New Mexico Mathematics Instructional Scope for Fifth Grade

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

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[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

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[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see [Standards Aligned Instructionally Embedded Formative Assessment Resources](#).

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can
Full text of the standard	The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	**Students who demonstrate understanding can:** The cognitive skills students perform to demonstrate to comprehension of a standard. **Depth Of Knowledge:** Correlation of standard to Webb's Depth of Knowledge **Bloom's Taxonomy:** Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities	Cross Curricular Connections
Relevance to Families and Communities: Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	**Cross Curricular Connections:** Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

TABLE OF CONTENTS:

Operations & Algebraic Thinking

- 5.OA.A
- 5.OA.B

Number & Operations in Base Ten

- 5.NBT.A
- 5.NBT.B

Number & Operations – Fractions

- 5.NF.A
- 5.NF.B

Measurement & Data

- 5.MD.A
- 5.MD.B
- 5.MD.C

Geometry

- 5.G.A
- 5.G.B

5.OA.A: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: Write and interpret numerical expressions

Additional Cluster This standard represents additional work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.

Standard Text	Standards for Mathematical Practice	Students who Demonstrate Understanding Can:
5.OA.A.1: Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	**SMP 6:** Students can attend to precision to describe their work with grouping symbols and order of operations.	• Understand the use of parentheses, expressions inside parentheses/brackets must be completed first when solving the equation. • Apply rules and solve problems for orders of operations (not to include exponents). • Solve problems and equations that employ parentheses. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember, Understand and Apply
Standard Text **5.OA.A.2:** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	**Standards for Mathematical Practice** SMP 7: Students can look for and make use of structure by exploring order of operations and apply the rules in a variety of situations, they look for patterns and the structure of expressions.	**Students who Demonstrate Understanding Can:** • Explain where to use parentheses to write numerical expressions to represent real world problems. • Interpret real world problems and write it as a numerical expression. **Webb's Depth of Knowledge:** 1, 2 **Bloom's Taxonomy**: Apply

<table>
<tr>
<th><u>Previous Learning Connections</u></th>
<th><u>Current Learning Connections</u></th>
<th><u>Future Learning Connections</u></th>
</tr>
<tr>
<td>Connect to fluently adding and subtracting within 1,000 (3.NBT.2)
Connect to recalling from memory products of two 1-digit numbers. (4.OA.1. B)</td>
<td>Connect to using knowledge of parentheses as a building block for order of operations.</td>
<td>Connect to performing arithmetic operations following the order of operations with and without parentheses, including those involving whole number exponents. (6.EE.2. D)
Connect to applying the properties of operations to generate equivalent expressions with an emphasis on the distributive property. (6.EE.3)
Connect to writing, reading, and evaluating expressions in which letters stand for numbers. (6.EE.A.2)
Connect to applying the properties of operations to generate equivalent expressions. (6.EE.A.3)
Connect to identifying when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. (6.EE.A.4)
Connect to finding the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2). (6.NS.B.4)</td>
</tr>
<tr>
<td colspan="3">Clarification Statement:
5.OA.A.1: In fourth grade, students used comparison of multiplication and division problems; thinking about solutions in terms of reasonableness by using estimation in order to determine if the solutions were reasonable.</td>
</tr>
</table>

Listening to others and gathering a variety of strategies to solve problems. Used appropriate mathematical vocabulary and accurate units of measure begin solving more sophisticated problems.
The order of operations is introduced in third grade and is continued in fourth. This standard calls for students to evaluate expressions with parentheses ( ), brackets [ ] and braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However, at this level students are only to simplify the expressions because there are no variables.
In fifth grade, students work with exponents only dealing with powers of ten (5.NBT.2). Students are expected to evaluate an expression that has a power of ten in it.
5.OA.A.2: In fourth grade, students used quantitative reasoning to solve single and multi-step problems that included all four operations using models, pictures, words, and numbers. Students continues to develop problem solving strategies by using various representations and models and selecting appropriate tools. They started writing equations to represent the mathematics of the situation.
This standard refers to expressions. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equal's sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1).
This standard calls for students to verbally describe the relationship between expressions without calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations.

Common Misconceptions

- Students may be confused about the order of operations, thinking that all multiplications are calculated before division and additions before subtractions. Instead of solving first multiplications and/or division in order from left to right and continue with addition and/or subtraction in order from left to right.
- Students may misapply generalizations as they attempt to make sense of rules/patterns. A strategy that can be used is posing the question as a debate, "Is it always true?" students' groups are assigned to a group to prove or disprove that a given rule will or will not always apply.
- Students may believe the order in which a problem with mixed operations is written is the exact order to solve the problem. The use of mnemonic phrase "Please Excuse My Dear Aunt Sally" to remember the order of operations (Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction) can mislead students to always perform multiplication before division and addition before subtraction. This is incorrect thinking. Multiplication and division are always performed first in the order that they appear in the problem –unless there are grouping symbols. To help correct students' thinking, they need to understand that addition and subtraction are inverse operations and multiplication and division are inverse operations, as in they have the same "impact". At this level, students need opportunities to explore the "impact" of the various operations on numbers and solve equations starting with the operation of greatest "impact".
- Students may not understand the equal sign, for example when given 8+4=___+5, students may understand it as a 'balance'.
- Students often do not use the correct terminology for the operations. Frequently students say "times" for multiplication. This is NOT the action for multiplication. Students need to say and think "groups of" (or "of" when using fractions and decimals) when explaining multiplication. For addition, students can explain that it is "joining", "combining", "putting together", or other appropriate words for addition. The same for the rest of the operational symbols.
- Students may not realize that math symbols are just short cuts for using words but that ALL symbols represent words in mathematics.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *Pre-teach Targeted: What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., grouping symbols) when studying writing and interpreting numerical expressions because the concept of order of operations will be new to students.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.OA.A.3: This standard provides a foundation for work with writing and interpreting numerical expressions because students have previous experience writing expressions. In addition, students worked informally with order of operations in grades 3 and 4 as they solved multi-step problems through modeling and writing equations. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with writing and interpreting numerical expressions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for auditory information. *For example, express the calculation "add 8 and 7, then multiply by 2" as (8 + 7) X 2. Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product*. For students who are impaired provide text equivalents in the form of captions or automated speech-to-text (voice recognition) for spoken language; visual diagrams, charts, notations of music or sound; written transcripts for videos or auditory clips; American Sign Language (ASL) for spoken English; visual analogues to represent emphasis and prosody (e.g., emoticons, symbols, or images); visual or tactile (e.g., vibrations) equivalents for sound effects or alerts because this will allow students to be able to hear the words and see them written down.

Build

Effort and Persistence*: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with writing and interpreting numerical expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because this is a hard concept for 5th grade students to understand and the feedback will help this see patterns of errors and understand why their answer will not work correctly.

Language and Symbols*: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with writing and interpreting numerical expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge providing graphic symbols with alternative text descriptions because students need to understand the vocabulary words that go with the math symbols to be able to write out and interpret the numerical expressions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with writing and interpreting numerical expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students need to be able to understand that the way they interpret the numerical expression could lead to an incorrect answer.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with writing and interpreting numerical expressions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as elevating the frequency of self-reflection and self-reinforcements because students will be able to express themselves through self-reflection and this will allow them to go back over their thinking and double check their work.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on writing and interpreting numerical expressions by revisiting student thinking through a short mini-lesson because student misconceptions in thinking may lead to errors in calculation. Encourage students to explain and clarify their reasoning in solving equations.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit writing and interpreting numerical expressions by confronting student misconceptions because the order in which to calculate and knowing when to use parenthesis can be confusing to a number of students.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying writing and interpreting numerical expressions because some students will be able to write and solve more complicated equations. Offer opportunities to play games in which they must write equations to make a target number and explain their reasoning.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying writing and interpreting numerical expressions the use of mathematical representations within the classroom is critical because this cluster focuses on writing and evaluating mathematical expressions. Students are asked to solve multi-step problems using mathematical representations in the form of expressions that may include grouping symbols. In addition, students are expected to apply the rules of order of operations to evaluate, write, and interpret numerical expressions.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia

Each day Mia spends 20 minutes jogging and 35 minutes cycling.

A. How many minutes does Mia spend jogging and cycling in 4 days? Write an expression to show or explain how you know.

The expression 7[(35 + 20) + (10 X 2)] can be used to determine the total number of minutes that Mia spends exercising in a week, including warm-up and cool-down times.

B. Evaluate the expression 7[(35 + 20) + (10 X 2)]. Show or explain how you found your answer.

Answer Key

Constructed-Response Rubric	
Score	**Description**
4	4 points
3	3 points
2	2 points
1	1 point
0	The response is incorrect or irrelevant to the skill or concept being measured.
Blank	No Response.

Scoring Notes

Part a: 2 points for correct answer, **220** (minutes), with sufficient work or explanation shown involving the expression, 4(20 + 35) or equivalent

OR

1 point for correct answer with insufficient or no explanation or work shown

or

for correct strategy with incorrect or no answer

Part b: 2 points for correct answer, **525**, with sufficient work or explanation to indicate correct strategy

OR

1 point for correct answer with insufficient or no explanation or work shown

or

for correct strategy with incorrect or no answer

Sample Response

a. 220 minutes; $4(20 + 35) = 4 \times 55 = 220$ minutes in 4 days

b. 525; $7[(35 + 20) + (10 \times 2)] = 7(55 + 20) = 7 \times 75 = 525$

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on writing and interpreting numerical expressions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can write or verbally state mathematical expressions that represent real-life situations such as, "My brother is 2. I am five times older than my brother. My sister is 4 years older than me. How old is my sister?" [(2x5)+4=14].	Science: Students can create numerical expressions from data displayed in a table or graph

5.OA.B: OPERATIONS & ALGEBRAIC THINKING

Cluster Statement: Analyze patterns and relationships.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standards of Mathematical Practice	Students who Demonstrate Understanding Can:
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	SMP3: Students can construct viable arguments and critique the reasoning of other by comparing descriptions and looking for counterexamples, ordered pairs that do not fit the rule SMP 7: Students can look for and make use of structure when finding patterns, students are developing a deeper understanding of all 4 operations and beginning to make generalizations by constructing rules for their patterns.	• Identify the relationship between two patterns. • Given a starting point, apply two math rules to that number. • Graph data on a coordinate plane (positive numbers only). **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Analyze, Understand
<u>Previous Learning Connections</u> • Connect to following one rule and then determined what happened in that pattern. (4.OA.5)	**<u>Current Learning Connections</u>** • Connect to graphing points on a coordinate plane. (5.G.1, 5.G.2)	**<u>Future Learning Connections</u>** • Connect to applying the use of variables to represent two quantities in real world problems. Students will write equations to represent the dependent and independent variables. (6.EE.9) • Connect to describing the relationship in ratio rates to solve real world problems. (6.RP.2, 6.RP.3)

Clarification Statement:
This standard is closely related to graphing points in the first quadrant of a coordinate plane (5.G.1-2) This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule.
In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines.

