NEW MEXICO
Public Education Department

5.NF.B: NUMBERS & OPERATIONS - FRACTIONS

Cluster Statement: Apply and extend previous understandings of multiplication and division.

Major Cluster This standard represents major work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.

Standard Text	**Standards of Mathematical Practice**	**Students who Demonstrate Understanding Can:**
5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	SMP 2: Students can reason abstractly and quantitatively to solve and write problems that include dividing a fraction by a whole number and a whole number by a fraction using models and verbal explanations. It is important for students to understand the meaning of the "remainder" when it is expressed as a fraction, which means the fraction is the part of the whole that it is left over.	• Interpret a fraction as division of the numerator by the denominator. • Interpret the remainder as a fractional part of the problem. • Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Apply
Standard Text	**Standards of Mathematical Practice**	**Students who Demonstrate Understanding Can:**

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. • 5.NF.B.4.a. Interpret the product (a/b) × q as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* • 5.NF.B.4.b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.	SMP 8: Students look for and express regularity in repeated reasoning by identifying patterns that develop when solving multiplication problems involving fractions.	• Extend previous understandings of multiplication to multiply a fraction or a whole number by a fraction. • Explain that the product (a/b) x q is the same as a x q ÷ b. • Multiply a fraction or a whole number by a fraction. • Create a story context to multiply a fraction or a whole number by a fraction. • Explain that finding the area of a rectangle with fractional side lengths by filling with tiles is the same as would be found by multiplying the side lengths. • Find the area of a rectangle by tiling it with unit squares. • Multiply fractional side lengths to find the area of a rectangle. **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Apply
Standard Text **5.NF.B.5 Interpret multiplication as scaling (resizing), by:** • 5.NF.B.5.a: Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. • 5.NF.B.5.b: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole	**Standards of Mathematical Practice** SMP 2: Students can reason abstractly and quantitatively by reasoning that multiplying by a fraction less than 1 results in a smaller value.	**Students who Demonstrate Understanding Can:** • Interpret multiplication by scaling, comparing the size of a product to the size of one factor based on the size of the other factor. • Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number and why multiplying a given number by a fraction less than one results in a product smaller than the given number

numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1		**Depth of Knowledge:** 1, 2, 3 **Bloom's Taxonomy:** Apply, Analyze
Standard Text **5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.**	**Standards of Mathematical Practice** SMP 2: Students can reason abstractly and quantitatively. by applying their understanding of multiplication of fractions to contextualize and decontextualize real world problems.	**Students who Demonstrate Understanding Can:** • Represent word problems involving multiplication of fractions and mixed numbers. • Solve real world problems involving multiplication of fractions and mixed numbers **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Apply
Standard Text **5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.** 5.NF.B.7.a: Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* 5.NF.B.7.b: Interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for 4*	**Standards of Mathematical Practice** SMP 1: Students make sense of problems and persevere in solving them by using multiplicative reasoning and transfer their understanding of multiplication to division by whole numbers and fractions. SMP 3: Students can construct viable arguments and critique the reasoning of others by engaging in debates to defend their problem-solving reasoning to deepen their understanding of the relationship between division of unit fractions by whole numbers and whole numbers by unit fractions.	**Students who Demonstrate Can:** • Know the relationship between multiplication and division. • Interpret division of a unit fraction by a whole number and justify your answer using the relationship between multiplication and division, by creating story problems, using visual models, and relationship to multiplication. • Interpret division of a whole number by a unit fraction and justify your answer using the relationship between multiplication and division, and by representing the quotient with a visual fraction model. • Solve real world problems involving division of unit fractions by whole numbers other than 0 and division of whole numbers by unit

<table>
<tr>
<td rowspan="3">÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

5.NF.B.7.c: Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of fraction by a fraction is not a requirement at this grade.</td>
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<td>fractions using strategies such as visual fraction models and equations.</td>
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<td>Depth of Knowledge: 1, 2</td>
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<td>Bloom's Taxonomy: Apply</td>
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<td><u>Previous Learning Connections</u>

• Connect to understanding concepts of area and relate area to multiplication and to addition. (3.MD.7)
• Connect to using the four operations with whole numbers to solve problems. (4.OA.1,2,3)
• Connect to understanding the concept of equivalent fractions by using visual fraction models. (4.NF.1)
• Connect to multiplying a fraction by a whole number. (4.NF.4)</td>
<td><u>Current Learning Connections</u>
• Connect to understanding of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. (5.NBT.5,6,7)
• Connect to knowledge of writing simple expressions to solve real problems with fraction. They will also interpret expressions without actually evaluating them. (5.OA.2)
• Connect to using operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems involving information presented in line plots. (5. MD.2)</td>
<td><u>Future Learning Connections</u>

• Connect to using ratios written as fractions and divide into decimal form 3 ÷ 4 = 3⁄4 = 0.75. (6.RP .1,3)
• Connect to solving multiplication equations that include non-negative rational numbers. (6.EE.7)
• Connect to multiplying and divide fractions by fractions. (6.NS.1)</td>
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<td colspan="3">Clarification Statement:
5.NF.B.3: Fifth grade students should connect fractions with division, understanding that 5 ÷ 3 = 5⁄3 * Students should explain this by working with their understanding of division as equal sharing.
Students should also create story contexts to represent problems involving division of whole numbers.
This standard calls for students to extend their work of partitioning a number line from third and fourth grade.
Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities.</td>
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5.NF.B.4: Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction (e.g., 2 x (1⁄4) = 1⁄4 + 1⁄4).
This standard extends student's work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as 3⁄5 could be represented as 3 pieces that are each one-fifth (3 x (1⁄5)). These standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.
This standard extends students' work with area. In third grade students determine the area of rectangles and composite rectangles. In fourth grade students continue this work. The fifth-grade standard calls students to continue the process of covering (with tiles).
5.NF.B.5: These standards ask students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.
5.NF.B.6: This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.
5.NF.B.7: This is thefirst time that students are dividing with fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. The concept unit fraction is a fraction that has a one in the denominator. For example, the fraction 3⁄5 is 3 copies of the unit fraction 1⁄5. 1⁄5 + 1⁄5 + 1⁄5 = 3⁄5 = 1⁄5 x 3 or 3 x 1⁄5.
This standard asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.

