NEW MEXICO
Public Education Department

New Mexico Mathematics Instructional Scope for Sixth Grade

June 2020

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands
[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.
[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<table>
<tr><td colspan="3"><GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME></td></tr>
<tr><td colspan="3">Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.</td></tr>
<tr><td rowspan="3">Standard Text
Full text of the standard</td><td rowspan="3">Standard for Mathematical Practices
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.</td><td>Students who demonstrate understanding can: The cognitive skills students perform to demonstrate to comprehension of a standard.</td></tr>
<tr><td>Depth Of Knowledge: Correlation of standard to Webb's Depth of Knowledge</td></tr>
<tr><td>Bloom's Taxonomy: Correlation of standard to Bloom's Taxonomy</td></tr>
<tr><td>Connections to Previous Learning:
Supports student connections to learning from previous grade levels.</td><td>Connections to Current Learning
Supports student connections to learning within the grade level.</td><td>Connections to Future Learning
Supports student connections to learning in a future grade.</td></tr>
<tr><td colspan="3">Clarification Statement: Clarifies the language of the standard.</td></tr>
<tr><td colspan="3">Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.</td></tr>
<tr><td colspan="3">Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies
Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive) : Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.</td></tr>
<tr><td colspan="3">Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards</td></tr>
<tr><td colspan="3">Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.</td></tr>
<tr><td colspan="2">Relevance to Families and Communities:
Connecting with families and communities to create relevant connections between mathematics inside and outside of school.</td><td>Cross Curricular Connections: Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.</td></tr>
</table>

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

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6.RP: RATIO AND PROPORTIONAL RELATIONSHIPS

Cluster Statement: A: Understand ratio concepts and use ratio reasoning to solve problems.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	SMP 6: Students can attend to precision by communicating precisely with others and using clear mathematical language when describing a ratio relationship between quantities. SMP 8: Students look for and express regularity in repeated reasoning by identifying patterns of regularity in a ratio relationship that demonstrate the proportionality. They can extend the pattern to create additional equivalent ratios and rates in various forms, including an equation, a table, a graph, etc. They can use the patterns to discuss the ratio relationship, explain their strategy and/or solution to a given problem.	• Understand and explain that a ratio is a comparison of two quantities. • Describe what a ratio illustrates using ratio language. • Write a ratio relationship in the forms a:b, a to b, a/b. • Translate a ratio relationship into words. • Understand the differences between part:part and part:whole relationships. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand

<table>
<tr>
<td>Standard Text

6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.</td>
<td>Standard for Mathematical Practices

SMP 2: Students can reason abstractly and quantitatively by analyzing and comparing ratios and unit rates in tables, equations, and graphs from a variety of situations.

SMP 4: Students can model with mathematics by analyzing real-life ratio situations with mathematics and creating mathematical representation to model the situation.

SMP 6: Students can attend to precision by communicating precisely with others and use clear mathematical language when describing a ratio relationship between quantities.

SMP 7: Students can look for and make use of structure by making connections between covariance, rates, and representations showing the relationships between quantities.</td>
<td>Students who demonstrate understanding can:
• Define a unit rate in relation to the concept of a ratio.
• Calculate unit rates from scenarios.
• Read and hear contexts involving unit rates and interpret them.
• Represent units rates symbolically, in contexts, and through visuals.
• Use precise language of unit rate to describe ratio relationships both orally and in writing.

Webb’s Depth of Knowledge: 1-2

Bloom’s Taxonomy:
Understand, Apply</td>
</tr>
<tr>
<td>Standard Text

6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td>Standard for Mathematical Practices

SMP 2: Students reason abstractly and quantitatively by representing a wide variety of real-world contexts through the use of real numbers and variables in mathematical terms. Students consider the ratio relationship contained in the problem to understand the meaning of the number or variable. They use multiplicative reasoning when finding the missing element in a proportion. They will manipulate symbolic representations such as tables, tape diagrams, graphs, etc. by applying properties of operations. They will interpret their solution in the context of the original ratio relationship and/or the equivalent rate.

SMP 3: Students construct viable arguments, in both a verbal and written format, to support and defend their solution, strategy, reasoning, and interpretation. They will use mathematical and proportional</td>
<td>Students who demonstrate understanding can:
• Create and interpret tables of equivalent ratios
• Plot values from a table on a coordinate plane
• Examine tables in order to compare ratios.
• Solve real-world unit rate problems
• Calculate the percent of a quantity as a rate per 100.
• Reason with ratios to convert, manipulate and transform units of measure

Webb’s Depth of Knowledge: 1-2

Bloom’s Taxonomy:
Understand, Apply</td>
</tr>
</table>

	reasoning (in addition to possible visual representations such as a table, double number line, graph, etc) to support and defend their argument. They will critically analyze and evaluate their reasoning and strategies, as well as those of their peers. They will ask and answer questions such as: "How does your answer relate to the ratio relationship in the problem?" "How do you know your answer is correct?" "Can you use another strategy or show that in a different way?" SMP 4: Student model a ratio relationship in various forms such as an equation or inequality, a table, a tape diagram, a double number line diagram, a graph. In addition, they can use a model to demonstrate when a relationship is not proportional. They can use mathematical symbols and visual diagrams. They can use the models to support and defend their reasoning.	
Previous Learning Connections • Connect students' previous understandings of conversion tables, graphing points, and how these ideas connect to the real-world. These previous understandings will support students in their understanding of number relationships, specifically when comparing numbers. • In Grade 4, learners were taught to multiply or divide to solve word problems involving multiplicative comparison. In Grade 5, learners had to interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b) AND interpret	**Current Learning Connections** • Connect student understandings of ratio relationships and number relationship as they move to use variables to represent two quantities that change in relationship to one another in the 6.EE.9 CCSS.	**Future Learning Connections** • Connect student understanding of ratios and rate from Grade 6 to compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. • These skills from this cluster are connected in Grade 7 when learners will recognize and represent proportional relationships between quantities. This includes student understanding of proportional relationships to solve multistep ratio and percent problems.

multiplication as scaling or resizing. These skills will need to be explicitly reviewed to support student success with this domain.		

Clarification Statement:
Students are introduced to ratios, a relationship or comparison of two quantities or measures. They will represent ratios in various forms (a:b, a to b, a/b) and compare types of ratios. They will use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students will learn how and where ratios and rates are used in the real world.

Common Misconceptions
When working to solve ratio problems, students may run into confusion with the order of quantities (i.e: the ratio of triangles to squares requires students to write the quantity of triangles first as the numbers are not interchangeable). Students may have similar difficulties when understanding when to create a part-to-part ratio vs. a part-to-whole ratio.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding ratio concepts and use reasoning to solve problems because students need to understand the difference between ratio and fractions which is a huge misconception.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.NF.B.3 This standard provides a foundation for work in this cluster because it is interpreting a fraction of which students need a solid foundation as to not confuse fractions and ratios. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception*: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with understand ratio concepts and use reasoning to solve problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as <descriptions (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because using different visual information will allow students to use their learning style to access information such as reading the concept, listening to concept, or having visual animation that allow students to see and using physical manipulative to touch the concept.

Build

Effort and Persistence*: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understand ratio concepts and use reasoning to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because this will give students specific information on what you expect them to be able to accomplish and gives them a place to look for information that can help them to relook at the concepts.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with understand ratio concepts and use reasoning to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because ratios is about comparing where they learned in fourth grade to determine equivalence. Ratios is also a multiplicative comparison therefore looking at fifth grade where they learned to interpret a fraction will activate students prior understanding and reasoning about ratios.

Expression and Communication*: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understand ratio concepts and use reasoning to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.)because there are three ways to look at a ratio and understanding that no matter the form used the outcome is the same.