Common Misconceptions
- Students may reverse the points when plotting them on a coordinate plane. They count up first on the y-axis and then count over on the x-axis. The location of every point in the plane has a specific place. Have students plot points where the numbers are reversed such as (4, 5) and (5, 4). Begin with students providing

a verbal description of how to plot each point. Then, have them follow the verbal description and plot each point.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teaching

Pre-teach (Targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying analyzing patterns and relationships because students will generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane.

Pre-teach (Intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.OA.C.5: This standard provides a foundation for work with analyzing patterns and relationships because the students will be able to generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with analyzing patterns and relationships benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because students who are allowed to express their creativity when solving and analyzing patterns and relationships will become more engaged in their learning. Allowing students to be creative will help the students become interested in topics that they once felt were boring.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with *analyzing* patterns and relationships benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students who are provided feedback on their effort or areas of improvement will become more engaged in their learning. When they are given tips and ideas on how to improve their work, they will persevere to become more successful.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*.

- For example, learners engaging with *analyzing* patterns and relationships benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because students learn by being led by example. Students learn by different means. Some need to see examples that tie into what they are learning. If they can visually see what the problem is asking them to solve for, then they will understand how to access the problems to come to a solution.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with *analyzing* patterns and relationships benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because students need feedback to help them understand the skills or concepts. Students make their own mistakes and need to learn from those mistakes, so providing feedback that is good for one student may not be good for another. Make the feedback meaningful to each individual student to help them be more successful.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with *analyzing* patterns and relationships benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing guides and checklists for scaffolding goal-setting because students are eager learners. If you provide them with guidance on how to set goals, they will eventually learn to do so on their own. If the students are provided with guides and checklists, the students will learn what is vital and important for setting their goals for success.

Re-Teaching

Re-teach (Targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on analyzing patterns and relationships by clarifying mathematical ideas and/or concepts through a short mini-lesson because the students will be able to create, analyze and solve patterns and practice "PEMDAS" in order for them to create their pattern while getting the correct response. This enables the students to practice order of operations.

Re-teach (Intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit analyzing patterns and relationships by confronting student misconceptions because students will need to be able to walk through the PEMDAS process. The students will need to understand the process of multiplication and division and addition and subtraction do not necessarily need to be performed in that order. Students need to remember that the order goes from the operation on the left to the right. These misconceptions will give the students incorrect answers for their problems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying analyzing patterns and relationships because students will be able to explore generating patterns and creating graphs and charts to exhibit their responses to the problems. It would also allow students to explore different topics and develop their own specifications for solving problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying analyzing patterns and relationships supporting productive struggle is critical because the process develops a sense of perseverance and creative problem solving. When students face problems they don't immediately know how to solve, we don't want them to give up because we want them to continue to work towards a possible solution that helps them understand the problem on their own way of thinking.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: PARCC Released Item 2019

Two Patterns are shown.

Pattern A	2	4	6	8	10
Pattern B	12	24	36	48	60

Which Statement about the corresponding terms in Pattern A and Pattern B is always true?

A. The terms in Pattern B are 6 times the corresponding terms in Pattern A.
B. The terms in Pattern B are 10 times the corresponding terms in Pattern A.
C. The terms in Pattern B are 10 more than the corresponding terms in Pattern A.
D. The terms in Pattern B are 20 more than the corresponding terms in Pattern A.

Answer Key A

Relevance to families and communities:

During a unit focused on analyzing patterns and relationships , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, during community events, students and families can create charts and graphs that will show price/cost analysis when selling products during these events. Families could develop a sense of determining which type of snack or drink would sell more at different prices in order to determine how much to charge for their products.

Cross-Curricular Connections:

Science: Give students data represented in a table. Have students discuss the relationship between the numbers in the table.

5.NBT: NUMBER & OPERATIONS IN BASE 10

Cluster Statement: Understand the place value system.

Major Cluster This standard represents major work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.

Standard Text	Standards for Mathematical Practice	Students who Demonstrate Understanding Can:
5.NBT.A.1 **Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.**	SMP 2: Students can reason abstractly and quantitatively by understanding the relationship between adjacent places in both whole numbers and decimals to reinforce conceptual understanding of individual places as well as the magnitude of a number across place values on both sides of the decimal point. SMP 7: Students can look for and make use of structure by attending to and understanding the relationship between adjacent place values in both whole numbers and decimals.	• Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1⁄10 of what it represents in the place to its left. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Understand

Standard Text	Standards for Mathematical Practices:	Students who can Demonstrate Understanding Can:
5.NBT.A.2 **Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.**	SMP 7: Students can look for and make use of structure by identifying and using patterns when multiplying decimals.	• Represent powers of 10 using whole number exponents. • Translate between powers of 10 written as 10 raised to a whole number exponent, the expanded form, and standard notation. • Explain the patterns in the number of zeros of the product when multiplying a number by powers of 10. • Explain the relationship of the placement of the decimal point when a decimal is multiplied or divided by a power of 10. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standards for Mathematical Practices:**	**Students who Demonstrate Can:**
5.NBT.A.3: Read, write, and compare decimals to thousandths • 5.NBT.A.3a: Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). • 5.NBT.A.3a: Compare two decimals to the thousandths based on the meanings of the digits in each place, using >,=,< symbols to record results of comparisons.	SMP 5: Students can use appropriate tools strategically by selecting a combination of tools such as a number line, writing an equation, and/or using a graphical representation to solve place value problems and determine which tool is most efficient to represent a solution.	• Read and write decimal to thousandths using base-ten numerals, number names, and expanded form. • Use >, =, and < symbols to record the results of comparisons between decimals. • Compare two decimals to the thousandths, based on the place value of each digit. • Use knowledge of base ten and place value to round decimals to any place. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Understand, Analyze

Standard Text	**Standards for Mathematical Practice**	**Students Who Demonstrate Understanding Can:**
5.NBT.A.4: Use place value understanding to round decimals to any place	SMP 7: Students can look for and make sure of structure to understand the connection between rounding decimals and whole numbers.	• Explain why the value of digits depends on its place. • Round decimals to any place.
Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• 4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. • 4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. * For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. • 4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. • 4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	• 5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm. • 5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. • 5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	• 6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • 6.NS.B.3Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. • 6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.
Clarification Statement: 5.NBT.A.1: Students extend their understanding of the base-ten system to the relationship between adjacent		

places, how numbers compare, and how numbers round for decimals to thousandths. This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is 1⁄10th the size of the tens place. Based on the base-10 system, digits to the left are 10 times as great as digits to the right; likewise, digits to the right are 1⁄10th of digits to the left.

5.NBT.A.2: Multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. Patterns in the number of 0s in products of a whole number and a power of 10 and the location of the decimal point in products of decimals with powers of 10 can be explained in terms of place value. Because students have developed their understandings of and computations with decimals in terms of multiples rather than powers, connecting the terminology of multiples with that of powers affords connections between understanding of multiplication and exponentiation. (Progressions for the CCSSM, Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 16) This standard includes multiplying by multiples of 10 and powers of 10, including 102 which is 10 x 10=100, and 103 which is 10 x 10 x 10=1,000. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10. Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.

5.NBT.A.3: This standard reference expanded form of decimals with fractions included and comparing decimals builds on work from fourth grade. This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers.

5.NBT.A.4: Students have a deep understanding of place value and number sense by explaining and give reasons about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. When rounding a decimal to a given place, students may identify the two possible answers, and use their understand of place value to compare the given number to the possible answers.

Common Misconceptions

- Students may try to extend a shallow understanding of whole number place value to decimal place.
- Students may think the more digits in the number the greater the number.
- Students can confuse the language describing the relationship between place values for whole numbers and decimal numbers. 5
- Students memorize a rule of “adding zeros” to make the powers of 10 and then misapply this “rule”.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach Targeted: *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying understanding the place value system because students are extending prior knowledge of place value from previous years to include place value patterns, reading, writing, and comparing decimal numbers, and rounding decimals.

Pre-teach Intensive: *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.NBT.A.2: This standard provides a foundation for work with understanding the place value system because reading and writing whole numbers in expanded notation reinforces understanding of the value of each digit in a number and how those values relate to one another. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding the place value system benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students apply place value concepts in a variety of real-life contexts, such as money and measurement. Developing conceptual understanding through models and relevant realistic tasks will reinforce the meaning of whole number and decimal place values and their relationships.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding the place value system benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because place value understanding requires a high level of precision. Timely, specific feedback from both teachers and peers will reduce student confusion and misconceptions.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*.

- For example, learners engaging with understanding the place value system benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students in fifth grade are extending their knowledge of whole number place value to working with decimal numbers. They will be working with understanding the relationship between places both to the left *and* to the right of a given place value.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding the place value system benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as concept mapping tools, such as a place value chart or number lines because using structural models will help students develop an understanding of the individual places, the magnitude of a number and the relationship between adjacent places.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding the place value system benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students require multiple opportunities to build on previously learned and new concepts. In addition, encouraging students to explain their thinking process around solving place value problems will improve their understanding.

Re-teach

Re-teach Targeted: *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding the place value system by clarifying mathematical ideas and/or concepts through a short mini lesson because students may benefit from additional modeling. Provide a variety of experiences and activities in which students model and write base-ten numerals on a place value chart. Modeling reading the decimal numbers correctly will support the meaning of number place value.

Re-teach Intensive: *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding the place value system by <addressing conceptual understanding because students require both concrete experiences and written activities to build their comprehension of decimals.

Extension Ideas

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying understanding the place value system because it leads the students to more generalized thinking about place patterns. Use question stems to help students make connections, for example, "What do you notice about…?" "Why do you think that works?" "Will that always be true when you…?" "Can you find an example of when that is not true?"

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

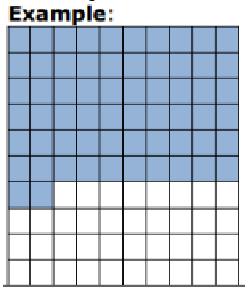
Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying understanding the place value system the types of mathematical tasks are critical because building conceptual understanding for place value is essential to fifth grade mathematics. For example, when multiplying 32 x 1000, students should understand that the product represents 32 groups of 1000, or "thirty-two thousands," which is written as 32,000. When teachers focus on the procedure of "adding zeros," students miss the opportunity to build the conceptual understanding which is critical for working with decimals.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: PARCC Released Item 2018
Round 14.235 to the nearest tenth.

- Students recognize that the possible answer must be in tenths thus, it is either 14.2 of 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).

Students should use numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0, 0.5, 1, 1.5 are examples of benchmark numbers.

Example:



Which benchmark number is the best estimate of the shaded amount in the model to the left?

Explain your thinking.

*http://www.dusd.net/cgi/files/2013/04/5th-flipbook.pdf

Resource 2
Which numbers or expressions have the same value as twenty-nine thousandths?
Select **two** correct answers.

A. 0.29

B. 2.9

C. 0.029

D. $2 \times \frac{1}{10} + 9 \times \frac{1}{1000}$

E. $2 \times \frac{1}{10} + 9 \times \frac{1}{100}$

F. $2 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

Answer Key
C and F

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding the place value system, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways decimals are used in the home and community can a be a great way to connect schools tasks with home tasks.	STEM: Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems. Science: Provide students opportunities to take precise measurements. Have students round these measurements to the nearest tenth, hundredth, or thousandths.

