NEW MEXICO
Public Education Department

5.MD: MEASUREMENT & DATA

Cluster Statement: C: Geometric measurement: understand concepts of volume.

Major Cluster This standard represents major work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.

Standard Text	**Standard for Mathematical Practices**	**Students who Demonstrate Understanding Can:**
5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement. • 5.MD.C.3.A: A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. • 5.MD.C.3.B: A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	SMP 6: Students can attend to precision by using specific vocabulary to describe the dimensions when measuring volume. SMP 7: Students can look for and make use of structure by using their knowledge of the mathematical structure of area and applying that knowledge to volume.	• Explain that volume is the measurement of the space inside a solid three-dimensional figure. • Explain that a unit cube has 1 cubic unit of volume and is used to measure volume of three-dimensional shapes. • Explain that any solid figure packed without gaps or overlaps and filled with *n* unit cubes indicates the total cubic units or volume. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember
Standard Text **5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.**	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by finding the volume of a rectangular prism by counting unit cubes, using cubic cm, cubic in., and cubic ft. SMP 5: Students can use tools by using manipulatives to build cubes and rectangular prisms without gaps or overlaps and discover the formula for volume of a rectangular prism. SMP 6: Students can attend to precision by using specific vocabulary to describe the dimensions when measuring volume.	**Students who demonstrate understanding can:** • Measure volume by counting unit cubes, cubic cm, cubic in, cubic ft, and improvised units. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand and Apply

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Standard Text	Standard for Mathematical Practices	Students who Demonstrate Understanding Can:
5.MD.C.5: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. • 5.MD.C.5.A: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. • 5.MD.C.5.B: Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. • 5.MD.C.5.C: Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	SMP 1: Students can make sense of problems and persevere in solving them by solving real-world and mathematical problems involving volume. SMP 4: Students can model with mathematics by applying the formula $V=l \times w \times h$ and $V= B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths. SMP 7: Students can look for and make use of structure by recognizing volume as additive.	• Identify a right rectangular prism. • Multiply the three dimensions in any order to calculate volume (Commutative and Associative properties). • Recognize that "B" refers to the area of the base. • Recognize volume as additive. • Develop volume formula for a rectangle prism by comparing volume when filled with cubes to volume by multiplying the height by the area of the base, or when multiplying the edge lengths (l x w x h). • Apply the following formulas to right rectangular prisms having whole number edge lengths in the context of real-world mathematical problems: Volume = length x width x height or Volume = area of base x height. • Solve real world problems by decomposing a solid figure into two non-overlapping right rectangular prisms and adding their volumes. • Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes. **Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Understand, Apply and Analyze
Previous Learning Connections • Connect to creating 3-D shapes. **(1.G.2b)** • Connect to learning to measure area using unit squares **(3.MD.6)** • Connect to applying the formulas to determine area and perimeter of rectangles. **(4.MD.3**)	**Current Learning Connections** • Connect to fluently multiplying multi-digit whole numbers. **(5.NBT.5)**	**Future Learning Connections** • Connect to finding the volume of right rectangular prisms with fractional dimensions in the context of solving real-world and mathematical problems. **(6.G.2)**

Clarification Statement:
5.MD.C.3: "Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube.

5.MD.C.4: They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build.

5.MD.C.5: Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism.

Common Misconceptions

- Students might try to measure volume with square or linear units.
- Students may labele volume with the wrong unit or read the shorthand for volume as 32 feet cubed rather than accurately reading it as 32 cubic feet.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *Pre-teach Targeted: What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying understanding of volume concepts because "this is the first time that students begin exploring the concept of volume. In previous grades students worked with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of a cube) with a layer of unit cubes and then adding layers of unit cubes on top of the bottom layer. Students should have ample experiences with concrete manipulatives before moving to pictorial representations."[1] Students will then derive the formula for calculating volume from their concrete understanding based on model representations.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement: This standard provides a foundation for work with understanding volume concepts because students use their understanding of area to make sense of volume. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with understanding concepts of volume benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as concrete (physical models) and abstract (formulas or drawings) representations of volume. Students are able to understand volume when represented with concrete, physical models in which they touch and manipulate, to develop an understanding of what volume represents at a basic level. Students begin by counting cubes that represent the volume of a solid, rectangular prism. They then move into modeling with cubes to show their understanding of volume. Once this understanding is established, students can create, or apply a formula to represent their thinking. Drawings and graphs can be used in addition to concrete models to represent understanding of volume because students need a concrete, visual

[1] http://www.dusd.net/cgi/files/2013/04/5th-flipbook.pdf

representation of volume in order to relate the concept to a formula. Students may be able to understand volume when related to a physical model but may struggle to understand the concept of the formula for calculating volume without a visual/physical representation of the concept to attach the formula to.

Build

Effort and Persistence*: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding of volume benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because students may have trouble understanding the concept of volume without a visual representation. The concept of volume is difficult for students to understand when only given the formula for calculating volume. Multiple representations of how to physically manipulate volume will help students gain the understanding necessary for working with the concept of volume in a concrete way, in the beginning, moving to a more abstract understanding as students gain more experience and understanding of concepts associated with volume.

Language and Symbols*: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding concepts of volume benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students have experience with concepts of area and the area model, they are able build upon this understanding to relate to the idea of volume. Students apply previously learned concepts to develop an understanding of a new more complex concept.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding concepts of volume benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks and 3D models) because students must understand the concept of volume at a concrete/physical level before being able to think of volume in an abstract way. This understanding supports the use of formulas to quickly compute volume.