Internalize

Comprehension*: How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understand ratio concepts and use reasoning to solve problems benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need to practice ratio understanding as to not confuse it with fractions problems. The more the students have the opportunity to review and practice comparing ratios the increase reasoning skills and higher order thinking skills.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding ratio concepts and use ratio reasoning to solve problems by clarifying mathematical ideas and/or concepts through a short mini lesson because students often confuse ratios and fractions. By clarifying the ratio concepts, misconceptions will be reduced, and it will allow students to explore ratios.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding ratio concepts and use ratio reasoning to solve problems by addressing conceptual understanding because it allows students to attend to two quantities simultaneously. The students will be able to form a multiplicative comparison of two quantities and increase understanding of equivalent concepts.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying understanding ratio concepts and use ratio reasoning to solve problems because open ended tasks that link multiple disciplines will allow students to make connections and broaden their understanding of the concept and when and where to use it. Thus, increasing higher order thinking skills.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Equity Based Practice (Facilitating Meaningful Mathematical Discourse): Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying understanding ratio concepts and using ratio reasoning to solve problems, facilitating meaningful mathematical discourse is critical because it improves students' reasoning abilities which builds their higher order thinking skills.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 6 Ratios

6.RP.01.03.c: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

- Learning Target: I can find the percent of a whole and use it to solve problems.
- Webb's Depth of Knowledge: 2

A state park trail is 2.4 kilometers long.

a. This year the park closed 624 meters of the trail for repairs. What percent of the trail was closed for repairs? Show your work or explain how you know.

The park plans to close a different section of the trail each year for the next 3 years. Each year 20% of the total length of the trail will be repaired.

b. How many meters of trail will still need to be repaired after the planned repairs are completed? Show your work or explain how you know.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding ratio concepts and using ratio reasoning to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about ratios used in the home and community can a be a great way to connect schools tasks with home tasks.	Science: Students can apply this to science by creating a ratio of the model of the solar system to the actual size of the solar system. In addition, students can use their knowledge of ratios to help them interpret the ratios of time, space, and energy to determine a ratio. MS-PS3-1 (Energy), MS-ESS1-3 (Earth's Place in the Universe)[1] https://www.nextgenscience.org/pe/ms-ps3-1-energy https://www.nextgenscience.org/pe/ms-ess1-3-earths-place-universe Social Studies: Students can apply the idea of ratios to social studies. They can determine ratios of populations and other types of ratios that are associated with their study of social studies.

[1] https://www.nextgenscience.org/pe/ms-ps3-1-energy
https://www.nextgenscience.org/pe/ms-ess1-3-earths-place-universe

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6.NS: THE NUMBER SYSTEM

Cluster Statement: A: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.NS.A.1: Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi.?*	SMP 2: Students reason abstractly and quantitatively by making sense of the mathematical quantities in a problem to determine the relationship between the value to be divided (the dividend) and the size or quantity of groups (divisor). They understand how to estimate the quotient when dividing by a fraction, whether it is smaller or larger than 1. SMP 6: Students use correct mathematical terms when referring to the quotient, dividend, divisor, remainder, types of fractions. They communicate using clear and precise language in their discussions and in their mathematical written responses. They precisely represent the division of fractions numerically and/or visually, attend to precision when calculating the quotient, correctly label parts of diagrams, and correctly label the quotient. SMP 7: Students make use of the structure of fractions to understand what is being asked in the problem. They can estimate the solution based on previously learned patterns of scaling fractions with multiplication and extend that knowledge to interpret dividing by fractions that are less than, equal to, or greater than 1.[1]	• Describing, writing, or verbally explaining the relationship between multiplication and division of fractions. • Interpret and compute quotients of fractions using visual models and equations. • Create visual fraction models and equations to represent the problem. • Solve word problems involving division of fractions by fractions. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

[1] https://achievethecore.org/aligned/wp-content/uploads/2016/06/Math-Practices-Question-Prompts-2016.pdf

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect student's 3rd and 5th grade understandings of division as an unknown factor problem. A student's ability to interpret whole number by whole number quotients and whole number by fraction quotients will be applied within this cluster. (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.)	• Students will need to understand how to complete operations with rational numbers to help demonstrate their conceptual understanding of the distributive property	• Connect the understandings from this cluster to the 7.NS standards in 7th grade when students are required to demonstrate understanding of multiplication and division and of fractions to multiply and divide rational numbers. In Grade 7, learners solve real-world and mathematical problems involving the four operations with rational numbers. In HS Algebra standards, learners continue to use their understanding of division of fraction knowledge when solving more complex algebraic equations.

Clarification Statement:

Students will continue their previous understanding of the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to explain why the procedures for dividing fractions make sense. They use visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems.

Common Misconceptions

- Students may think dividing by 1/2 is the same as dividing something in half. Dividing by 1/2 means to find how many one halves there are in a quantity. Dividing in half means to take a quantity and divide it into two equal parts. Thus 6 divided by 1/2 = 12 and 6 divided in half equals 3.
- Students may not realize how to apply the problem to a real-life situation in which they must know which quantity represents which part of the division problem.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the application and extension of previous understandings of multiplication and division to divide fractions by fractions because as students work in interpreting quotients and solving real world problems involving

dividing fractions, it will help to have a solid understanding in multiplying and dividing fractions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.NF.B.7: This standard provides a foundation for work with the application and extension of previous understandings of multiplication and division to divide fractions by fractions because in 5th grade students extend their knowledge of multiplication and division to work with fractions. This major work prepares them to be able to solve real world problems with fractions and interpret quotients. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with applying and extending previous understandings of multiplication and division to divide fractions by fractions benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because students will make connections within the mathematics when they can optimize relevance, value and authenticity in using real world situations that hold meaning and can support visual and contextual models for them to make sense of.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with applying and extending previous understandings of multiplication and division to divide fractions by fractions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because in solving word problems and interpreting quotients, it is important for students to move beyond the algorithm for dividing fractions and understand the concept of what it means to divide a fraction into equal parts. Students need practice using multiple representations and situations to understand what this means. They may also need support in experiencing this cluster in different ways.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with applying and extending previous understandings of multiplication and division to divide fractions by fractions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because as students are interpreting quotients and solving word problems, they need to understand the parts of both a

fraction and division problem so that consistency can be maintained as students decide how to dissect a word problem. For example, students often confuse the divisor and dividend. This would be important to clarify with multiple representations and examples.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with applying and extending previous understandings of multiplication and division to divide fractions by fractions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because as students develop their use of the algorithm to divide fractions by fractions they are expected to interpret quotients and solve real word problems. This requires that students understand what dividing fractions means in context of multiple situations. It would be helpful to model and use many situations (diagrams and drawings) to help students understand how to apply this understanding to new situations.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with applying and extending previous understandings of multiplication and division to divide fractions by fractions benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) because if students are able to take previous experiences in division situations and apply them to fractions, they will be able to make sense of problems and highlight patterns, and connect these same ideas later with algebraic reasoning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on the application and extension of previous understandings of multiplication and division to divide fractions by fractions by critiquing student approaches/solutions to make connections through a short mini-lesson because as students think through and process their own work and work of others they form a deeper understanding of the concept. This would be a good opportunity to have kids work in groups to solve problems and present solutions for discussion.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit in the application and extension of previous understandings of multiplication and division to divide fractions by fractions by addressing conceptual understanding because students need to have a good foundation in both multiplication and division as well as fraction concepts to be able to apply these skills to solving real world problems and truly being able to understand what the solution means .

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying the application and extension of previous understandings of multiplication and division to divide fractions by fractions because students benefit from visual representations such as a model showing division of a fraction by a fraction and what the model actually represents. An activity where students can apply a visual representation with a real-world problem and interpret the solution would help students make important connections.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying the application and extension of previous understandings of multiplication and division to divide fractions by fractions eliciting and using student thinking is critical because as students apply and extend from previous learning it is natural for them to develop errors or make mistakes in thinking and at the same time have solid thinking that needs to be built upon. It is important to develop and create a culture within the classroom that not only allows for these mistakes but values them and finds pathways for students and teachers to affirm ideas while building correct conceptual understanding.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 6 Number System

1. Coach Walker has a cooler that can hold $5\frac{1}{2}$ gallons of a sports drink. He fills $\frac{3}{4}$ of the cooler with the sports drink to bring to football practice.

a. How many gallons of the sports drink did Coach Walker put into the cooler?

