are they acute, obtuse, or right angles? Are any of the line segments perpendicular? Are any of the line segments parallel?

b. We can build all of these letters from line segments and arcs of circles. Build all of the capital letters with the smallest number of "pieces," where each piece is either a line segment or an arc of a circle.

c. Which letters have perpendicular line segments?

d. Which letters have parallel line segments?

e. Which letters have no line segments?

f. Do any letters contain both parallel and perpendicular lines?

g. What makes the lower case letters "i" and "j" different than all of the capital letters?

This type of assessment question requires students to work with correct geometric terminology for this standard. This standard asks the student to categorize letters dependent on their attributes, which is seen in this cluster with 2 dimensional shapes. This task can be used in within lessons for this standard or at the end of the unit. This task will allow teacher to determine vocabulary or misconceptions that will need reteach. This task can also be extended to other standards within this cluster, for example, finding and drawing lines of symmetry for letters.

Relevance to families and communities:

During a unit focused on drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles, consider options for learning from your families and communities. The cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways we use lines, angles and shapes within different careers (construction, farming, engineering). This could also be extended to shapes found in different cultural aspects (pottery, ceremonial dress, etc.)

Cross-Curricular Connections:

Science: In fourth grade the NGSS recommends students will study waves and their application in technology for transfer. Students will identify rays and angles in drawings of wave propagation. The NGSS also recommends students will recognize symmetry, or lack of symmetry, in the internal and external structures of plants and animals.

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Student Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction⁹

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the [Universal Design Learning \(UDL\) Framework](#)

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the [Learning Goal](#), and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
<p>ENGAGEMENT</p> <p><input type="checkbox"/> How will you provide multiple options for recruiting interest?</p>	<p>Recruiting Student Interest:</p> <p><input type="checkbox"/> What do you anticipate in the range of student interest for this lesson?</p> <p><input type="checkbox"/> Plan for options for recruiting student interest:</p> <ul style="list-style-type: none"> <input type="checkbox"/> provide choice (e.g. sequence or timing of task completion) <input type="checkbox"/> set personal academic goals <input type="checkbox"/> provide contextualized examples connected to their lives <input type="checkbox"/> support culturally relevant connections (i.e home culture) <input type="checkbox"/> create socially relevant tasks <input type="checkbox"/> provide novel & relevant problems to make sense of complex ideas in creative ways

⁹ Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from <http://udlguidelines.cast.org>

	<ul style="list-style-type: none"> <input type="checkbox"/> provide time for self-reflection about content & activities <input type="checkbox"/> create accepting and supportive classroom climate <input type="checkbox"/> utilize instructional routines to involve all students
<p>REPRESENTATION</p> <p><input type="checkbox"/> How will you reduce barriers to perceiving the information presented in this lesson?</p>	<p>Perception:</p> <p><input type="checkbox"/> What do you anticipate about the range in how students will perceive information presented in this lesson?</p> <ul style="list-style-type: none"> <input type="checkbox"/> Plan for different modalities and formats to reduce barriers to learning: <ul style="list-style-type: none"> <input type="checkbox"/> display information in a flexible format to vary perceptual features <input type="checkbox"/> offer alternatives for auditory information <input type="checkbox"/> offer alternatives for visual information
<p>ACTION & EXPRESSION</p> <p><input type="checkbox"/> How will the learning for students provide a variety of methods for navigation to support access?</p>	<p>Physical Action:</p> <p><input type="checkbox"/> What do you anticipate about the range in how students will physically navigate and respond to the learning experience?</p> <ul style="list-style-type: none"> <input type="checkbox"/> Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: <ul style="list-style-type: none"> <input type="checkbox"/> requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies <input type="checkbox"/> physically indicating selections <input type="checkbox"/> interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding	
<p>ENGAGEMENT</p> <p><input type="checkbox"/> How will the learning for students provide options for sustaining effort and persistence?</p>	<p>Sustaining Effort & Persistence:</p> <p><input type="checkbox"/> What do you anticipate about the range in student effort?</p> <ul style="list-style-type: none"> <input type="checkbox"/> Plan multiple methods for attending to student attention and affect by: <ul style="list-style-type: none"> <input type="checkbox"/> prompting learners to explicitly formulate or restate learning goals <input type="checkbox"/> displaying the learning goals in multiple ways <input type="checkbox"/> using prompts or scaffolds for visualizing desired outcomes <input type="checkbox"/> engaging assessment discussions of what constitutes excellence <input type="checkbox"/> generating relevant examples with students that connect to their cultural background and interests <input type="checkbox"/> providing alternatives in the math representations and scaffolds <input type="checkbox"/> creating cooperative groups with clear goals, roles, responsibilities <input type="checkbox"/> providing prompts to guide when and how to ask for help <input type="checkbox"/> supporting opportunities for peer interactions and supports (e.g. peer tutors) <input type="checkbox"/> constructing communities of learners engaged in common interests <input type="checkbox"/> creating expectations for group work (e.g., rubrics, norms, etc.) <input type="checkbox"/> providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge <input type="checkbox"/> providing feedback that: <ul style="list-style-type: none"> <input type="checkbox"/> emphasizes effort, improvement, and achieving a standard rather than on relative performance <input type="checkbox"/> is frequent, timely, and specific <input type="checkbox"/> is informative rather than comparative or competitive