5.NBT: NUMBER & OPERATIONS IN BASE 10

Cluster Statement: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Major Cluster This standard represents major work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.

Standard Text	Standards of Mathematical Practices	Student who Demonstrate Understanding Can:
5.NBT.B.5: Fluently multiply multi-digit whole numbers using the standard algorithm.	SMP 7: Students can look for and make use of structure when using standard algorithm and explain how it works.	• Multiply multi-digit whole numbers. • Use multiple strategies including traditional algorithm. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Apply
Standard Text	**Standards for Mathematical Practices:**	**Student who Demonstrate Understanding Can:**
5.NBT.B.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	SMP 1: Students can make sense of problems and persevere in solving them by using the relationship between addition and subtraction, and between multiplication and division.	• Explain calculations using equations or models that represent understanding of division. • Find whole number quotients of whole numbers with four-digit dividends and vtwo-digit divisors. • Use multiple strategies to solve division problems. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standards for Mathematical Practices:**	**Student who Demonstrate Understanding Can:** • Justify reasoning with written explanation.

5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	SMP 3: Students can construct viable arguments and critique the reasoning of others by justifying their calculations with written explanations.	• Explain how place value affects how to use the four operations. • Use the four operations with decimals to the hundredths. • Use models or drawings. **Depth of Knowledge:** 1 **Bloom's Taxonomy:** Apply, Analyze
Previous Learning Connections • Connect to using place value understanding and properties of operations to perform multi-digit arithmetic. (4.NBT.4,5,6)	**Current Learning Connections** • Connect to understanding the place value concept that the number to the left is 10 times larger and the number to the right is 10 times smaller, will use exponents to express powers of 10 and can understand the patterns of zeros and decimal placement related to powers of 10. (5.NBT.1,2) • Connect to applying and extend their previous understandings of multiplication and division to multiply and divide fractions. (5.NF.1,3,4,6,7) • Connect to converting customary and metric measurement units within a given measurement system. (5.MD.1)	**Future Learning Connections** • Connect to fluently adding, subtracting, multiplying, and dividing decimals using the standard algorithm. (6.NS.2,3) • Connect to recognizing that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.A.1 • Connect to read, write, and compare decimals to thousandths. 5.NBT.A.3

Clarification Statement:
5.NBT.B.5: In fifth grade, students fluently compute products of whole numbers using the standard algorithm. Underlying this algorithm are the properties of operations and the base-ten system. Division strategies in fifth grade involve breaking the dividend apart into like base-ten units and applying the distributive property to find the quotient place by place, starting from the highest place. (Division can also be viewed as finding an unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the unknown factor.) Students continue their fourth-grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted.
Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.
Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.
This standard refers to **fluency** which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart also using strategies according to the numbers in the problem, 26 x 4 may lend itself to (25 x 4) + 4 where as another problem might lend itself to making an equivalent problem 32 x 4 = 64 x 2)). This standard builds upon students' work with multiplying numbers in third and fourth grade. In fourth grade, students developed

understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding. The size of the numbers should NOT exceed a three-digit factor by a two-digit factor.
5.NBT.B.6: This standard reference various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.
5.NBT.B.7: In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Common Misconceptions

- Students who only memorize steps for algorithms without understanding will confuse the "steps" in the addition algorithm with the "steps" in the multiplication algorithm.
- Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted): *Pre-teach Targeted: What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying operations with multi-digit whole numbers and with decimals to hundredths because in previous grade levels, students began with modeling and exploring the meaning of whole and two-digit number multiplication. At this point, students need to continue multiplying and dividing multi-digit numbers to make the connections between whole numbers and decimal numbers.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.B.5: This standard provides a foundation for work with performing operations with multi-digit whole numbers and with decimals to hundredths because students start applying the property of operations as strategies to multiply and divide by using the commutative property of multiplication, associative property of multiplication, and distributive property.. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with performing operations with multi-digit whole numbers and decimals to hundredths benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as applying previous experiences using models, strategies, place value, and problem context in multiplication and division operations because students will be able to add and subtract whole numbers fluently, convert multiplication contexts to efficient algorithm, explore division examples to find efficient procedures for division and apply the understanding of whole numbers to using decimals in performing operations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with performing operations with multi-digit whole numbers and decimals to hundredths benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because students must be able to understand the solving of multiplication and division in order to continue the scaffolding of decimals . It is important to obtain feedback in order to correctly guide students to the procedures of each operation, how to correctly solve them and apply that understanding to using decimals. For example, knowing that if they multiply whole numbers the answer will be a whole number. However, when they multiply tenths by tenths, the answer will be in the hundredths.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*.

- For example, learners engaging with performing operations with multi-digit whole numbers and decimals to hundredths benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students will have the opportunity to make explicit connections from concrete and pictorial models to solving written equations, by using estimation, models, and place value structure.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with performing operations with multi-digit whole numbers and decimals to hundredths benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students will be able to use different models and tools such as area models, number line, and partial products to connect conceptual understanding to procedural skills. Students will also use the structure of mathematics, including the use of place value and properties to use efficiency algorithm.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with performing operations with multi-digit whole numbers and decimals to hundredths benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because students need to use previous knowledge of multiplication and division in order to understand the use of decimals.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on performing operations with multi-digit whole numbers and with decimals to hundredths by revisiting student thinking through a short mini-lesson because it is important to ensure students are comprehending the relationship between multiplication and division with decimal numbers. In the same way, students will be encouraged to explain their thinking about a specific problem.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit performing operations with multi-digit whole numbers and with decimals to hundredths by confronting student misconceptions because students need to understand the importance of place value, regrouping, and remainders when solving operations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying performing operations with multi-digit whole numbers and with decimals to hundredths because students could continue with the division algorithm which is exposed in sixth grade.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying to perform operations with multi-digit whole numbers and with decimals to hundredths the use of mathematical representations within the classroom is critical because students' affirmation and validations of home language and culture is used by allowing them to use different representations for effective algorithm form. They can use models, strategies, place value, problem contexts, area models, number lines, and partial products to solve whole number problems and make the connection to decimal numbers.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia

A math contest will be held at Riverdale School. The costs for the math contest are shown in this table.

Type of Costs	Cost
Copy of the test	$ 1.00 for each student
Snack	$ 2.00 for each person
Lunch	$ 4.00 for each person
Required fee	$75.00

Each team has 5 students and 1 coach for a total of 6 people.

A. What is the total cost, in dollars, for the copies, snacks, and lunches for one team?

There are 12 teams coming to the math contest.

B. What is the total cost, in dollars, to the school for the math contest? Show your work or explain how you know.

Riverdale School's budget for the math contest is $800. Each team will bring one or more alternative students. The alternate students will each get a copy of the test, a snack, and a lunch. All teams must bring the same number of alternative students.

C. What is the greatest number of alternate students that each team can bring without going over the $800 budget? Show your work or explain how you know.

Answer Key

Constructed-Response Rubric	
Score	**Description**
4	5 points
3	4 points
2	2 or 3 points
1	1 point
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Scoring Notes

Part a: 1 point for correct answer, ($)**41**

Part b: 2 points for correct answer, ($)**567**, or correct answer based on work in previous part, with correct strategy shown

OR

1 point for correct answer with insufficient or no explanation or work shown

or

for correct strategy with incorrect or no answer

Part c: 2 points for correct answer, **2** (alternates), or correct answer based on work in previous parts, with correct strategy shown

OR

1 point for correct answer with insufficient or no explanation or work shown

or

for correct strategy with incorrect or no answer

or

for concluding that no alternates can be added, based on an incorrect part b that has a total that would not allow for alternates

Sample Response

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a. 6 x 4 + 6 x 2 + 5 = 41 for each team. b. \$41.00 × 12 = \$492 for person costs. Then add \$75 for the required fee. The total is \$567. c. \$800 – \$567 = \$233. 1 + 2 + 4 = \$7. Each student costs \$7. \$7 × 12 = \$84. \$84 for 1 alternate per team. 233 ÷ 84 = $2\frac{65}{84}$. So, only 2 alternates per team.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on operations with multi-digit whole numbers and with decimals to hundredths, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about money and how it breaks down into decimals when paying for something. Making a grocery list and adding up the total amount a person needs to pay and subtract that from a specific amount that will be paid to see what the change (difference) will be. Understanding that we use dollars in the form of whole numbers and cents in the form of decimal numbers.	STEM: Using given or collected data, round numbers to a given whole number or decimal place to solve real-world problems.

5.NF.A: NUMBERS AND OPERATIONS - FRACTIONS

Cluster Statement: Use equivalent fractions as a strategy to add and subtract fractions.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who Demonstrate Understanding Can:**
5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	SMP 4: Students can model with mathematics by using appropriate models including are models, fraction bars, and the number line will help students to develop efficient strategies and subtracting fractions and mixed numbers.	• Explain why fractions with unlike denominators need to be replaced with equivalent fractions with like denominators when adding or subtracting • Generate equivalent fractions to find the like denominator. • Solve addition and subtraction problems involving fractions (including mixed numbers) with like and unlike denominators using an equivalent fraction strategy.
		Depth of Knowledge: 1
		Bloom's Taxonomy: Apply

Standard Text	**Standard for Mathematical Practices**	**Students who Demonstrate Understanding Can:**
5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. Use equivalent fractions as a strategy to add and subtract fractions.*	SMP 1: Students can make sense of problems and persevere in solving them when developing their conceptual understanding of addition and subtraction of fractions using both mixed numbers and unlike denominators. SMP 2: Students can reason abstractly and quantitatively by assessing the reasonableness of answers.	• Assess the reasonableness of answers, using mental estimation. • Add and subtract fractions, including those with unlike denominators. • Solve word problems using addition and subtraction of fractions, including those with unlike denominators.
		Depth of Knowledge: 1
		Bloom's Taxonomy: Apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to comparing fractions with different denominators by creating common denominators. (4.NF.1,2) • Connect to adding and subtracting fractions with like denominators. (4.NF.3) • Connect to making a line plot to display a data set of measurements in fractions of a unit. (4. MD.4)	• Connect to making a line plot to display a data set and will add and subtract fractions of a unit to solve problems involving the information presented in the line plot. (5.MD.2)	• Connect to solving algebraic equations and real-world problems using rational numbers. (6.EE.7)

Clarification Statement:

5.NF.A.1: Builds on the work in fourth grade where students add fractions with like denominators. In fifth grade, the example provided in the standard 2⁄3 + 3⁄4 has students find a common denominator by finding the product of both denominators. This process should come after students have used visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm described in the standard. The use of these visual fraction models allows students to use reasonableness to find a common denominator prior to using the algorithm. Fifth grade students will need to express both fractions in terms of a new denominator with adding unlike denominators.

5.NF.A.2: This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as 7⁄8 is greater than 3⁄4 because 7⁄8 is missing only 1⁄8 and 3⁄4 is missing 1⁄4 so 7⁄8 is closer to a whole. Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. Example here such as 5⁄8 is greater than 6⁄10 because 5⁄8 is 1⁄8 larger than 1⁄2 (4⁄8) and 6⁄10 is only 1⁄10 larger than 1⁄2 (5⁄10).