Each player on the football team fills his water bottle with the sports drink from the cooler. Each water bottle can hold $1\frac{1}{2}$ pints of the sports drink.

b. How many players can fill their water bottles with the sports drink before the cooler is empty?
[1 gallon = 8 pints]

6.NS.1 Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions (e.g.), by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

- Learning Target: I can divide fractions to solve a problem.

- Webb's Depth of Knowledge: 2
- This type of assessment question requires students to write a division equation, solve the equation and interpret the meaning of the quotient. This task will provide a teacher insight into a student's comprehension of dividing fractions. Students must create the equation placing the correct fraction in the dividend and divisor and then using the quotient for the next part of the problem. A teacher can check in with the student after the question to see what they notice and still wonder. Students might even draw a visual to represent the problem before moving on to part a and b.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on the application and extension of previous understandings of multiplication and division to divide fractions by fractions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, learning about the different ways division of fractions is used in the home and community (cooking, sharing, distance problems) can a be a great way to connect school tasks with home tasks.	English: • RST.6.8.3- following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. • RST.6.8.4- demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics. • RST.6.8.7- distinguish among facts, reasoned judgment based on research findings, and speculations in a text. • SL.6.1- engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly. Social Studies: CCSS.ELA-LITERACY.RH.6-8.1/CCSS.ELA-LITERACY.RH.6-8.7-Students can determine growth in different contexts related to social studies. Students can apply their knowledge of number operations to create a claim for a question.

6.NS: THE NUMBER SYSTEM

Cluster Statement: B: Compute fluently with multi-digit numbers and find common factors and multiples.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Fluently divide multi-digit numbers. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply
6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.	SMP 6: Students attend to precision by precisely represent the steps of the division algorithm, attend to precision when calculating the quotient, and correctly labeling the quotient, if needed.	
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Fluently add, subtract, multiply and divide multi-digit decimals. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply
6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	SMP 6: Students attend to precision by precisely represent the steps of the appropriate algorithm, attend to precision when calculating the quotient, and correctly labeling the quotient, if needed.	
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Find the GCF of two whole numbers less than or equal to 100. • Find the LCM of two whole numbers less than or equal to 12. • Use the distributive property to express a sum of two whole numbers (1-100) with a common factor as a multiple of a sum of two whole numbers with no common factor . **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	SMP 6: Students can attend to precision by communicating precisely with others and use clear mathematical language when discussing the algorithms SMP 7: Students can look for and make use of structure by using tree diagrams and Venn Diagrams to show LCM and GCF.	

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Students will need to reflect on their previous understanding of factor pairs from 4th grade. They will connect their previous learning around multiples to finding LCMs and GCFs in this cluster. • This cluster also connects to instruction from Grade 5 where students found whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value. These same skills will be utilized when dividing decimals.	• In this cluster students use the distributive property to express a sum of whole numbers. This connects to future 6th grade learning when they explore the conceptual understanding of the distributive property in the 6.EE.A cluster.	• Students will connect their skills with the standard algorithm in order to successfully multiply and divide rational numbers. This will be connected in the standard algorithm as well as in application to real-world contexts. In high school, learners continue to use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor as they learn factorization.

Clarification Statement:
Students will continue to build on their previous understanding of adding, subtracting, multiplying, and dividing to fluently use algorithms to solve problems. They will also work with finding the GCF to begin the early stages of factoring.

Common Misconceptions

- Students may misplace the decimal point when representing the product or quotient of decimals.
- Students may confuse the concepts of factors and multiples.
- Student may have difficulty in finding LCM and GCFs. They may misunderstand when to apply LCM and when to apply GCF to solve a problem.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying computing fluently with multi-digit numbers and finding common factors and multiples because students were asked in 5th grade to perform operations with multi-digit whole numbers and decimals to hundredths and in 4th grade to gain familiarity with factors and multiples. The basic work of both grades will be vital to developing fluency.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 4.OA.B.4: This standard provides a foundation for work with computing fluently with multi-digit numbers and finding common factors and multiples because students are asked to determine factors and if a number is composite or prime. This will help them in their grade level work of expressing factors in different ways using

distributive property. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with computing fluency with multi-digit numbers and finding common factors and multiples benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because students are often not interested in activities that have no relevance to their lives especially in abstract math concepts and math skills so for this standard one might provide relevancy by using the students' culture in the math problems. For example, Pueblo students could create multiple problems around Feast Day.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with computing fluently with multi-digit numbers and finding common factors and multiples benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because students will stay engaged if the context of the activities, examples, tasks, problems, etc. are culturally relevant to them such as real world statistics about Type 2 diabetes which could connect in a cross-curricular unit to history/social studies, science, health/PE, and language arts.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with computing fluently with multi-digit numbers and finding common factors and multiples benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages because in creating culturally relevant curriculum should include the use of the students' heritage language which will help build their understanding of the concepts and skills and help with their engagement with the curriculum. For example, for Spanish learners when pre teaching key vocabulary you would connect Spanish to English words like "compute" and "calcular" and possibly use both in context.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with computing fluently with multiplication and division and finding common factors and multiples benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips because sentence

starters allow for greater participation and equity in participation as well as deeper discussions. For example, provide a graphic organizer for sequencing their steps, provide a word bank with words like first, second, third; provide guiding questions like, what are the major steps in this sequence.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with computing fluently with multi-digit numbers and finding common factors and multiples benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because progressing towards independence is highly motivating to students and develops the propensity towards building lifelong learners .

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on computing fluently with multi-digit numbers and finding common factors and multiples by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may confuse operations with decimals and need a reminder of how to work within the algorithm and/or look at different models for factors to determine if they could both be correct.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on computing fluently with multi-digit numbers and finding common factors and multiples by addressing conceptual understanding because it is important for students to understand why an algorithm works if they are going to use it with fluency. This helps students to catch mistakes and understand if a solution is reasonable or not.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying computing fluently with multi-digit numbers and finding common factors and multiples because problem solving and modeling using a variety of interesting topics can be used to give students experience in applying the skills they are now fluent with.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying computing fluently with multi-digit numbers and finding common factors and multiples the types of mathematical tasks are critical because all students need a well-developed conceptual understanding of operations with decimals, factors and multiples. It is important to make sure that opportunities are given to develop this understanding so that some students are not at a disadvantage when using the algorithm and developing fluency.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Illustrative Mathematics http://tasks.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/274

6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

- Learning Target: I can identify and use the appropriate algorithm to solve a real-world problem.
- This type of assessment question requires students to know how to apply the division algorithm and to correctly use the algorithm with decimals. This task will provide a teacher with insight into how a student identifies the appropriate algorithm needed to solve the task, how to set up the algorithm correctly, and then how to use the algorithm correctly to solve the task.

Relevance to families and communities:

During a unit focused on computing fluently with multi-digit numbers and finding common factors and multiples, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways decimals, factors or multiples are used in the home and community can a be a great way to connect schools tasks with home tasks.

Cross-Curricular Connections:

English:

- RST.6.8.3- following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- RST.6.8.4- demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- RST.6.8.7- distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- SL.6.1- engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:
CCSS.ELA-LITERACY.RH.6-8.1/CCSS.ELA-LITERACY.RH.6-8.7-Students can determine growth in different contexts

	related to social studies. Students can apply their knowledge of number operations to create a claim for a question.

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6.NS: THE NUMBER SYSTEM

Cluster Statement: C: Apply and extend previous understandings of numbers to the system of rational numbers.