Common Misconceptions

- Students may initially think that you can not divide a "smaller number" by a "bigger number" since this will be a new situation for them to consider.
- Students may believe that multiplication always results in a larger number.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted): *Pre-teach Targeted: What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying apply and extend previous understandings of multiplication and division to multiply and divide fractions because use of models to multiply a fraction by a whole number will help student connect to the meaning of whole number multiplication .

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.A.1. This standard provides a foundation for work with multiplying and dividing fractions because this standard has students represent and solve problems involving multiplication and division, conceptual models of understanding multiplication and division. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences include ways to recruit interest such as providing choices in their learning give an example such as the sequence or timing of task completion because students worked with concrete models for multiplying a fraction by a whole number in Grade 4. They will continue to extend this work to additional situations for multiplying a whole number by a fraction. They use area models to connect their understanding of multiplication of whole numbers to multiplication of fractions. Students will explore division of a whole number by a fraction and a fraction by a whole. They will model through visual models and contexts to make sense of what multiplication and division of fractions entails.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because Following many opportunities to model, explain, and solve problems, students use their experiences to recognize patterns and develop efficient strategies. .

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because students will make connections between division with fractions and multiplication with fractions using previous experiences with the relationship between multiplication and division of whole numbers .

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with apply and extend previous understandings of multiplication to multiply and divide fractions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because this will allow students to visualize situations and connect those experiences to writing multiplication and division equations, they can talk about the equations in terms of a missing factor or dividend and make generalizations. .

Internalize

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from

mistakes), such as elevating the frequency of self-reflection and self-reinforcements because students will be able to support their learning through opportunities to share ideas, clarify their understanding, develop mathematical arguments, and make generalizations about multiplying and dividing fractions .

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on connecting multiplication and division of whole numbers to multiplication and division of fractions by giving students connected situations they can model by clarifying mathematical ideas and/or concepts through a short mini-lesson because exploration using various representations including concrete and pictorial models.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit Apply and extend previous understanding of multiplication and division to multiply and divide fractions by confronting student misconceptions because students may initially think that they cannot divide a "smaller number by a bigger number" since this will be a new situation for them to consider. It is important they understand this concept in a way that makes sense to them rather than be shown how to do it.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying Apply and extend previous understanding of multiplication and division to multiply and divide fractions because as students work with various models of multiplication and division of whole numbers, fractions, and mixed numbers, visual representations will help them understand the size of the product/quotient when they multiply/divide a fraction by a whole number, a whole number by a fraction, or a fraction by a fraction.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying Apply and extend previous understanding of multiplication and division to multiply and divide fractions the types of

mathematical tasks are critical because students are demonstrating their conceptual understanding, procedural fluency, and problem solving and reasoning. Students use a variety of problem-solving situations to develop an understanding of multiplication and division of fractions.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia

Students made cards using paper.

- They made 9 cards from 38 pieces of paper.
- They used the same amount of paper for each card.

A. Write a fraction to show the amount of paper used for each card. Show your work or explain how you know.

The students sold their cards. They earned a total of $18. Four students divided the money evenly.

B. Between what two consecutive whole dollar amounts did each student earn? Show your work or explain how you know

Answer Key

Constructed-Response Rubric	
Score	**Description**
4	4 points
3	3 points
2	2 points
1	1 point
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Scoring Notes

Part a	2 points	for correct answer, $\frac{38}{9}$ **(pieces of construction paper) or equivalent**, with sufficient work or explanation to indicate correct strategy
	or	
	1 point	for correct answer with insufficient or no explanation or work shown or for correct strategy with incorrect or no answer
Part b	2 points	for correct answer, **between 4 and 5 (dollars)**, with sufficient work or explanation to indicate correct strategy
	or	
	1 point	for correct answer with insufficient or no explanation or work shown or for correct strategy with incorrect or no answer

Sample Response

a. Since each card used the same amount, I divided. Fractions are the same as division, so they used $\frac{38}{9}$ pieces of construction paper for each greeting card.

b. By using fraction models, division, or other methods, the students each received $4\frac{1}{2}$ dollars, which is between 4 and 5. For example, 18 divided by 4 is 4 with remainder 2.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on Apply and extend previous understandings of multiplication and division to multiply and divide fractions , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, students can create story contexts for multiplying and dividing fractions and include a visual model. For example, How much pie? After a family function, Emily has 3 equally sized pies and wants to divide them equally into eight equal portions to give to family members that want to take some home. How much pie does each family member receive?	Social Studies: Connect fractions to studies of geography including scaling graphs and cross-sections, changes in measure (population, GDP) Health: connect fractions to food sharing, cooking, serving portions, nutrition, medical doses, heart beats per minutes, steps per day. Present students with real-world problems using these topics.