Common Misconceptions

- Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and same size.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *Pre-teach Targeted: What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the use of equivalent fractions as a strategy to add and subtract fractions because "to add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them."

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size: This standard provides a foundation for work with the use of equivalent fractions as a strategy to add and subtract fractions because students need to understand what an equivalent fraction is, in order to understand why it is important to first create equivalent fractions when adding and subtracting fractions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with the use of equivalent fractions as a strategy for adding and subtracting fractions benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives that allow students to connect a visual representation to the procedure because allowing students to use multiple forms of representation will allow students to connect the concrete model to the representational drawing to the abstract number, equation, and procedure. As such, multiple forms of representation allow for alternative learner responses and alternative ways of navigating through a problem-solving experience. In this case, concrete and representational models are crucial to the learner’s foundational understanding of how the model and procedure are connected to the creation of equivalent fractions as a strategy to add and subtract fractions.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with the use of equivalent fractions as a strategy for adding and subtracting fractions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students will have multiple ways of representing the problems mathematically. By engaging in peer interactions students are able to make sense of and understand multiple representations that lead to the understanding of the procedure. Students who are struggling to create a model can gain support from students who have a deeper understanding of creating equivalent fractions and using them to add and subtract fractions. Students who are already proficient in creating models can gain understanding of other forms of representation and ways of thinking.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with the use of equivalent fractions as a strategy for adding and subtracting fractions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (the procedure) with an alternative form (a tape diagram, drawing, physical or virtual manipulative) because students must understand what the procedure is doing in order to fully understand why it works. Students must be able to visually see the creation of equivalent fractions in order to understand the procedure involved in creating equivalent fractions. They must also be able to see why we need to create equivalent fractions before adding and subtracting fractions in order to understand the importance of that step of the procedure.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with the use of equivalent fractions as a strategy for adding and subtracting fraction benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because students' understanding will vary based on their experiences. Some students may be able to make quick connections to the concepts and others may struggle to make a connection between the concrete (model/representation) and the abstract (procedure). Differentiated feedback is important to ensure students are validated in their thinking, and misconceptions are addressed before becoming solidified in their thinking.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with the use of equivalent fractions as a strategy for adding and subtracting fraction benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing scaffolds that connect new information to prior knowledge because students have prior knowledge of adding and subtracting fractions with common denominators. The teacher can use this as an entry point into the new concepts of creating equivalent fractions as a strategy to add and subtract fractions. Students also have experience creating equivalent fractions and modeling with fractions. These skills can be combined to aid in the understanding of new concepts.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on the use of equivalent fractions as a strategy to add and subtract fractions by revisiting student thinking through a short mini-lesson because "students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions."

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on the use of equivalent fractions as a strategy to add and subtract fractions by addressing conceptual understanding because students need to understand what the procedure is doing in order to develop fluency and proficiency with the procedure for using equivalent fractions as a strategy for adding and subtracting fractions. Some students may need practice representing fractions visually or physically before understanding the idea of equivalent fractions and why they are needed when adding and subtracting fractions.

Extension Ideas

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying the use of equivalent fractions as a strategy to add and subtract fractions because some students may understand the concepts quickly and easily. These students will not benefit from the continued creation of models, if they already understand the reasoning behind the procedure. Allow these students to communicate their thinking through images, concepts, facts, language and procedures (ICFLP Dr. Lorenzo Gonzales). Expose these students to more complex problems involving mixed numbers, fractions with denominators that are not compatible, and problems that require changing both fractions. Allow these students to explore and create procedures for creating equivalent fractions as a strategy to add and subtract fractions. Encourage these students to explain their thinking, test hypothesis, and modify procedures as necessary. Valid their thinking and address any misconceptions that arise quickly.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying the use of creating equivalent fractions as a strategy to add and subtract fractions the types of mathematical tasks are critical because students must connect their mathematical models to the development of procedures used to add and subtract fractions with unlike denominators, in order to fully understand the concepts that make up the procedure. Students can create models that represent items they see and interact with daily. From those models, students can connect the procedural routine of creating equivalent fractions as a strategy to add and subtract fractions. By using objects that students are familiar with, the teacher can build fluency with a connection to procedural understanding. Students who understand the reason behind the procedure are more likely to build fluency and precision when using the procedure involved in creating equivalent fractions as a strategy to add and subtract fractions.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: PARCC Released Item 2018
Part A

Robin and Josie shared a bottle of green paint for an art project. Robin used $\frac{3}{5}$ of the bottle of green paint. Together they used $\frac{17}{20}$ of the bottle of green paint.

What fractional part of the bottle of green paint did Josie use? Write your answer as a fraction.
Part B

Josie chose a bottle of red paint with some paint missing. During art class, she used $\frac{1}{5}$ of the whole bottle of red paint. At the end of class, $\frac{2}{3}$ of the whole bottle of red paint was left.

What fractional part of red paint was in the bottle at the beginning of the art class? Write your answer as a fraction.

Answer Key

Part A: $\frac{1}{4}$ (or equivalent fraction) Part B: $\frac{13}{15}$ (or equivalent fraction)	
Relevance to families and communities: During a unit focused on the use of creating equivalent fractions as a strategy to add and subtract fractions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, connect fraction addition and subtraction to cooking within the home. Students may also be familiar with carpentry and may be able to connect this mathematical concept to this task seen within the home and/or community. By allowing students to interact with fractions on a personal level, students see the relevance to their everyday lives and can connect with the mathematical concepts.	**Cross-Curricular Connections:** STEM: Students add fractions from given or collected data to find the total.

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5.NF.B: NUMBERS & OPERATIONS - FRACTIONS

Cluster Statement: Apply and extend previous understandings of multiplication and division.

Major Cluster This standard represents major work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.

Standard Text	**Standards of Mathematical Practice**	**Students who Demonstrate Understanding Can:**
5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	SMP 2: Students can reason abstractly and quantitatively to solve and write problems that include dividing a fraction by a whole number and a whole number by a fraction using models and verbal explanations. It is important for students to understand the meaning of the "remainder" when it is expressed as a fraction, which means the fraction is the part of the whole that it is left over.	• Interpret a fraction as division of the numerator by the denominator. • Interpret the remainder as a fractional part of the problem. • Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Apply
Standard Text	**Standards of Mathematical Practice**	**Students who Demonstrate Understanding Can:**

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. • 5.NF.B.4.a. Interpret the product (a/b) × q as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* • 5.NF.B.4.b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.	SMP 8: Students look for and express regularity in repeated reasoning by identifying patterns that develop when solving multiplication problems involving fractions.	• Extend previous understandings of multiplication to multiply a fraction or a whole number by a fraction. • Explain that the product (a/b) x q is the same as a x q ÷ b. • Multiply a fraction or a whole number by a fraction. • Create a story context to multiply a fraction or a whole number by a fraction. • Explain that finding the area of a rectangle with fractional side lengths by filling with tiles is the same as would be found by multiplying the side lengths. • Find the area of a rectangle by tiling it with unit squares. • Multiply fractional side lengths to find the area of a rectangle. **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Apply
Standard Text	**Standards of Mathematical Practice**	**Students who Demonstrate Understanding Can:**
5.NF.B.5 Interpret multiplication as scaling (resizing), by: • 5.NF.B.5.a: Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. • 5.NF.B.5.b: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole	SMP 2: Students can reason abstractly and quantitatively by reasoning that multiplying by a fraction less than 1 results in a smaller value.	• Interpret multiplication by scaling, comparing the size of a product to the size of one factor based on the size of the other factor. • Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number and why multiplying a given number by a fraction less than one results in a product smaller than the given number

numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1		**Depth of Knowledge:** 1, 2, 3 **Bloom's Taxonomy:** Apply, Analyze
Standard Text **5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.**	**Standards of Mathematical Practice** SMP 2: Students can reason abstractly and quantitatively. by applying their understanding of multiplication of fractions to contextualize and decontextualize real world problems.	**Students who Demonstrate Understanding Can:** • Represent word problems involving multiplication of fractions and mixed numbers. • Solve real world problems involving multiplication of fractions and mixed numbers **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Apply
Standard Text **5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.** 5.NF.B.7.a: Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* 5.NF.B.7.b: Interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for 4*	**Standards of Mathematical Practice** SMP 1: Students make sense of problems and persevere in solving them by using multiplicative reasoning and transfer their understanding of multiplication to division by whole numbers and fractions. SMP 3: Students can construct viable arguments and critique the reasoning of others by engaging in debates to defend their problem-solving reasoning to deepen their understanding of the relationship between division of unit fractions by whole numbers and whole numbers by unit fractions.	**Students who Demonstrate Can:** • Know the relationship between multiplication and division. • Interpret division of a unit fraction by a whole number and justify your answer using the relationship between multiplication and division, by creating story problems, using visual models, and relationship to multiplication. • Interpret division of a whole number by a unit fraction and justify your answer using the relationship between multiplication and division, and by representing the quotient with a visual fraction model. • Solve real world problems involving division of unit fractions by whole numbers other than 0 and division of whole numbers by unit

÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. 5.NF.B.7.c: Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* *Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of fraction by a fraction is not a requirement at this grade.*		fractions using strategies such as visual fraction models and equations. **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Apply
Previous Learning Connections • Connect to understanding concepts of area and relate area to multiplication and to addition. (3.MD.7) • Connect to using the four operations with whole numbers to solve problems. (4.OA.1,2,3) • Connect to understanding the concept of equivalent fractions by using visual fraction models. (4.NF.1) • Connect to multiplying a fraction by a whole number. (4.NF.4)	**Current Learning Connections** • Connect to understanding of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. (5.NBT.5,6,7) • Connect to knowledge of writing simple expressions to solve real problems with fraction. They will also interpret expressions without actually evaluating them. (5.OA.2) • Connect to using operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems involving information presented in line plots. (5. MD.2)	**Future Learning Connections** • Connect to using ratios written as fractions and divide into decimal form 3 ÷ 4 = 3⁄4 = 0.75. (6.RP .1,3) • Connect to solving multiplication equations that include non-negative rational numbers. (6.EE.7) • Connect to multiplying and divide fractions by fractions. (6.NS.1)
Clarification Statement: 5.NF.B.3: Fifth grade students should connect fractions with division, understanding that 5 ÷ 3 = 5⁄3 * Students should explain this by working with their understanding of division as equal sharing. Students should also create story contexts to represent problems involving division of whole numbers. This standard calls for students to extend their work of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities.		

5.NF.B.4: Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction (e.g., 2 x (1⁄4) = 1⁄4 + 1⁄4).
This standard extends student's work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as 3⁄5 could be represented as 3 pieces that are each one-fifth (3 x (1⁄5)). These standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.
This standard extends students' work with area. In third grade students determine the area of rectangles and composite rectangles. In fourth grade students continue this work. The fifth-grade standard calls students to continue the process of covering (with tiles).
5.NF.B.5: These standards ask students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.
5.NF.B.6: This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.
5.NF.B.7: This is thefirst time that students are dividing with fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. The concept unit fraction is a fraction that has a one in the denominator. For example, the fraction 3⁄5 is 3 copies of the unit fraction 1⁄5. 1⁄5 + 1⁄5 + 1⁄5 = 3⁄5 = 1⁄5 x 3 or 3 x 1⁄5.
This standard asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.