Major Cluster: Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	SMP 1: Students make sense of problems and persevere in solving them by interpreting the meaning of positive values, negative values, and zero in real world contexts. Students see the relationship between the location of a number on a number line or model and the value of that number. Students may ask themselves questions such as: "What information is given?" "What do you notice about the situation in the context?" "What is a strategy or model that you can use to represent the information?" SMP 2: Students reason abstractly and quantitatively by making sense of quantities in relation to zero. They may use mathematical reasoning, visual cues or models. They will understand the meaning of the quantities by integrating their knowledge of integers with their background knowledge of varying real-world contexts. SMP 4: Students model integers using mathematical and visual representations. They can use manipulatives or a visual model to display the value of a given quantity. They have a flexibility that allows them to consider whether a horizontal or vertical number line is the best representation in a given context.	• Understand that positive and negative numbers are used to describe amounts having opposite values. • Represent quantities in real-world contexts and explain the meaning of 0 in each situations. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Understand, Apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. • **6.NS.C.6.A: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.** • **6.NS.C.6.B: Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.** • **6.NS.C.6.C: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.**	SMP 4: Students model with mathematics by determining what type of model they need to represent a number or ordered pair. They interpret numbers on number lines and in coordinate planes to understand the value or meaning of a number based on its location**.** SMP.6: Students attend to precision by expressing numerical values with precision using mathematical or visual representations. They label all parts of a number line or coordinate plane accurately. They can interpret various scales when reading or creating a model. SMP.8: Students look for and express regularity in repeated reasoning by noticing a pattern in ordered pairs that are reflected across the x-axis, across the y-axis, or across the origin. They can make predictions or generalizations about the location of an ordered pair or its reflection within a particular quadrant.	• Explain the concept of rational numbers by understanding that a rational number is a point on a number line and extending number line diagrams to show positive and negative numbers on the line and in the coordinate plane. • Express orally and in writing that opposite signs of a number indicate opposite places on a number line. • Understand where positive and negative numbers in an ordered pair appear on a coordinate plane and identify quadrants. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Understand

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Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
6.NS.C.7: Understand ordering and absolute value of rational numbers. • **6.NS.C.7.A: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.** ***For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*** • **6.NS.C.7.B: Write, interpret, and explain statements of order for rational numbers in real-world contexts.** ***For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C.*** • **6.NS.C.7.C: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.** ***For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.*** • **6.NS.C.7.D: Distinguish comparisons of absolute value from statements about order.** ***For example, recognize that an account balance less***	SMP 1: Students make sense of problems and persevere in solving them by analyzing a situation in a problem or context and interpret the meaning of the rational numbers utilizing their knowledge of numbers and the structure of various models. They see relationships between the values of rational numbers and their corresponding absolute values. They may themselves questions such as: "How does this number relate to another number in this context?" "What are some strategies I could use to understand or interpret this statement or problem?" "What information is given and what am I being asked to find?" SMP 2: Students reason and abstractly and quantitatively by making sense of the values of negative integers and rational numbers by relating it to their prior knowledge and experiences with positive integers and rational numbers. Students use mathematical, verbal, and visual cues to interpret and explain the value or meaning of numbers, inequalities, lists (order), location, and magnitude. **SMP.3** Students construct viable arguments and critique the reasoning of others, in both a verbal and written format, to support and defend their solution, strategy, reasoning, and interpretation. They will use mathematical reasoning (in addition to possible visual representations to support and defend their argument. They will critically analyze and evaluate their reasoning and strategies, as well as those of their peers. They will ask and answer questions such as: "What mathematical evidence would support your solution?" "How can we be sure that...?" "How could you prove that...?"	• Understand the absolute value of rational numbers. • Interpret and explain the meanings behind inequality statements. • Show understanding of rational numbers by giving them context in a real-life situation. • Understand that absolute value is a number's distance from zero on a number line. • Understand the difference between absolute value from order statements. • Explain the reasoning that as a value of a negative rational number decreases its absolute value increases. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Understand, Apply

than -30 dollars represents a debt greater than 30 dollars.		
Standard Text **6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.**	**Standard for Mathematical Practices** SMP 1: Students make sense of problems and persevere in solving them by interpreting the meaning of positive values, negative values, and zero in real world contexts. Students see the relationship between the location of a number on a number line or model and the value of that number. Students may ask themselves questions such as: "What information is given?" "What do you notice about the situation in the context?" "What is a strategy or model that you can use to represent the information?" SMP 2: Students reason abstractly and quantitatively by making sense of quantities in relation to zero. They may use mathematical reasoning, visual cues or models. They will understand the meaning of the quantities by integrating their knowledge of integers with their background knowledge of varying real-world contexts. SMP 4: Students model integers using mathematical and visual representations. They can use manipulatives or a visual model to display the value of a given quantity. They have a flexibility that allows them to consider whether a horizontal or vertical number line is the best representation in a given context.	**Students who demonstrate understanding can:** • Graph points in all four quadrants solving real-world problems. • Find distance between points using coordinates and absolute value. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand
<u>Previous Learning Connections</u> • This cluster is connected to what students previously learned in third grade, when they marked off units on a horizontal scale or number line. Students will recall that a fraction	**<u>Current Learning Connections</u>** • There are connections between this cluster and the 6.EE.B cluster when learners recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Also, in 6.G.3, there are connections made when students	**<u>Future Learning Connections</u>** • The skills from this cluster are applied in 7th grade when students make connections between their 6th grade understanding of what rational numbers are to include the addition and subtraction of integers. Students will need to represent addition and

can be represented on a number line, in the space between whole numbers. They will also recall the skills from Grade 5 when they graphed points on a coordinate plan and interpreted what the points represent.	use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.	subtraction of integers on a horizontal &/or vertical number line.

Clarification Statement:

Students will extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical. Horizontal and vertical number lines help students move from number lines to coordinate grids. They will focus on the relationship between negative and positive numbers and the meaning of absolute value. This cluster will lay the foundation for working with rational numbers, algebraic expressions and equations, functions and the coordinate plane in seventh and eighth grade.

Common Misconceptions

- Students may confuse the idea that greater the magnitude of a negative number the greater the number.
- Students may confuse the placement of rational numbers on number line.
- Students may confuse the absolute value bar with the number 1.
- Students may confuse the absolute value bar with parenthesis.
- Students may think that absolute value makes things positive and not understand it is about distance from 0

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying apply and extend previous understandings of numbers to the system of rational numbers because students can build understanding of positive and negative integers, reinforce concepts of distance and location on number lines.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.NS.C. 5 - This standard provides a foundation for work with applying and extending previous understandings of numbers to the system of rational numbers because it establishes the foundation of conceptual understanding of positive and negative numbers including zero. Ordering and comparing numbers can be easily done when visualized on a number line. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with applying and extending previous understandings of numbers to the system of rational numbers benefit when learning experiences include ways to recruit interest such as providing choices in their learning (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge) by using visual examples then extending use of the number line because students can be engaged by information and activities that are relevant and valuable to their interests.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with applying and extending previous understandings of numbers to the system of rational numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because understanding the idea of rational numbers are introduced in this grade level starting with positive and negative numbers. This is where students firm understanding of rational numbers should be fixed by using relevant applications like giving an example of negative numbers in real life (loss of yards in football, temperature, sea level, withdrawals from accounts) in supporting their learning.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with applying and extending previous understandings of numbers to the system of rational numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because this will give students easy access to unknown and not-so-understood information.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with applying and extending previous understandings of numbers to the system of rational numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because the concept of rational numbers are not limited to integers.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with applying and extending previous understandings of numbers to the system of rational numbers benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning

goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because understanding situations with positive and negative numbers can be difficult for students to connect to at first.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on applying and extending previous understanding of numbers to the system of rational numbers by critiquing student approaches/solutions to make connections through a short mini lesson because starting from what they know will make connections easier to approach like the concepts of losing and winning, going backward and moving forward, below freezing and above freezing.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on applying and extending previous understanding of numbers to the system of rational numbers by addressing conceptual understanding because the idea of positive numbers, negative numbers and zero are abstract in nature.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying apply and extend previous understanding of numbers to the system of rational numbers because students are expected to use their conceptual understanding in solving word problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying how to apply and extend previous understandings of numbers to the system of rational numbers the types of mathematical tasks are critical because it allows students to work on their specific interests and strengths. For example, differentiated tasks can be Banking and Finance (deposits and withdrawals), Is it hot or cold? (temperatures), The Extra Mile (moving forward or backward), etc. Assigning different tasks is important and crucial because it is directly linked to students learning. Also, the level of task complexity can vary by creating or adapting materials to the students' level of thinking and understanding. Teachers should at first know and understand the uniqueness of every student and plan accordingly.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Illustrative Mathematics https://tasks.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/277