	<ul style="list-style-type: none"> <input type="checkbox"/> models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
<p>REPRESENTATION</p> <p>[?] How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?</p>	<p>Language & Symbols:</p> <p>[?] What do you anticipate about the range of student background experience and vocabulary?</p> <ul style="list-style-type: none"> <input type="checkbox"/> Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: <ul style="list-style-type: none"> <input type="checkbox"/> pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge <input type="checkbox"/> graphic symbols with alternative text descriptions <input type="checkbox"/> highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure <input type="checkbox"/> embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) <input type="checkbox"/> embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) <input type="checkbox"/> highlighting structural relations or make them more explicit <input type="checkbox"/> making connections to previously learned structures <input type="checkbox"/> making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) <input type="checkbox"/> allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) <input type="checkbox"/> allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) <input type="checkbox"/> clarification of notation through lists of key terms <input type="checkbox"/> making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf <input type="checkbox"/> linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages <input type="checkbox"/> defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms <input type="checkbox"/> electronic translation tools or links to multilingual web glossaries <input type="checkbox"/> embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) <input type="checkbox"/> presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) <input type="checkbox"/> making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
<p>ACTION & EXPRESSION</p> <p>[?] How will the learning provide multiple</p>	<p>Expression & Communication:</p> <p>[?] What do you anticipate about the range in how students will express their thinking in the learning environment?</p> <ul style="list-style-type: none"> <input type="checkbox"/> Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

<p>modalities for students to easily express knowledge, ideas, and concepts in the learning environment?</p>	<ul style="list-style-type: none"> <input type="checkbox"/> options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video <input type="checkbox"/> use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) <input type="checkbox"/> flexibility in using a variety of problem solving strategies <input type="checkbox"/> spell or grammar checkers, word prediction software <input type="checkbox"/> text-to-speech software, human dictation, recording <input type="checkbox"/> calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper <input type="checkbox"/> sentence starters or sentence strips <input type="checkbox"/> concept mapping tools <input type="checkbox"/> Computer-Aided-Design (CAD) or mathematical notation software <input type="checkbox"/> virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) <input type="checkbox"/> multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) <input type="checkbox"/> multiple examples of novel solutions to authentic problems <input type="checkbox"/> different approaches to motivate, guide, feedback or inform students of progress towards fluency <input type="checkbox"/> scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) <input type="checkbox"/> differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)
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<h2 style="text-align: center;">Optimizing INTERNALIZATION of the Learning Goal</h2>	
<p>ENGAGEMENT</p> <p><input type="checkbox"/> How will the design of the learning strategically support students to effectively cope and engage with the environment?</p>	<p>Self-Regulation:</p> <p><input type="checkbox"/> What do you anticipate about barriers to student engagement?</p> <p><input type="checkbox"/> Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals:</p> <ul style="list-style-type: none"> <input type="checkbox"/> metacognitive approaches to frustration when doing mathematics <input type="checkbox"/> increase length of on-task orientation through distractions <input type="checkbox"/> frequent self-reflection and self-reinforcements <input type="checkbox"/> address subject specific phobias and judgments of “natural” aptitude (e.g., “how can I improve on the areas I am struggling in?” rather than “I am not good at math”) <input type="checkbox"/> offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving <input type="checkbox"/> use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
<p>REPRESENTATION</p> <p><input type="checkbox"/> How will the learning support transforming accessible information into usable knowledge</p>	<p>Comprehension:</p> <p><input type="checkbox"/> What do you anticipate about barriers to student comprehension?</p> <p><input type="checkbox"/> Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,</p>