Common Misconceptions

- Students may initially think that you can not divide a "smaller number" by a "bigger number" since this will be a new situation for them to consider.
- Students may believe that multiplication always results in a larger number.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted): *Pre-teach Targeted: What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying apply and extend previous understandings of multiplication and division to multiply and divide fractions because use of models to multiply a fraction by a whole number will help student connect to the meaning of whole number multiplication .

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.A.1. This standard provides a foundation for work with multiplying and dividing fractions because this standard has students represent and solve problems involving multiplication and division, conceptual models of understanding multiplication and division. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences include ways to recruit interest such as providing choices in their learning give an example such as the sequence or timing of task completion because students worked with concrete models for multiplying a fraction by a whole number in Grade 4. They will continue to extend this work to additional situations for multiplying a whole number by a fraction. They use area models to connect their understanding of multiplication of whole numbers to multiplication of fractions. Students will explore division of a whole number by a fraction and a fraction by a whole. They will model through visual models and contexts to make sense of what multiplication and division of fractions entails.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because Following many opportunities to model, explain, and solve problems, students use their experiences to recognize patterns and develop efficient strategies. .

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because students will make connections between division with fractions and multiplication with fractions using previous experiences with the relationship between multiplication and division of whole numbers .

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with apply and extend previous understandings of multiplication to multiply and divide fractions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because this will allow students to visualize situations and connect those experiences to writing multiplication and division equations, they can talk about the equations in terms of a missing factor or dividend and make generalizations. .

Internalize

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

- For example, learners engaging with apply and extend previous understandings of multiplication and division to multiply and divide fractions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as elevating the frequency of self-reflection and self-reinforcements because students will be able to support their learning through opportunities to share ideas, clarify their understanding, develop mathematical arguments, and make generalizations about multiplying and dividing fractions .

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on connecting multiplication and division of whole numbers to multiplication and division of fractions by giving students connected situations they can model by clarifying mathematical ideas and/or concepts through a short mini-lesson because exploration using various representations including concrete and pictorial models.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit Apply and extend previous understanding of multiplication and division to multiply and divide fractions by confronting student misconceptions because students may initially think that they cannot divide a "smaller number by a bigger number" since this will be a new situation for them to consider. It is important they understand this concept in a way that makes sense to them rather than be shown how to do it.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying Apply and extend previous understanding of multiplication and division to multiply and divide fractions because as students work with various models of multiplication and division of whole numbers, fractions, and mixed numbers, visual representations will help them understand the size of the product/quotient when they multiply/divide a fraction by a whole number, a whole number by a fraction, or a fraction by a fraction.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying Apply and extend previous understanding of multiplication and division to multiply and divide fractions the types of mathematical tasks are critical because students are demonstrating their conceptual understanding, procedural fluency, and problem solving and reasoning. Students use a variety of problem-solving situations to develop an understanding of multiplication and division of fractions.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia
Students made cards using paper.

- They made 9 cards from 38 pieces of paper.
- They used the same amount of paper for each card.

A. Write a fraction to show the amount of paper used for each card. Show your work or explain how you know.
The students sold their cards. They earned a total of $18. Four students divided the money evenly.
B. Between what two consecutive whole dollar amounts did each student earn? Show your work or explain how you know

Answer Key

Constructed-Response Rubric	
Score	**Description**
4	4 points
3	3 points
2	2 points
1	1 point
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Scoring Notes

Part a 2 points for correct answer, $\frac{38}{9}$ **(pieces of construction paper) or equivalent**, with sufficient work or explanation to indicate correct strategy

or

1 point for correct answer with insufficient or no explanation or work shown

or

for correct strategy with incorrect or no answer

Part b 2 points for correct answer, **between 4 and 5 (dollars)**, with sufficient work or explanation to indicate correct strategy

or

1 point for correct answer with insufficient or no explanation or work shown

or

for correct strategy with incorrect or no answer

Sample Response

a. Since each card used the same amount, I divided. Fractions are the same as division, so they used $\frac{38}{9}$ pieces of construction paper for each greeting card.

b. By using fraction models, division, or other methods, the students each received $4\frac{1}{2}$ dollars, which is between 4 and 5. For example, 18 divided by 4 is 4 with remainder 2.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on Apply and extend previous understandings of multiplication and division to multiply and divide fractions , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, students can create story contexts for multiplying and dividing fractions and include a visual model. For example, How much pie? After a family function, Emily has 3 equally sized pies and wants to divide them equally into eight equal portions to give to family members that want to take some home. How much pie does each family member receive?	Social Studies: Connect fractions to studies of geography including scaling graphs and cross-sections, changes in measure (population, GDP) Health: connect fractions to food sharing, cooking, serving portions, nutrition, medical doses, heart beats per minutes, steps per day. Present students with real-world problems using these topics.

NEW MEXICO Public Education Department

5.MD: MEASUREMENT & DATA		
Cluster Statement: A: Convert like measurement units within a given measurement system. **Supporting Cluster:** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text **5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.**	**Standard for Mathematical Practices** SMP 1: Students can make sense of problems and persevere in solving them by solving word problems involving customary and standard measurement conversions. SMP 2: Students can reason abstractly and quantitatively by making sense of the number of units in relationship to the size of the unit when converting. SMP 7: Students can look for and make use of structure by discovering the relationship of base-ten conversions within the metric system.	**Students who Demonstrate Understanding Can:** • Recognize units of measurement within the same system. • Convert units of measurement within the same system by multiplying or dividing • Solve multi-step, real world problems that involve converting units. **Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand and Apply
<u>Previous Learning Connections</u> • Connect to understanding the relative sizes of measurement units within a system. **(4.MD.1)** • Connect to using the four operations to solve word problems including problems involving fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. **(4.MD.2)**	**<u>Current Learning Connections</u>** • Connect to the powers of 10, which relates to converting metric measurements. **(5.NBT.2)** • Connect to working to perform operations with multi-digit whole numbers and with decimals to hundredths. **(5.NBT.5-7)**	**<u>Future Learning Connections</u>** • Connect to using ratios to convert measurement units. Connect to manipulating and transforming units appropriately when multiplying or dividing quantities. **(6.RP.3d)**
Clarification Statement: 5.MD.A.1: In Grade 5, students extend their abilities from Grade 4 (4.MD.A.1) to express measurements in larger or smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and make connections between fractions and decimals (e.g., 2 1/2 meters can be expressed as 2.5 meters or 250 centimeters).		
Common Misconceptions • Students may not pay attention to the units of measurement and try to perform operations without converting to a common unit first. • Students may overgeneralize the base-10 structure and applying it to measurement conversions, such as when subtracting 4 inches from 3 feet, taking one foot from the 3 feet and regrouping it as 10 inches.		

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying conversion of like measurement units within a given measurement system, because in this cluster students begin with using a table to make conversions. They will convert both customary and standard measurements within the same system of measurement and solve multistep word problems. 5th graders will discover base 10 conversions within the metric system, 1 kilometer= 1,000 meters.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.C.7: This standard provides a foundation for work with conversion of like measurement units within a given measurement system, because students multiply and divide within 100. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with conversion of like measurement units within a given measurement benefit when learning experiences include ways to recruit interest such as, providing novel and relevant problems to make sense of complex ideas in creative ways, because this promotes student practice with the use of conversions in solving multistep, real world problems. Begin the problem solving with simple problems that focus on renaming units to represent the solution before experiencing problems that require renaming to find the solution.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with conversion of like measurement units within a given measurement benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as, encouraging and supporting opportunities for peer interactions and supports, because these interactions will allow students to interpret and make sense of the word problems they solve using customary and standard measurement conversions. Students will make sense of the number of units in relation to the size of the unit when converting and discover the relationship of base ten conversions within the metric system.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with conversion of like measurement units within a given measurement benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as. pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge, because students will apply academic vocabulary associated with the metric system when explaining their mathematical reasoning about measurement tools and the ideas they are learning, (convert, conversion, metric, customary unit, relative size, liquid, mass, volume…).

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with conversion of like measurement units within a given measurement benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as, providing multiple examples of novel solutions to authentic problems, because students will need practice and experience solving real world problems involving conversions of metric and customary units.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with conversion of like measurement units within a given measurement benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as, providing options for organizing and possible approaches (tables and representations for processing mathematical operations) because this will help students to convert measurements into larger or smaller units within a measurement system by reinforcing place value for whole numbers and decimals and then focus on the connection between fractions and decimals.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on conversion of like measurement units within a given measurement system by clarifying mathematical ideas and/or concepts through a short mini-lesson because focus should be on how to convert measurements into larger or smaller units within a measurement system by reinforcing place value for whole numbers and decimals.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on conversion of like measurement units within a given measurement system by confronting student misconceptions because some students may not pay attention to the unit of measurement when subtracting. For example, when subtracting 5 inches from 2 feet (2ft-5in), students may incorrectly think the answer is 1 ft. 5 inches instead of 1 foot and 7 inches. To address this misconception, talk about and show the example of using 2 twelve-inch rulers, then subtract.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying the conversion of like measurement units within a given measurement system because it promotes student practice to solve real world problems involving conversions, use the vocabulary associated with the metric and customary conversions, and gain understanding on the relationship between units and how to do conversions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying conversion of like measurement units within the given measurement system the use of mathematical representations within the classroom is critical because students' knowledge and experiences will be used as resources for mathematical learning. Students will utilize their experience with conversions while using tools such as conversion charts and models of base ten conversions within the metric system. Students will discover the relationship between base ten conversions within the metric system to make connections to their background knowledge. They will use this experience to make sense of word problems they solve using customary and standard measurement conversions.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: PARCC Released Item 2017

Tanya buys 12 water bottles. Of these bottles, 5 hold 300 milliliters each and 7 hold 1.5 liters each.

Part A

How much water, in milliliters, does Tanya buy?

Part B

How much water, in liters, does Tanya buy?

Answer Key

Part A: 12,000 Part B: 12

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on conversion of like measurements within a given measurement system, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can measure the length of three items at home and express the length in standard and customary units.	Science: In fifth grade the NGSS recommends students work with measurement related to conservation of mass. Consider providing a connection for students to determine the mass of an object in different states in two different units and then convert one unit unto the other to discover that they are equivalent. Art: Making a model of an object involves having to convert from larger to small units. Consider providing a connection for students to make a scaled model of something involving simple polygons or polyhedrons.

NEW MEXICO
Public Education Department

5.MD: MEASUREMENT & DATA

Cluster Statement: B: Represent and interpret data.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who Demonstrate Understanding Can:
5.MD.B.2: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	SMP 1: Students can make sense of problems and persevere in solving them by interpreting and making sense of word problems involving information presented in line plots. SMP 2: Students can reason abstractly and quantitatively by attending to the meaning of the measured objects and plots on the number line by using operations involving fractions. SMP 5: Students can use tools by measuring objects to the nearest 1/8, 1/4, and 1/2 inch using a ruler.	• Identify benchmark fractions. • Make a line plot to display a data set of measurements in fractions of a unit. • Solve problems involving information presented in line plots which use fractions of a unit by adding, subtracting, multiplying, and dividing fractions. **Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember and Apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to generating data by measuring lengths and making line plots using that data. **(3.MD.4)** • Connect to solving addition and subtraction problems using the data on line plots. **(4.MD.4)**	• Connect to growing in their skill and understanding of fraction arithmetic. **(5.NF)**	• Connect to displaying numerical data in plots on number lines, dot plots, histograms, and boxplots and choosing the most appropriate graph/plot for the data. **(6.SP.4)**

Clarification Statement:
5.MD.B.2: Grade 5 students grow in their skill and understanding of fraction arithmetic, including multiplying a fraction by a fraction, dividing a unit fraction by a whole number or a whole number by a unit fraction, and adding and subtracting fractions with unlike denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. For example, given five graduated cylinders with different measures of liquid in each, students might find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally. (Students in Grade 6 will view the answer to this question as the mean value for the data set in questions.)