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

- Learning Target: I can apply my knowledge of integers in a real-world context.
- Webb's Depth of Knowledge: 2
- This type of assessment question requires students to apply their knowledge of integers in a real-world context. This task will inform the teacher's instruction based on their ability to determine the difference using a variety of strategies, since integer operations aren't introduced until 7th grade. Students can use a number line, reasoning or apply their knowledge of absolute value to solve the task.

Relevance to families and communities:

During a unit focused on applying and extending previous understandings of numbers to the system of rational numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different representations of positive numbers, negative numbers, and the meaning of zero across the languages in the classroom brings about a wide array of conceptual understanding that can be referenced in diverse cultures.

Cross-Curricular Connections:

English:

- RST.6.8.3- following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- RST.6.8.4- demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- RST.6.8.7- distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- SL.6.1- engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

Social Studies:

CCSS.ELA-LITERACY.RH.6-8.7-Students can use this idea of plotting points in a coordinate plane to adjusting it to the longitude and latitude lines on a map. They can use this to track a traveling pattern and discuss it further. They can track a voyage over time.

NEW MEXICO
Public Education Department

6.EE: EXPRESSIONS & EQUATIONS

Cluster Statement: A: Apply and extend previous understandings of arithmetic to algebraic expressions.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.	SMP 6: Students can attend to precision by using appropriate vocabulary and translate between verbal and numerical expressions fluently and accurately. Students must also set up expressions, equations, and/or inequalities that represent the correct interpretation of the problem at hand	• Write and evaluate numerical expressions involving whole number exponents using the correct terminology • Evaluate numerical expressions using their knowledge of order of operations from previous years. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. • **6.EE.A.2.A: Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.*** • **6.EE.A.2.B: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as***	SMP 1: Students can make sense of problems and persevere in solving them by looking for meaning in the problems and find effective ways to represent and solve them. Students must understand what the variable is represented in the problem in front of them stands for in order to make sense of the problem and solve for it. They will be able to explain what the variable represents and how their answer makes sense.	• Express orally and in writing that variables represent unknown quantities. • Write expressions using variables that represent unknown numbers. • Identify context to write algebraic expressions. • Translate verbal expressions into numerical expressions. • Use information from real world examples to evaluate expressions with variables **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. • **6.EE.A.2.C: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).** ***For example, use the formulas V = s^3 and A = 6 s^2 to find the volume and surface area of a cube with sides of length s = ½.***		
Standard Text **6.EE.A.3** **Apply the properties of operations to generate equivalent expressions.** ***For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.***	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by reasoning with symbolic representations in equations and manipulating algebraic expressions while maintaining equality.	**Students who demonstrate understanding can:** • Create an equivalent expression through the use of properties of operations • Apply the distributive, commutative, identity, and distributives properties to expressions that include variables **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

<table>
<tr>
<td>Standard Text

6.EE.A.4
Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.</td>
<td>Standard for Mathematical Practices

SMP 7: Students can look for and make use of structure by applying the properties to generate equivalent expressions. Students also use the structure of the properties to generate the expressions and will need to prove that their expressions are equivalent by using substitution.</td>
<td>Students who demonstrate understanding can:
• Identify equivalent expressions.
• Combine like terms
• Reason that two expressions are equivalent through the use of substitution

Webb’s Depth of Knowledge: 1-2

Bloom’s Taxonomy:
Understand</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
• Students will connect their prior knowledge on using whole-number exponents to denote the powers of 10 in order to properly set-up exponents and identify the base. Additionally, in 5th grade learners have already been taught the commutative and associative property of both addition and multiplication.</td>
<td><u>Current Learning Connections</u>
• Students will connect what they were previously taught in 6th grade finding the greatest common factor of two whole numbers and using the distributive property to express sums of whole numbers to this cluster. These skills will be needed when students create and identify equivalent expressions.</td>
<td><u>Future Learning Connections</u>
• In 7th grade, learners will learn to apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Learners will develop an understanding of operations with rational numbers when working with expressions and linear equations. In 8th grade, students will know and apply the properties of integer exponents to generate equivalent numerical expressions. In high school, students will need to interpret parts of an expression, such as terms, factors, and coefficients.</td>
</tr>
<tr>
<td colspan="3">Clarification Statement:

The focus for this cluster is writing and evaluating numerical expressions involving whole number exponents, finding the value of an expression using exponential notation such as $4^2 = 4 \times 4$ or $d\ 3 = d \times d \times d$, and using the appropriate terminology to explain how to evaluate an expression. Students are applying the properties of operations to generate equivalent expressions including the distributive property to produce equivalent representation.</td>
</tr>
<tr>
<td colspan="3">Common Misconceptions
• When given an expression with an exponent, students may misinterpret the base and the exponent as factors and multiply the two numbers. For example, show that 5 x 3=15, which is much smaller than 5x5x5 which equals 125.
• Students may use distributive property incorrectly in that students will often multiply the first term, but forget to do the same to the second term.
• Students may misuse the communitive property by applying it to subtraction and/or division problems.</td>
</tr>
</table>

- Students confuse variables with letters for unit of measure.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions because this cluster requires the acquisition of a considerable amount of new vocabulary. The terms that are used to identify the parts and types of expressions will support students in becoming proficient in explaining and discussing many new concepts encompassed in expressions, equations, and inequalities. This is the first formal experience students have with variables, coefficients, and constants. Students will also be extending previous learning of exponents, order of operations, sums, differences products, quotients, equivalent, like and unlike terms.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.NBT.B.7: This standard provides a foundation for work with writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions because it ensures that students have a mastery of all operations with whole numbers and decimals. In addition, this standard focuses on the properties of operations and the relationship between addition and subtraction. If students understand that relationship, they can then make the connection to the relationship between multiplication and division. For students to successfully evaluate algebraic expressions and generate equivalent expressions, they need to have a mastery of all parts of this standard to use as an anchor for new learning. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions in which letters stand for numbers benefit when learning experiences include ways to recruit interest such as providing choices in their learning such as multiple modes of representation because students with various learning modalities will be better able to access the conceptual ideas relating to algebraic expressions and variables if they can share individual representations. For example, a student who learns visually or with tactile supports can draw or create a visual representation of a given expression, while a linear thinker can write an algebraic representation. As students share different representations of algebraic expressions, this will support them in making connections to equivalent expressions.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions in which letters stand for numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because students can work collaboratively on tasks that require them to represent algebraic expressions, apply new knowledge and concepts, and extend that learning to real-world situations. A community of learners can work collaboratively and learn from one another, as they bring different strengths and learning styles to the group.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions in which letters stand for numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because this cluster contains a great deal of mathematical vocabulary that is new to students. Since this is their first introduction to algebraic expressions and concepts, it is imperative to give them a firm foundation with the complex terms and structures that will support them in future mathematical experiences.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions in which letters stand for numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students have different learning styles and strengths. Allowing them to apply different strategies, as well as taking the time to share and discuss those strategies, will encourage students to acquire an assortment of strategies that they can employ when solving a problem or approaching a task.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions in which letters stand for numbers benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because this cluster encompasses a great deal of foundational algebraic knowledge and experience that students will need to support their learning in upcoming clusters and in future math courses. Students will need to be able to examine and reflect upon their thinking strategies, as well as