<p>that is accessible for future learning and decision-making?</p>	<p>providing a process for meaning making of new learning, and applying learning to new contexts:</p> <ul style="list-style-type: none"> <input type="checkbox"/> incorporate explicit opportunities for review and practice <input type="checkbox"/> note-taking templates, graphic organizers, concept maps <input type="checkbox"/> scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) <input type="checkbox"/> explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) <input type="checkbox"/> opportunities over time to revisit key ideas and connections <input type="checkbox"/> make explicit cross-curricular connections <input type="checkbox"/> highlight key elements in tasks, graphics, diagrams, formulas <input type="checkbox"/> outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships <input type="checkbox"/> multiple examples & non-examples <input type="checkbox"/> cues and prompts to draw attention to critical features <input type="checkbox"/> highlight previously learned skills that can be used to solve unfamiliar problems <input type="checkbox"/> options for organizing and possible approaches (tables and representations for processing mathematical operations) <input type="checkbox"/> interactive representations that guide exploration and new understandings <input type="checkbox"/> introduce graduated scaffolds that support information processing strategies <input type="checkbox"/> tasks with multiple entry points and optional pathways <input type="checkbox"/> “Chunk” information into smaller elements <input type="checkbox"/> remove unnecessary distractions unless essential to learning goal <input type="checkbox"/> anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) <input type="checkbox"/> pre-teach critical prerequisite concepts via demonstration or representations <input type="checkbox"/> embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) <input type="checkbox"/> advanced organizers (e.g., KWL methods, concept maps) <input type="checkbox"/> bridge concepts with relevant analogies and metaphors
<p>ACCESS ACTION & EXPRESSION</p> <p><input type="checkbox"/> How will the learning for students support the development of executive functions to allow them to take advantage of their environment?</p>	<p>Executive Functions:</p> <p><input type="checkbox"/> What do you anticipate about barriers to students demonstrating what they know?</p> <p><input type="checkbox"/> Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it:</p> <ul style="list-style-type: none"> <input type="checkbox"/> prompts and scaffolds to estimate effort, resources, difficulty <input type="checkbox"/> models and examples of process and product of goal-setting <input type="checkbox"/> guides and checklists for scaffolding goal-setting <input type="checkbox"/> post goals, objectives, and schedules in an obvious place <input type="checkbox"/> embed prompts to “show and explain your work” <input type="checkbox"/> checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps <input type="checkbox"/> embed coaches/mentors to demonstrate think-alouds of process <input type="checkbox"/> guides to break long-term goals into short-term objectives <input type="checkbox"/> graphic organizers/templates for organizing information & data <input type="checkbox"/> embed prompts for categorizing and systematizing <input type="checkbox"/> checklists and guides for note-taking <input type="checkbox"/> asking questions to guide self-monitoring and reflection <input type="checkbox"/> showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	<ul style="list-style-type: none"> <input type="checkbox"/> prompt learners to identify type of feedback or advice they seek <input type="checkbox"/> templates to guide self-reflection on quality & completeness <input type="checkbox"/> differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) <input type="checkbox"/> assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples
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Planning Guidance for Culturally and Linguistically Responsive Instruction¹⁰

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues¹¹ define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices¹²

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

¹⁰ This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, <https://www.culturallyresponsive.org/vabb>)

¹¹ Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

¹² Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.¹³

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

¹³ Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary¹⁴

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $3/4$ and $-3/4$ are additive inverses of one another because $3/4 + (-3/4) = (-3/4) + 3/4 = 0$.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.¹⁵

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

¹⁴ Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from <http://www.corestandards.org/Math/Content/mathematics-glossary/>

¹⁵ Adapted from Wisconsin Department of Public Instruction, <http://dpi.wi.gov/standards/mathglos.html>, accessed March 2, 2010.

First quartile. For a data set with median M , the first quartile is the median of the data values less than M . Example: For the data set $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the first quartile is 6.¹⁶ See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or $-a$ for some whole number a .

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the interquartile range is $15 - 6 = 9$. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.

Also known as a dot plot.¹⁷

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.¹⁸ Example: For the data set $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set $\{2, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set $\{2, 3, 6, 7, 10, 12, 14, 15, 22, 90\}$, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $3/4$ and $4/3$ are multiplicative inverses of one another because $3/4 \cdot 4/3 = 4/3 \cdot 3/4 = 1$.

¹⁶ Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

¹⁷ Adapted from Wisconsin Department of Public Instruction, op. cit.

¹⁸ To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $5/50 = 10\%$ per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b . The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.¹⁹

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

¹⁹ Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.¹

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN²
PUT TOGETHER / TAKE APART³	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5 = 0 + 5$, $5 + 0 = 5$, $5 = 1 + 4$, $5 = 4 + 1$, $5 = 2 + 3$, $5 = 3 + 2$
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

¹Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

²These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

³Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

⁴For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.¹

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN (“HOW MANY IN EACH GROUP?” DIVISION)	NUMBER OF GROUPS UNKNOWN (“HOW MANY GROUPS?” DIVISION)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS², AREA³	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

¹The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

²Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

³The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$

Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over additions	$a \times (b + c) = a \times b + a \times c$

Table 4: The properties of equality.

Here a , b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$.
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$ then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a .

Table 5. The properties of inequality.

Here a , b , and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.