Common Misconceptions
- Students may confuse various parts of the graph. Consider showing graphs that are incorrectly displayed and discuss why they are incorrect.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying representation and interpretation of data because this cluster focuses on solving problems using line plots created to display measurement data in fractions of a unit.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.MD.B.4: This standard provides a foundation for work with representation and interpretation of data because students begin to make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and solve problems involving addition and subtraction of fractions by using information presented in line plots. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with representation and interpretation of data benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because experience with what a line plot is, how the data was gathered, and how to read and interpret the data.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representation and interpretation of data benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because this will allow data and line plots to have some relevance to students and allow them to make connections to the standard.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with representation and interpretation of data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students will use appropriate vocabulary when working with line plots and fractional measurements.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representation and interpretation of data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of novel solutions to authentic problems because this will allow students opportunities to solve problems using operations on fractions from information presented in line plot.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with representation and interpretation of data benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because it will help students to build their abilities to construct a line plot with information gathered and display, analyze, and interpret their data.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representation and interpretation of da ta by clarifying mathematical ideas and/or concepts through a short mini-lesson because students are building their experience in measuring objects to one-eighth of a unit, constructing a line plot with information gathered, and display, analyze, and interpret their own line plot.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on representation and interpretation of data by helping students move from specific answers to generalizations for certain types of problems because some students may not know what measurement to use if the object measures between ⅛ and ¼ inch. To address this, help students understand that approximations can be used to measure to the closest ⅛ inch and ¼ inch.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying representation and interpretation of data because data is more meaningful to students if it is their own project or idea; students create their own data, measure objects to the nearest ⅛ inch, construct line plot, and display, analyze, and interpret their line plot to draw conclusions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. “Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence.” Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or “warm-demander” requires a strong relationship with students and an understanding of the culture of the students. For example, when studying representation and interpretation of data supporting productive struggle is critical because students will need to make sense of measured objects and plots on a number line to solve everyday problems. Students will use reasoning and connections to their background to display, interpret, and analyze their own line plots.

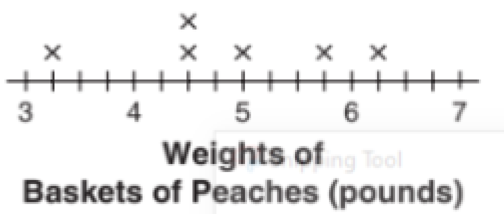
Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia

The owner of a produce stand recorded the weights, in pounds, of the baskets of peaches for sale.

$3\frac{1}{4}, 5\frac{3}{4}, 6\frac{1}{8}, 4\frac{1}{2}, 5, 3\frac{1}{8}, 6\frac{1}{4}$

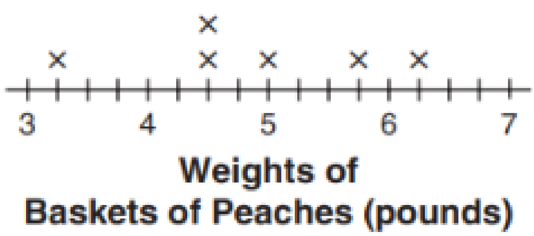
The owner made this line plot of data.



A. Why are the data on the line plot incorrect? Show your work or explain your answer.

The owner used the incorrect data from this line plot to find the total weight of the peaches sold.

$3\frac{1}{4}, 5\frac{3}{4}, 6\frac{1}{8}, 4\frac{1}{2}, 5, 3\frac{1}{8}, 6\frac{1}{4}$



B. By how many pounds would the total weight of the peaches change if he used the correct data from the list?

Answer Key

Constructed-Response Rubric	
Score	**Description**
2	for correct answers to part a., **The owner did not include all the data ($3\frac{1}{8}$ and $6\frac{1}{8}$)** and part b., **$9\frac{1}{4}$ or equivalent**
1	for correct answer to one part
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Sample Response

a. The line plot is the wrong scale. The number line needs to be divided in eighths. The owner left out the data that had eighths.

b. $9\frac{1}{4}$

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on representation and interpretation of data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, students can measure their families' hand to the nearest ⅛ of an inch, construct a line plot with the information gathered and display, analyze, and interpret their family line plot.	Science: In fifth grade the NGSS recommends students work with measurement related to conservation of mass. Consider providing a connection for students to determine the mass of various object in different states in that measure in fractional units. Then have students graph and analyze that data. Social Studies: In fifth grade the New Mexico Social Studies Standards state students should "gather, organize and interpret information using a variety of media and technology". Consider having students gather, graph and analyze data that contains measurements in fractions of a unit.

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<table>
<tr><th colspan="3">5.MD: MEASUREMENT & DATA</th></tr>
<tr><td colspan="3">Cluster Statement: C: Geometric measurement: understand concepts of volume.

Major Cluster This standard represents major work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.</td></tr>
<tr><td rowspan="3">Standard Text

5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
• 5.MD.C.3.A: A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
• 5.MD.C.3.B: A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.</td><td rowspan="3">Standard for Mathematical Practices

SMP 6: Students can attend to precision by using specific vocabulary to describe the dimensions when measuring volume.

SMP 7: Students can look for and make use of structure by using their knowledge of the mathematical structure of area and applying that knowledge to volume.</td><td>Students who Demonstrate Understanding Can:
• Explain that volume is the measurement of the space inside a solid three-dimensional figure.
• Explain that a unit cube has 1 cubic unit of volume and is used to measure volume of three-dimensional shapes.
• Explain that any solid figure packed without gaps or overlaps and filled with n unit cubes indicates the total cubic units or volume.</td></tr>
<tr><td>Webb’s Depth of Knowledge: 1</td></tr>
<tr><td>Bloom’s Taxonomy: Remember</td></tr>
<tr><td rowspan="3">Standard Text

5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.</td><td rowspan="3">Standard for Mathematical Practices

SMP 4: Students can model with mathematics by finding the volume of a rectangular prism by counting unit cubes, using cubic cm, cubic in., and cubic ft.

SMP 5: Students can use tools by using manipulatives to build cubes and rectangular prisms without gaps or overlaps and discover the formula for volume of a rectangular prism.

SMP 6: Students can attend to precision by using specific vocabulary to describe the dimensions when measuring volume.</td><td>Students who demonstrate understanding can:
• Measure volume by counting unit cubes, cubic cm, cubic in, cubic ft, and improvised units.</td></tr>
<tr><td>Webb’s Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom’s Taxonomy: Understand and Apply</td></tr>
</table>

<table>
<tr>
<td rowspan="3">Standard Text

5.MD.C.5: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

• 5.MD.C.5.A: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
• 5.MD.C.5.B: Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
• 5.MD.C.5.C: Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by solving real-world and mathematical problems involving volume.

SMP 4: Students can model with mathematics by applying the formula $V=l \times w \times h$ and $V= B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths.

SMP 7: Students can look for and make use of structure by recognizing volume as additive.</td>
<td>Students who Demonstrate Understanding Can:
• Identify a right rectangular prism.
• Multiply the three dimensions in any order to calculate volume (Commutative and Associative properties).
• Recognize that “B” refers to the area of the base.
• Recognize volume as additive.
• Develop volume formula for a rectangle prism by comparing volume when filled with cubes to volume by multiplying the height by the area of the base, or when multiplying the edge lengths (l x w x h).
• Apply the following formulas to right rectangular prisms having whole number edge lengths in the context of real-world mathematical problems: Volume = length x width x height or Volume = area of base x height.
• Solve real world problems by decomposing a solid figure into two non-overlapping right rectangular prisms and adding their volumes.
• Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes.</td>
</tr>
<tr>
<td>Depth of Knowledge: 2-3</td>
</tr>
<tr>
<td>Bloom’s Taxonomy:
Understand, Apply and Analyze</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
• Connect to creating 3-D shapes. (1.G.2b)
• Connect to learning to measure area using unit squares (3.MD.6)
• Connect to applying the formulas to determine area and perimeter of rectangles. (4.MD.3)</td>
<td><u>Current Learning Connections</u>
• Connect to fluently multiplying multi-digit whole numbers. (5.NBT.5)</td>
<td><u>Future Learning Connections</u>
• Connect to finding the volume of right rectangular prisms with fractional dimensions in the context of solving real-world and mathematical problems. (6.G.2)</td>
</tr>
</table>

Clarification Statement:
5.MD.C.3: "Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube.

5.MD.C.4: They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build.

5.MD.C.5: Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism.

Common Misconceptions
- Students might try to measure volume with square or linear units.
- Students may labele volume with the wrong unit or read the shorthand for volume as 32 feet cubed rather than accurately reading it as 32 cubic feet.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach
Pre-teach (targeted): *Pre-teach Targeted: What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*
- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying understanding of volume concepts because "this is the first time that students begin exploring the concept of volume. In previous grades students worked with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of a cube) with a layer of unit cubes and then adding layers of unit cubes on top of the bottom layer. Students should have ample experiences with concrete manipulatives before moving to pictorial representations."[1] Students will then derive the formula for calculating volume from their concrete understanding based on model representations.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*
- 3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement: This standard provides a foundation for work with understanding volume concepts because students use their understanding of area to make sense of volume. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction
Access
Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*
- For example, learners engaging with understanding concepts of volume benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as concrete (physical models) and abstract (formulas or drawings) representations of volume. Students are able to understand volume when represented with concrete, physical models in which they touch and manipulate, to develop an understanding of what volume represents at a basic level. Students begin by counting cubes that represent the volume of a solid, rectangular prism. They then move into modeling with cubes to show their understanding of volume. Once this understanding is established, students can create, or apply a formula to represent their thinking. Drawings and graphs can be used in addition to concrete models to represent understanding of volume because students need a concrete, visual

[1] http://www.dusd.net/cgi/files/2013/04/5th-flipbook.pdf

representation of volume in order to relate the concept to a formula. Students may be able to understand volume when related to a physical model but may struggle to understand the concept of the formula for calculating volume without a visual/physical representation of the concept to attach the formula to.

Build

Effort and Persistence*: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding of volume benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because students may have trouble understanding the concept of volume without a visual representation. The concept of volume is difficult for students to understand when only given the formula for calculating volume. Multiple representations of how to physically manipulate volume will help students gain the understanding necessary for working with the concept of volume in a concrete way, in the beginning, moving to a more abstract understanding as students gain more experience and understanding of concepts associated with volume.