those of their peers in order to begin making connections and generalizations about similar types of expressions and similar types of tasks. Many students will require support and explicit instruction, along with opportunities to apply the new learning and to test generalizations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions by examining tasks from a different perspective through a short mini-lesson because students often need to see multiple representations or approaches to interpreting and generating algebraic expressions. The conceptual way of thinking about mathematics is new to them, and many of them need opportunities to engage with their peers who may offer a different perspective or approach to understanding.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions by addressing conceptual understanding because some students will need explicit instruction, in order to make connections between their prior knowledge and experiences working with numerical expressions and the conceptual way of understanding algebraic expressions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying writing, reading, evaluating algebraic expressions because students can extend their learning of evaluating algebraic expressions by applying it to Geometry topics. They should be given tasks and opportunities to apply the concept of evaluating using geometric formulas. They can make the connection between the variables represented in the formulas for Volume, Area, and Surface Area with the conceptual way of calculating them without the use of concrete objects.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural

competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions the use of mathematical representations within the classroom is critical because it allows students to use various strategies or representations that are familiar or logical to them, in order to make sense of verbal expressions and algebraic expressions. Many students need the support of physical or visual representations to connect their understanding of mathematical concepts and language that are new and foreign to them. This cluster introduces students to what will be the critical foundation to their conceptual understanding of algebraic concepts and patterns/relationships applied within the properties of operations, so it is important to allow students time and opportunities to connect these concepts to various mathematical representations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: http://tasks.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/532

6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.

- Learning Target: I can write an expression, using variables, to represent exponential growth in a task.
- Webb's Depth of Knowledge: 2
- This type of assessment question requires students to apply and extend previous understandings of arithmetic to algebraic expressions specifically with exponents. This task can be used to determine if students can solve the problem and write an expression to represent a repeated calculation. Students can get an answer without writing an expression; however, mastery of the cluster requires students to write and evaluate a numerical expression.

Relevance to families and communities:

During a unit focused on writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, learn about the habits and experiences that your students have at home or other settings away from school. Create or modify tasks to reflect situations or topics that will be interesting or familiar to your students and their concept of the world around them.

Cross-Curricular Connections:

Science: https://www.nextgenscience.org/pe/ms-ps2-1-motion-and-stability-forces-and-interactions Students can work to create, read, and evaluate expressions that result from the forces at work. Students will have to be able to create and support their argument. (MS-PS2-1, Motion and Stability: Forces and Interactions)

English:

- RST.6.8.3 Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. RST.6.8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 6-8 texts and topics*.
- RST.6.8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
- SL.6.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

NEW MEXICO
Public Education Department

6.EE: EXPRESSIONS & EQUATIONS

Cluster Statement: B: Reason about and solve one-variable equations and inequalities.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	SMP 1: Students will make sense of problems and persevere in solving them by using algebraic reasoning to understand real-world and mathematical problems. They will interpret the unknown variable and what it represents in context. They identify an appropriate strategy to apply when solving the problem. Students will build upon their prior experiences and background knowledge with numerical expressions to make sense of the problem. They may ask themselves questions such as: "What are you trying to find?" "What do you know from the problem?" "What is the unknown?" "What is the relationship between the known and unknown numbers?" SMP 2: Students reason abstractly and quantitatively by representing a wide variety of real-world contexts through the use of real numbers and variables in mathematical terms. Students consider the context contained in the problem to understand the meaning of the number or variable. They use algebraic and mathematical reasoning when writing an expression. SMP 4: Students will model with mathematics by using numerical and algebraic expressions and symbols such as variables, numbers, parentheses, operators, etc. to represent a mathematical or real-world problem. SMP 6: Students will attend to precision when communicating their	• Reason to find the single value that makes an equation true • Explain what a variable is representing in a particular situation or context • Use substitution to simplify numerical expressions and determine if solution is true. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Understand, Apply

	solutions using precise and accurate mathematical language. They will calculate the solution to the expression accurately. They will correctly label any numbers in a real-world problem, and they will correctly label any visual models they use to support or defend their answers. SMP 7: Students will look for and make use of structure when interpreting a real-world problem using their understanding of the word meanings and the structure of mathematical and algebraic expressions.	
Standard Text **6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.**	**Standard for Mathematical Practices** SMP 1: Students will make sense of problems and persevere in solving them by using algebraic reasoning to understand real-world and mathematical problems. They will interpret the unknown variable and what it represents in context. They identify an appropriate strategy to apply when solving the problem. Students will build upon their prior experiences and background knowledge with numerical expressions to make sense of the problem. They may ask themselves questions such as: "What are you trying to find?" "What do you know from the problem?" "What is the unknown?" "What is the relationship between the known and unknown numbers?" SMP 2: Students reason abstractly and quantitatively by representing a wide variety of real-world contexts through the use of real numbers and variables in mathematical terms. Students consider the context contained in the problem to understand the meaning of the number or variable. They use algebraic and mathematical reasoning when writing an expression. SMP 4: Students will model with mathematics by using numerical and	**Students who demonstrate understanding can:** • Explain that a variable represents a number or a specified set of numbers. • Identify what the variable represents quantitatively and in context. • Represent real world scenarios with variable expressions. **Webb's Depth of Knowledge:** 1-2

	algebraic expressions and symbols such as variables, numbers, parentheses, operators, etc. to represent a mathematical or real-world problem. SMP 6: Students will attend to precision when communicating their solutions using precise and accurate mathematical language. They will calculate the solution to the expression accurately. They will correctly label any numbers in a real-world problem, and they will correctly label any visual models they use to support or defend their answers. SMP 7: Students will look for and make use of structure when interpreting a real-world problem using their understanding of the word meanings and the structure of mathematical and algebraic expressions.	**Bloom's Taxonomy:** Understand, Apply
Standard Text **6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q* for cases in which *p*, *q* and *x* are all nonnegative rational numbers.**	**Standard for Mathematical Practices** SMP 1: Students will make sense of problems and persevere in solving them by using algebraic reasoning to understand real-world and mathematical problems. They will interpret the unknown variable and what it represents in context. They identify an appropriate strategy to apply when solving the problem. Students will build upon their prior experiences and background knowledge with numerical expressions to make sense of the problem. They may ask themselves questions such as: "What are you trying to find?" "What do you know from the problem?" "What is the unknown?" "What is the relationship between the known and unknown numbers?" SMP 2: Students reason abstractly and quantitatively by representing a wide variety of real-world contexts through the use of real numbers and variables in mathematical terms. Students	**Students who demonstrate understanding can:** • Write and solve one step addition equations (x + p= q) when x, p and q are positive • Write and solve one step multiplication equations (px=q) form when x, p and q are positive. • Model real-world situations with equations

	consider the context contained in the problem to understand the meaning of the number or variable. They use algebraic and mathematical reasoning when writing an expression. SMP 4: Students will model with mathematics by using numerical and algebraic expressions and symbols such as variables, numbers, parentheses, operators, etc. to represent a mathematical or real-world problem. SMP 6: Students will attend to precision when communicating their solutions using precise and accurate mathematical language. They will calculate the solution to the expression accurately. They will correctly label any numbers in a real-world problem, and they will correctly label any visual models they use to support or defend their answers. SMP 7: Students will look for and make use of structure when interpreting a real-world problem using their understanding of the word meanings and the structure of mathematical and algebraic expressions.	**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
6.EE.B.8: Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < *c* have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	SMP 1: Students will make sense of problems and persevere in solving them by using algebraic reasoning to understand real-world and mathematical problems. They will interpret the unknown variable and what it represents in context. They identify an appropriate strategy to apply when solving the problem. Students will build upon their prior experiences and background knowledge with numerical expressions to make sense of the problem. They may ask themselves questions such as: "What are you trying to find?" "What do you know from the problem?" "What is the unknown?" "What is the relationship between the known and unknown numbers?" SMP 2: Students reason abstractly and quantitatively by representing a wide variety of real-world contexts through the use of real numbers and variables in mathematical terms. Students consider the context contained in the problem to understand the meaning of the number or variable. They use algebraic and mathematical reasoning when writing an expression. SMP 4: Students will model with mathematics by using numerical and algebraic expressions and symbols such as variables, numbers, parentheses, operators, etc. to represent a mathematical or real-world problem. SMP 6: Students will attend to precision when communicating their solutions using precise and accurate mathematical language. They will calculate the solution to the expression accurately. They will correctly label any numbers in a real-world problem, and they will correctly label any visual models they use to support or defend their answers.	• Represent a real-world problem with an inequality (x > c or x < c) • Explain that an inequality can have infinite solutions and show it on a number line. • Understand the difference between >, ≥, and <, ≤ and graphing with the appropriate open or closed circle. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply

	SMP 7: Students will look for and make use of structure when interpreting a real-world problem using their understanding of the word meanings and the structure of mathematical and algebraic expressions.	
Previous Learning Connections • Students connect their previous understandings of what the equal sign is and that it shows equivalence to this cluster. The idea of equivalence is most aligned to their work in grades 4 and 5 with visual fraction models and understanding basic properties of operations to solve.	**Current Learning Connections** • This cluster really expands on the previous cluster of 6.EE.A.2 where students learned how to read, write and evaluate expressions in which letters stand for numbers.	**Future Learning Connections** • In Grade 7, students begin to formally apply the properties of operations. They will solve two step equations in the form of px + q = r and p(x + q) = r. In Grade 8, students solve linear equations in one variable that include one solution, no solution, or infinitely many solutions. They include equations that require the distributive property or combining like terms. In Grade 8, the variable can be on both sides of the equation. In high school, students further their knowledge of solving equations with multistep equations that require the distributive property or combining like terms.

Clarification Statement:
Students focus on the meaning of an equation and use reasoning and prior knowledge to solve it. They use variables to represent numbers and write expressions when solving problems. Students learn to write inequalities in the form of $x > c$, $x \geq c$, $x < c$ or $x \leq c$ and use of number line representation to show the solutions of inequalities.

Common Misconceptions
- Students may have difficulty conceptualizing that an inequality can have more than one solution.
- Students may assume if there is no coefficient in front of the variable, then the variable does not have a value. They do not see that y=1y.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g. keeping equations balanced when solving, displaying solutions to equations and inequalities on a number line) when studying reasoning about and solving one-variable equations and inequalities because students have experience solving one step numerical equations, but this is the first time they will be introduced to the concept of solving equations and inequalities using inverse operations. This is also the first introduction to solving inequalities that have a solution set containing infinitely many solutions. Although students have used number lines in the past, this will be their first experience with displaying solutions and solution sets on a number line. When reading a number line, students will need to be pre-taught how to read and interpret the circles and arrows.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.A2: This standard provides a foundation for working with reasoning about and solving one-variable equations and inequalities because students need to have a firm grasp of how to read, interpret, write, and evaluate algebraic expressions containing variables before they will be able to clearly understand the connection between the parts of an equation or inequality. Students won't understand that an equation is two equivalent expressions if they don't have a clear understanding of expressions. In order for students to be able to interpret, explain, and discuss algebraic expressions, equations, and inequalities, they need to have a proficient understanding of the parts of expressions and different types of expressions (e.g. sum, difference, product, quotient, or a combination). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with reasoning about and solving one-variable equations and inequalities benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because algebraic concepts and working with variables specifically is new to students in the 6th grade, and many students find the conceptual way of thinking to be complex. Providing students with culturally relevant, age appropriate, and engaging problems will allow them to connect concrete ideas with conceptual strategies.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with reasoning about and solving one-variable equations and inequalities benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success. This type of feedback will support students in identifying problems and situations that model additive and multiplicative relationships. Students' learning will benefit from multiple opportunities to access equations, inequalities, and real-world problems and receive feedback as they reflect and/or analyze their errors. This type of feedback will allow them to build improved strategies for examining and solving more complex problems in the future.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with reasoning about and solving one-variable equations and inequalities benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as making connections to previously learned structures because students have previously learned how to reason and solve numerical equations and inequalities, and the structure is similar to the new algebraic equations and inequalities. In addition, students have used inverse operations informally in previous grades to check the accuracy of their computation, and they can make a connection to that prior learning and apply the use of inverse operations to solve one-variable equations and inequalities.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with reasoning about and solving one-variable equations and inequalities benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g. cubes, tiles, algebra blocks, "Hands-on Equations") because the use of manipulatives can support visual learners as they develop strategies to connect concrete ideas with conceptual strategies. As students manipulate the concrete objects to represent solving for the unknown variable, they will discover how it represents the balance of the two sides of the equation or inequality.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with reasoning about and solving one-variable equations and inequalities benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with one-step equations or inequalities) because with experience and support, students will begin to make generalizations about problems and situations that represent additive or multiplicative relationships. Students will also develop an understanding of the difference between a solution to an equation and a solution set for an inequality. As students begin to deduce the connections between similar types of problems, they will be able to more fluently apply strategies to solve, model, and/or discuss problems involving equations and inequalities.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on reasoning about and solving one-variable equations and inequalities by clarifying mathematical ideas and/or concepts through a short mini-lesson because the algebraic concepts that are introduced in this cluster are a necessary foundation for students' success in all subsequent math courses. When examining the coherence map, this cluster has a direct correlation to major clusters in Math 7, Math 8, Algebra I, Geometry, and Algebra II. Taking the time to revisit, reteach, or practice the 6.EE.B cluster, will support students in becoming proficient in foundational concepts and skills that will improve their chances of success as mathematical thinkers and

problem solvers.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit: reasoning about and solving one-variable equations and inequalities by confronting student misconceptions because if students have a misconception about the correct way to solve, check or represent a one-step equation or inequality, that will cause them more confusion when they are introduced to more complex equations and inequalities in the future. If they have a misconception about when to apply a particular operation or in what order to write the parts of an equation or inequality from a word problem, addressing and reteaching that can help students master this major cluster and its concepts to a higher degree. Students can be guided through scaffolding or collaborative discussions to examine a multitude of similar problems, in order to reflect, analyze errors, and create generalizations that will correct their misconceptions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- Students are NOT learning how to solve one step equations using the properties of operations yet. To make it more difficult for students, add in fractions and decimals. The cluster is truly about reasoning. Students need to understand how to maintain equivalence when working with equations. They may use the properties of operations to solve but are not explicitly being told that is what they are doing.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for other methods of solving tasks that occur outside of school mathematics. For example, when studying reasoning about and solving one-variable equations and inequalities the types of mathematical tasks are critical because as students are introduced to new algebraic concepts using variables, they will need to think of ways to use strategies to connect to their prior knowledge of operations and number sense. Since students are beginning to transition from a literal understanding of numbers to a conceptual one, giving them opportunities to make connections and build meaning with the use of variables conceptually first will serve to strengthen their subsequent mastery of the procedural fluency that will be critical to their success in future experiences with algebraic topics. Deepening students' conceptual understanding will also aid in their ability to fluently apply equations and inequalities in a variety of real-world contexts.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: http://tasks.illustrativemathematics.org/content-standards/6/EE/B/5/tasks/673
6.EE.5: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- Learning Target: I can interpret the meaning of a solution to an inequality.
- Webb's Depth of Knowledge: 2
- This type of assessment question requires students to evaluate a solution set in a real-world context. This task provides a great opportunity for students to engage in mathematical discourse and for teachers to hear the reasoning behind strategies and solutions presented by the students. Students can take different approaches to the problem, but the focus should be on interpreting the solution, what does it mean in the context of the inequality.