Internalize

Executive Functions*: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with understanding of volume concepts benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing models or examples of the process and product of goal-setting because interacting with concepts of volume physically (creating models to represent understanding) supports understanding of concepts and formulas. Allowing students the opportunity to make a plan, carry out the plan, and reflect on its effectiveness teaches students higher level thinking skills required for complex problem solving. Using models as tools to represent volume will provide students with a deep understanding of what volume is. This will help students solidify these concepts in their nonconscious, in order to develop higher level thinking skills involving volume. This will support students' understanding of more complex volume concepts introduced in 6th grade, for example understanding volume involving fractional measurements. Students' experiences setting goals, creating plans, and reflecting will guide their learning and give them a foundation to build from when encountering problems that are harder to visualize.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding volume concepts by examining tasks from a different perspective through a short mini lesson because students can gain a better understanding of concepts by analyzing models created by other students. There are multiple ways models can be constructed and used to calculate volume. Students will gain a deeper understanding of volume by engaging with models created by other students. Give students the opportunity to analyze, engage and interact with multiple perspectives/models of representation. Allow students the time to explain their thinking and make connections between different methods of representation.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of understanding volume concepts by addressing conceptual understanding because students must understand what volume is before they can interact with real-world problems involving volume. Allow students time to deconstruct pre-made models in an attempt to understand that "volume is the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters"; hence the model is constructed of 3-dimensional cubes (measuring 1in. X 1in. X 1in. or 1 cm. X 1cm. X 1cm.).

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying understanding of volume concepts because they are able to grasp the concepts of volume easily. Allow these students to derive and use formulas for calculating volume based on their conceptual understanding of volume. These students will benefit from interacting with real-world problems involving volume in which they need to use a formula to solve. Allow students to show their thinking through images, concepts, facts, language, and procedures[2]. Expose students to questions that require them to calculate multiple numerical volumes and combine or decompose them, in order to arrive at a solution. Expose students to real-world mathematical problems that are connected to other discipline areas (e.g., science/social studies). Students may also benefit from problems that have multiple solutions based on the strategy the student applies.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Using and Connecting Mathematical Representations): The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding volume

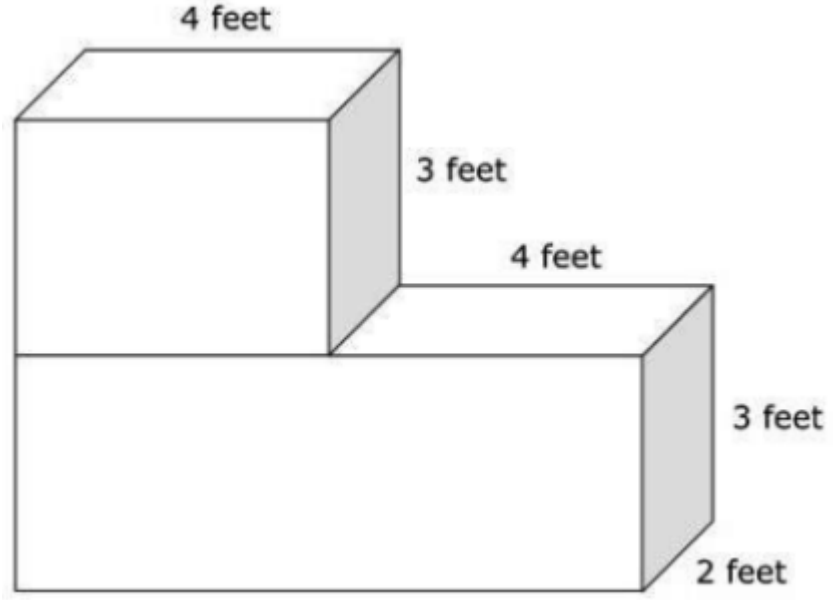
[2] ICFLP Dr. Lorenzo Gonzales (need to get full reference from 5th grade team)

concepts, the use of mathematical representations within the classroom is critical because students can use a variety of mathematical representations that they are already familiar with. This helps students connect to prior knowledge and allows them to use what they already know to connect to new concepts. Encourage students to use examples of things they see and experience in their everyday lives as mathematical representations of volume. Validate students' thinking, as they make connections to volume in the real-world and within their own environments. Encourage students to use multiple representations to show their mathematical thinking around volume and their everyday lives. Allowing students the time to share ideas, thoughts, and representations will give students an insight into the lives of other students.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: PARCC released item 2018

Cement was poured to make two rectangular prisms. The prisms were stacked, as shown.



Part A

What are the length, width, and height, in feet, of the smaller rectangular prism?

Part B

What is the total amount of cement, in cubic feet, used to make the two rectangular prisms?

Answer Key

Part A: 2, 3, 4 in any order Part B: 72

Relevance to families and communities:

During a unit focused on understanding volume concepts, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, comparing the volume of multiple structures/objects found at home, or in the community will help students gain a deeper understanding of volume. Reasoning about the capacity of different size rooms in the home or around the community may help students make a connection to the significance of volume. Connecting packing cubes into a rectangular prism

Cross-Curricular Connections:

Science: In fifth grade the NGSS states students should "describe and graph quantities such as area and volume to address scientific questions." Consider providing a connection for students to determine the volume of cubes or rectangular prisms as part of their investigation.

Art: Drawing boxes is connected to developing the ability to indicate perspective in a drawing. Consider providing an opportunity for students to sketch various boxes with the same volume but different dimensions. Also, consider allowing students to make boxes to pack inside of larger boxes (measuring 1in. X 1in. X 1in. or 1 cm. X 1cm. X 1cm.). Have students predict how many boxes can fit inside of the

and packing items into a storage shed, or packages into a mail delivery truck may help students connect schoolwork to real-world examples found within the home or community.	premade larger boxes. Connect the number of boxes used to the volume of the box. Allowing students to cut, and construct boxes will help with their fine motor skills.