Language and Symbols*: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding concepts of volume benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students have experience with concepts of area and the area model, they are able build upon this understanding to relate to the idea of volume. Students apply previously learned concepts to develop an understanding of a new more complex concept.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding concepts of volume benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks and 3D models) because students must understand the concept of volume at a concrete/physical level before being able to think of volume in an abstract way. This understanding supports the use of formulas to quickly compute volume.

Internalize

Executive Functions*: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with understanding of volume concepts benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing models or examples of the process and product of goal-setting because interacting with concepts of volume physically (creating models to represent understanding) supports understanding of concepts and formulas. Allowing students the opportunity to make a plan, carry out the plan, and reflect on its effectiveness teaches students higher level thinking skills required for complex problem solving. Using models as tools to represent volume will provide students with a deep understanding of what volume is. This will help students solidify these concepts in their nonconscious, in order to develop higher level thinking skills involving volume. This will support students' understanding of more complex volume concepts introduced in 6th grade, for example understanding volume involving fractional measurements. Students' experiences setting goals, creating plans, and reflecting will guide their learning and give them a foundation to build from when encountering problems that are harder to visualize.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding volume concepts by examining tasks from a different perspective through a short mini lesson because students can gain a better understanding of concepts by analyzing models created by other students. There are multiple ways models can be constructed and used to calculate volume. Students will gain a deeper understanding of volume by engaging with models created by other students. Give students the opportunity to analyze, engage and interact with multiple perspectives/models of representation. Allow students the time to explain their thinking and make connections between different methods of representation.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of understanding volume concepts by addressing conceptual understanding because students must understand what volume is before they can interact with real-world problems involving volume. Allow students time to deconstruct pre-made models in an attempt to understand that "volume is the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters"; hence the model is constructed of 3-dimensional cubes (measuring 1in. X 1in. X 1in. or 1 cm. X 1cm. X 1cm.).

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying understanding of volume concepts because they are able to grasp the concepts of volume easily. Allow these students to derive and use formulas for calculating volume based on their conceptual understanding of volume. These students will benefit from interacting with real-world problems involving volume in which they need to use a formula to solve. Allow students to show their thinking through images, concepts, facts, language, and procedures[2]. Expose students to questions that require them to calculate multiple numerical volumes and combine or decompose them, in order to arrive at a solution. Expose students to real-world mathematical problems that are connected to other discipline areas (e.g., science/social studies). Students may also benefit from problems that have multiple solutions based on the strategy the student applies.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Using and Connecting Mathematical Representations): The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding volume

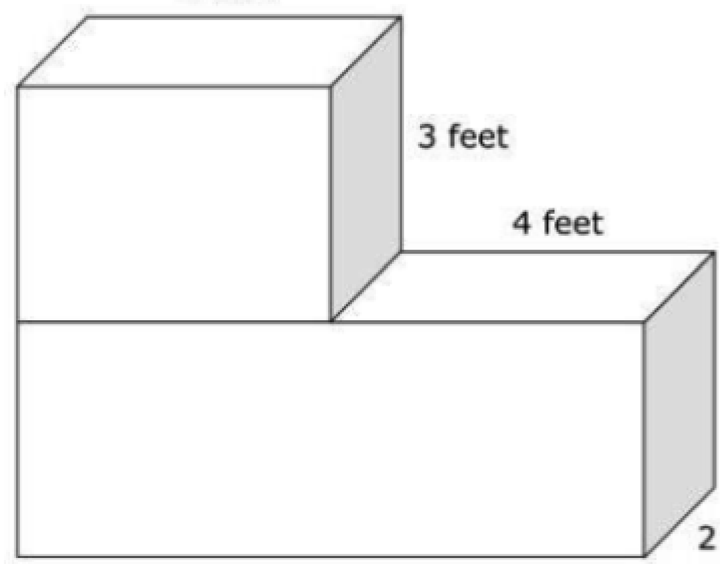
[2] ICFLP Dr. Lorenzo Gonzales (need to get full reference from 5th grade team)

concepts, the use of mathematical representations within the classroom is critical because students can use a variety of mathematical representations that they are already familiar with. This helps students connect to prior knowledge and allows them to use what they already know to connect to new concepts. Encourage students to use examples of things they see and experience in their everyday lives as mathematical representations of volume. Validate students' thinking, as they make connections to volume in the real-world and within their own environments. Encourage students to use multiple representations to show their mathematical thinking around volume and their everyday lives. Allowing students the time to share ideas, thoughts, and representations will give students an insight into the lives of other students.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: PARCC released item 2018

Cement was poured to make two rectangular prisms. The prisms were stacked, as shown.



Part A

What are the length, width, and height, in feet, of the smaller rectangular prism?

Part B

What is the total amount of cement, in cubic feet, used to make the two rectangular prisms?

Answer Key

Part A: 2, 3, 4 in any order Part B: 72

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on understanding volume concepts, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, comparing the volume of multiple structures/objects found at home, or in the community will help students gain a deeper understanding of volume. Reasoning about the capacity of different size rooms in the home or around the community may help students make a connection to the significance of volume. Connecting packing cubes into a rectangular prism	Science: In fifth grade the NGSS states students should "describe and graph quantities such as area and volume to address scientific questions." Consider providing a connection for students to determine the volume of cubes or rectangular prisms as part of their investigation. Art: Drawing boxes is connected to developing the ability to indicate perspective in a drawing. Consider providing an opportunity for students to sketch various boxes with the same volume but different dimensions. Also, consider allowing students to make boxes to pack inside of larger boxes (measuring 1in. X 1in. X 1in. or 1 cm. X 1cm. X 1cm.). Have students predict how many boxes can fit inside of the

and packing items into a storage shed, or packages into a mail delivery truck may help students connect schoolwork to real-world examples found within the home or community.	premade larger boxes. Connect the number of boxes used to the volume of the box. Allowing students to cut, and construct boxes will help with their fine motor skills.

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<table>
<tr><th colspan="3">5.G.A: GEOMETRY</th></tr>
<tr><td colspan="3">Cluster Statement: Graph points on the coordinate plane to solve real-world and mathematical problems.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

5.G.A.1 Graph points on the coordinate plane to solve real-world mathematical problems. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</td><td rowspan="3">Standards of Mathematical Practice

SMP 6: Students can attend to precision by using clear, specific directions when plotting points on the coordinate plane.</td><td>Students who Demonstrate Understanding Can:
Graph points in the first quadrant.
Interpret coordinate values of points in real world context and mathematical problems.
Represent real world and mathematical problems by graphing points in the first quadrant.</td></tr>
<tr><td>Depth of Knowledge: 1,2</td></tr>
<tr><td>Bloom's Taxonomy: Apply, Understand</td></tr>
<tr><td rowspan="3">Standard Text

5.G.A.2 Graph points on the coordinate plane to solve real-world mathematical problems. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.</td><td rowspan="3">Standards of Mathematical Practice

SMP 1: Students can make sense of problems and persevere in solving them when considering real-life situations that can be represented by the points they plot on a coordinate plane.

SMP 2: Students can reason abstractly and quantitatively by contextualizing graphed points.</td><td>Students who Demonstrate Can:
Graph points in the first quadrant.
Interpret coordinate values of points in real world context and mathematical problems.
Represent real world and mathematical problems by graphing points in the first quadrant.</td></tr>
<tr><td>Depth of Knowledge: 1</td></tr>
<tr><td>Bloom's Taxonomy: Apply</td></tr>
</table>

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to plotting points on a number line and constructed perpendicular lines. (4.G.1, 4. MD.4)	• Connect to forming ordered pairs from given rules and graph points on a coordinate plane. (5.OA.3)	• Connect to extending understanding of a coordinate plane to the negative number coordinates. (6.NS.6)

Clarification Statement:
5.G.A.1 and 5.G.A.2: These standards deal with only the first quadrant (positive numbers) in the coordinate plane. Although students can often "locate a point," these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: "right 2, up 3"; and as the point defined by being a distance 2 from the y-axis and a distance 3 from the x-axis. In these two descriptions the 2 is first associated with the x-axis, then with the y-axis.
5.G.A.2: This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms.

Common Misconceptions
- Students may think the order in plotting a coordinate point is not important.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying graphing points on the coordinate plane to solve real-world and mathematical problems because students must be able to understand domain specific vocabulary and should be able to access prior knowledge learned in previous grade levels.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.MD.B.4: This standard provides a foundation for work with graphing points on the coordinate plane to solve real-world and mathematical problems because students should be able to show data by making a line plot, where the horizontal scale is marked off in appropriate units. Without knowledge of the vocabulary and prior knowledge, the students will continue to struggle. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*
- For example, learners engaging with graph points on the coordinate plane to solve real-world and mathematical problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for all images, graphics,

video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because it is important to recognize that all students do not learn the same. Students can learn visually, aurally, verbally, physically, logically, socially, and solitary. Learning styles have more influence than you may realize. Your preferred styles guide the way you learn. They also change the way you internally represent experiences, the way you recall information, and even the words you choose.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with *g*raphing points on the coordinate plane to solve real-world and mathematical problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because providing feedback helps perseverant students understand the value of hard work, hone their problem-solving skills and take responsibility for their own academic progress. They do not make excuses or blame others for failure.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with *g*raphing points on the coordinate plane to solve real-world and mathematical problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf because students who are presented key information in their first languages (Spanish, ASL, etal) will generally gain a better understanding of the activity being completed. Students learn more by understanding in their primary language before learning it in their second language. By presenting in their primary language it allows for access to prior knowledge and learning experiences.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with *g*raphing points on the coordinate plane to solve real-world and mathematical problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs because students learn best when they are given step by step instructions. As soon as the students learn the skills or concepts, it is important to pull away any scaffolds in order to ensure that the students are learning the skills or concepts being taught.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with *g*raphing points on the coordinate plane to solve real-world and mathematical problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") because students need to change their mindset from a fixed mindset to a more growth mindset. Changing one's mindset will only change if the student is willing to approach their fears and do what it takes to tame these fears.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on graphing points on the coordinate plane to solve real-world and mathematical problems by clarifying mathematical ideas and/or concepts through a short mini-lesson because confusion by students of key domain specific vocabulary which can cause students to reverse the data being presented. Reteaching key domain specific vocabulary and any prior knowledge will present comprehension of the standard being taught.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit graphing points on the coordinate plane to solve real-world and mathematical problems by confronting student misconceptions because students will be able to identify the difference between horizontal and vertical and its association with the variables x and y on a coordinate grid. Students will be able to distinguish between Horizontal (lying flat) vs Vertical (standing tall) which are commonly reversed.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying graphing points on the coordinate plane to solve real-world and mathematical problems because students will be able to develop an understanding of why coordinate grids are listed as x-axis, y-axis) and explain in full detail what would happen if mixed around.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

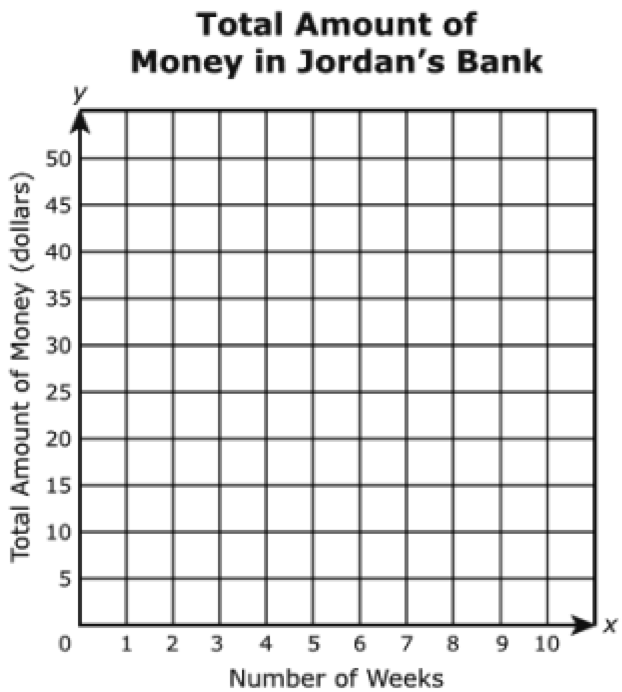
When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying graphing points on the coordinate plane to solve real-world and mathematical problems the types of mathematical tasks are critical because when students are given problems that they can relate to their everyday lives, they tend to develop a strong understanding of the concept or skill that is being taught. By allowing the students to have a part in developing the problem, it gives them ownership of the problem and it allows them to perform successfully on the task. The teacher should only provide the framework and allow the students to fill in the remaining information that is needed to complete the problem.

This allows the students to use their personal and real-life situations to create more meaningful tasks that will allow for more success.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: PARCC Released Item 2017

Jordan has $10 in the bank. Jordan earns $5 each week for doing chores and puts the money in the bank. After a certain number of weeks of doing chores, Jordan has $35. A graph is set up so Jordan can record the total amount of money in the bank after putting in $5.



Part A

Which ordered pair represents the amount of money Jordan has in the bank before doing any chores?

A. (0, 10)
B. (0, 35)
C. (10, 0)
D. (35, 0)

Part B

Which ordered pair represents the amount of money Jordan has after 4 weeks of doing chores?

A. (4, 20)
B. (4, 30)
C. (20, 4)
D. (30, 4)

Answer Key

Part A: A Part B: B

Relevance to Families and Communities:	**Cross-Curricular Connections:**
During a unit focused on graphing points on the coordinate plane to solve real-world and mathematical problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of	STEM: Plot on a coordinate system. For example: Plot stars, planets, moons, asteroids, and other celestial bodies on a diorama of the solar system. Plot stars of a constellation on a coordinate system. Identify the location of stars on a system map using ordered pairs.

school to create stronger home to school connections for students, for example, families and communities can create different charts and graphs to analyze various types of fundraiser sales to determine which items would be more efficient in selling during community events.	

5.G.B: GEOMETRY		
Cluster Statement: Classify two-dimensional figures into categories based on their properties. **Additional Cluster:** This standard represents additional work for this grade. As a reminder, 65-85% of instructional time over the course of the year should be focused on the major work of the grade.		
Standard Text **5. G.B.3 Classify two-dimensional figures into categories based on their properties. Understand** that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	**Standards of Mathematical Practice** **SMP 1: Make sense of problems and persevere in solving them.** - Students can become problem solvers as they reason about shapes to create a graphic organizer to classify shapes. Teacher must focus on helping students reason about attributes (properties) of shapes, including sides, angles, and symmetry. **SMP 6: Attend to precision.** - Students can use clear, specific language when discussing their reasoning about shapes. Focus on vocabulary associated with the properties of shapes, including attribute, category, subset/subcategory, properties, two-dimensional, polygon, rhombus, rectangle, square, triangle, quadrilateral, pentagon, hexagon, trapezoid, parallel, perpendicular, congruent angles, right angles, acute angles, obtuse angles, symmetry, line, and line segment. **SMP 8: Look for and make use of structure.** -Students can identify attributes to classify and create a graphic organizer to help them make sense of the hierarchy of shapes. Source: The Common Core Mathematics Companion: The Standards Decoded	**Students who Demonstrate Understanding Can:** • Recognize that some two-dimensional shapes can be classified into more than one category based on their attributes. • Recognize if a two-dimensional shape is classified into a category, that it belongs to all subcategories of that category. **Depth of Knowledge:** 1, 2, 3 **Bloom's Taxonomy:** Understand
Standard Text **5.G.B.4 Classify two-dimensional figures into categories based on**	**Standards of Mathematical Practice**	**Students who Demonstrate Can:** • Recognize the hierarchy of two-dimensional shapes based on their attributes.

their properties. Classify two-dimensional figures in a hierarchy based on properties.	**SMP 2: Reason abstractly and quantitatively.** Students use their understanding of shapes to defend their reasoning about shapes. For example, by asking how they might prove whether a square is *always* a rectangle and whether a rectangle is always a square. **SMP 3: Construct viable arguments and critique the reasoning of others.** Students engage in a debate to defend whether a parallelogram is also a trapezoid. Students use vocabulary to defend their thinking. Teachers can use the example in you tube *Constructing viable arguments-triangles or trapezoid.wmv* **SMP 7: Look for and make use of structure.** Students classify shapes by their features or characteristics, such as by the number of vertices, angle measurement, sets of parallel lines, whether they are polygons, etc. Students can notice a pattern that may be generalized as real-world problems are represented on the coordinate plane.	• Analyze properties of two-dimensional figures in order to place into a hierarchy. • Classify two-dimensional figures into categories and/or subcategories based on their attributes. **Depth of Knowledge:** 1, 2 **Bloom's Taxonomy:** Understand
Previous Learning Connections • Connect to learning that shapes in different categories share attributes. (2.G.1) • Connect to learning that shared attributes can define a larger category. For example, rectangles, squares, and rhombuses are all examples of quadrilaterals. (3.G.1) • Connect to classifying two-dimensional figures based on lines and angles. (4.G.2)	**Current Learning Connections**	**Future Learning Connections** • Connect to drawing shapes with given conditions. (7.G.2)

Clarification Statement:
5.G.B.3: This standard calls for students to reason about the attributes (properties) of shapes. Student should have experiences discussing the property of shapes and reasoning. The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do not appear until middle school.
5.G.B.4: This standard builds on what was done in 4th grade. Figures from previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle,

circle, kite. A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other. Student should be able to reason about the attributes of shapes by examining: What are ways to classify triangles? Why can't trapezoids and kites be classified as parallelograms? Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals, and How many lines of symmetry does a regular polygon have?
Note, in the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (Progressions for the CCSSM: Geometry, The Common Core Standards Writing Team, June 2012.)

Common Misconceptions

- Students may think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearsing new mathematical language when studying Classifying Two-Dimensional Figures Into Categories Based On Their Properties because this cluster is rich in mathematical vocabulary that may be confusing for some students.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 2.G.A.1: This standard provides a foundation for work with <mathematics of the assigned cluster> because its roots begin in 2nd grade where students recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: How will the learning for students provide a variety of methods for navigation to support access?

- For example, learners engaging with classifying two-dimensional figures into categories based on their properties benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as <varying methods for response and navigation by providing manipulatives because allowing the students to physically manipulate and sort the geometric shapes into categories and subcategories will deepen their understanding as they may need to touch and turn the shoes to help determine the properties.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with classifying two-dimensional figures into categories based on their properties benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because of the extensive amount of mathematics vocabulary in this cluster.

Students will benefit from working with a partner to talk through the reasoning behind their identification of the shapes and their attributes as they increase their vocabulary knowledge.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with classifying two-dimensional figures into categories based on their properties benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge and providing graphic symbols with alternative text descriptions because attribute descriptions may be confusing for some learners. It is recommended to create a series of anchor charts for students to refer to which differentiate and review the types of angles, types of lines, and categories of polygons.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with classifying two-dimensional figures into categories based on their properties benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because students may benefit from physically sorting shapes or shape cards into the various categories and subcategories as they discuss their reasoning with their peers.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with classifying two-dimensional figures into categories based on their properties benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on   Classifying Two-Dimensional Figures Into Categories Based On Their Properties by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may have a difficult time understanding that figures can belong to more than one category, based on their attributes. For example, squares also belong to the following categories: quadrilaterals, rectangles, and parallelograms.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit Classifying Two-Dimensional Figures Into Categories Based On Their Properties by helping students move from specific answers to generalizations because the more ways students can classify and reason about shapes, the better they will understand their properties. Lead students into answering questions like, "Why is a square always a triangle?" and "Why is a rectangle not always a square?".

Extension Ideas

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

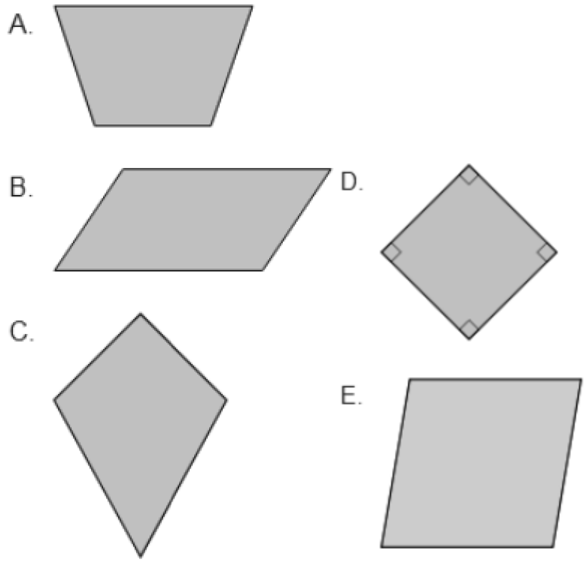
- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying Classifying Two-Dimensional Figures Into Categories Based On Their Properties because students can extend thinking using graphic organizers, such as flow charts or T-charts, to compare and contrast the attributes of geometric figures. (Students need not be limited to quadrilaterals).

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying classifying two-dimensional figures into categories based on their properties facilitating meaningful mathematical discourse is critical because this cluster requires students to reason about the attributes of shapes. Students need ample opportunity to discuss with peers the properties and attributes of shapes to develop understanding. Lead discussions asking students to not only talk about the properties of polygons, but also to reason about the attributes of each shape and how each shape should be classified. Students should also be able to explain why some shapes fit into subcategories.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: PARCC Released Item 2018
Which shapes are parallelograms but not rectangles?
Select **two** correct shapes



Answer Key: B and E

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on classifying two-dimensional figures into categories based on their properties, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, have your student identify different shapes in the home and community environment Ask your student to describe the shape based on the number of sides and angles and ask if they can tell you what categories(s) that shape fits into.	Art: Provide students with multiple colors and textures of paper. Have them work in groups to create collages based on the attributes of different shapes. Give students strips of paper that give examples of different shapes. Allow them to create their collages based on the attributes given. History and Architecture: Have students study the shapes of historical dwellings/buildings. Have students make connections to the building in their communities. Discuss why certain shapes may have been more fitting than others for various buildings. Have students describe the dwellings/building based on their attributes.

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ⍰ How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ⍰ What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ⍰ How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ⍰ What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ⍰ How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ⍰ What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding	
ENGAGEMENT ⍰ How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ⍰ What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols**: ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication**: ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: ? What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension**: ? What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ? How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ? What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* x 1/*a* = 1/*a* x *a* = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.