Relevance to families and communities:

During a unit focused on reasoning about and solving one-variable equations and inequalities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, learn about the habits and experiences that your students have at home or other settings away from school. Create or modify tasks to reflect situations or topics that will be interesting or familiar to your students and their concept of the world around them.

Cross-Curricular Connections:

Science:

- https://www.nextgenscience.org/pe/ms-ess2-2-earths-systems
- https://www.nextgenscience.org/pe/ms-ess1-4-earths-place-universe

Students can apply their study of geo sciences to math by creating expressions and variables to represent the changes that have occurred on Earth. As the Earth has changed, the rate at which it changed as well as the changes to the environment can be modeled with mathematics. This can be used to study the time, space, energy phenomena that may be too small or large to observe. Students can use their expressions to conduct experiments or analysis of the above phenomena. (MS-ESS2-2, Earth's Systems, MS-ESS1-4, Earth's Place in the Universe)

English:

- RST.6.8.3 Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. RST.6.8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 6-8 texts and topics*.
- RST.6.8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
- SL.6.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

NEW MEXICO
Public Education Department

6.EE: EXPRESSIONS & EQUATIONS

Cluster Statement: C: Represent and analyze quantitative relationships between dependent and independent variables.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	SMP 1: Students will make sense of problems and persevere in solving them by using algebraic reasoning to understand real-world and mathematical problems. They will interpret the unknown variable and what it represents in context. They identify an appropriate strategy to apply when solving the problem. Students will build upon their prior experiences and background knowledge with numerical expressions to make sense of the problem. They may ask themselves questions such as: "What are you trying to find?" "What do you know from the problem?" "What is the unknown?" "What is the relationship between the known and unknown numbers?" SMP 2: Students reason abstractly and quantitatively by representing a wide variety of real-world contexts through the use of real numbers and variables in mathematical terms. Students consider the context contained in the problem to understand the meaning of the number or variable. They use algebraic and mathematical reasoning when writing an expression. SMP 4**:** Students will model with mathematics by using numerical and algebraic expressions and symbols such as variables, numbers, parentheses, operators,	• Use variables to represent unknowns in a real-world problem and write an equation to show the relationship between two changing quantities. • Describe the variables in context of dependent and independent • Analyze the relationship between the dependent and independent variables using tables, graphs and equations. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Apply, Analyze

	etc. to represent a mathematical or real-world problem. SMP 6: Students will attend to precision when communicating their solutions using precise and accurate mathematical language. They will calculate the solution to the expression accurately. They will correctly label any numbers in a real-world problem, and they will correctly label any visual models they use to support or defend their answers. SMP 7: Students will look for and make use of structure when interpreting a real-world problem using their understanding of the word meanings and the structure of mathematical and algebraic expressions.	
Previous Learning Connections • In Grade 5, learners are taught how to generate patterns from rules that are given to them. This will connect when students are analyzing the relationship between the dependent and independent variables in this cluster.	**Current Learning Connections** • The students will expand their knowledge of 6.EE.7 in this cluster by continuing practice of writing equations in real-world situations. The students will expand their knowledge of 6.RP.3 by continuing to find relationships with numbers through rate reasoning.	**Future Learning Connections** • The students will continue using dependent and independent variables and noticing patterns throughout the rest of their mathematical career, showing up mainly in the RP clusters and as they dive in to linear and non-linear relationships. In high school, they will be using this knowledge as they construct and compare linear, quadratic, and exponential models.
Clarification Statement: The focus for this cluster is using variables to represent two quantities in a real-world problem that change in relationship to one another. Students write an equation and analyze the relationship between the dependent and independent variables using graphs and tables.		
Common Misconceptions • Students may confuse what the graph represents in context. For example, that moving up or down on a graph does not necessarily mean that a person is moving up or down. • Students may reverse the independent and dependent variable in an equation, graph or table.		
Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies **Pre-Teach**		

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit when studying relationships between dependent and independent variables because this standard introduces new information i.e., to understand the relationship between dependent and independent variables. The dependent variable is the variable that can be changed; one that is affected by the change in the independent variable.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.O.A.B.3. This standard provides a foundation for work with summarizing and analyzing relationships between dependent and independent variables because representing two quantities in a real-world problem can be generated from a pattern using given rules. Students represent quantitative relationships in different ways. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with quantitative relationships between dependent and independent variables benefit when learning experiences include ways to recruit interest such as providing choices in their learning, for example, given a problem set of 15 different situations, students will choose at least three items based on their interest to identify the dependent variable and the independent variable because students engage themselves in learning when they are interested in the topic and have student choice.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with relationships between dependent and independent variables benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because students need to understand how the expressions or equations relate to situations presented, as well as the process of solving them by exposing them to different representations.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with quantitative relationships between dependent and independent variables benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because the purpose of this cluster is for students to understand the

relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed and is graphed on the x-axis; the dependent variable is the variable that is affected by the change in the independent variable and is graphed on the y-axis. Students are expected to recognize and explain the effect on the dependent variable when the independent variable changes.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with quantitative relationships between dependent variable and independent variable benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips, for example, "the ___ variable depends of ___ variable" or " as the number of ___variable increases, the number of ___variable increases/decreases" because this strategy provides opportunities for students to explain their thinking and respond to the mathematical thinking of others.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with quantitative relationships between dependent and independent variables benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as provide tasks with multiple entry points and optional pathways because students connect the pieces together, students use multiple representations for the mathematical relationship. Students translate freely among the words, models, tables, graphs and equations. Students start with any of the representations and understand or explain the relationship to others.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on relationships of dependent and independent variables by critiquing student approaches/solutions to make connections through a short mini lesson because it allows students to receive immediate feedback of their work. It may even make sense for a student's critique of their peer's work to be part of making connections. It is important that it doesn't simply focus on the right or wrong solutions but presents a balanced view that allows improvement and redirection of students' learning.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on summarizing and analyzing dependent and independent variables by offering opportunities to understand and explore different strategies because quantitative relationships can be presented in different forms. Students mastery of identifying the dependent variable and the independent variable from real-world problems leads to a deeper understanding of the connections between the equation to a graph, table or written description that show the same relationship.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying relationships between the dependent and independent variables because providing multiple situations for the student to analyze and determine what unknown is dependent on the other components allows students' thinking and creativity to happen. One example is the use of technology, including computer apps and other hand-held technology that allows the collection of real-time data to create tables and charts. It is important for students to realize that although real-world data often is not linear, a line sometimes can model the data.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying how to represent and analyze quantitative relationships between dependent and independent variables the use of mathematical representations within the classroom is critical because students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 6: Expressions & Equations-Represent and analyze quantitative relationships

A zoo has two young alligators named Scales and Claw. This table compares the lengths of the two alligators at different times in their lives.

Length of Scales in inches (s)	Length of Claw in inches (c)
12	18
16	24
20	30

a. Based on the information in the table, write an equation that can be used to find s, the length in inches of Scales when Claw is c inches long.

b. Scales is now 32 inches long. How long is Claw?

6.EE.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem*

involving motion at constant speed, list and graph ordered pairs of distances and times and write the equation d = 65t to represent the relationship between distance and time.

- Learning Target: I can write an equation to represent a relationship and use the equation to find a value.
- Webb's Depth of Knowledge: 2
- This type of assessment question requires students to analyze a table and write an equation that represents the relationship between the length of scales and the length of inches and then to use the equation to calculate a variable that isn't represented on the table. The teacher can evaluate a student's ability to first analyze a relationship between the columns on the table, which variable is dependent, and which is independent. Then, the teacher can determine if a student can take the information to create an equation that fits the data reflected on the table but also data points beyond. A task like this allows students to enter the problem by first just noticing and wondering about the table but then parts A and B require elements of the standard and allow the teacher to evaluate a student's understanding based on completion.

Relevance to families and communities:

During a unit focused on representing and analyzing quantitative relationships between dependent and independent variables, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, the cost of making one Navajo ceremonial basket is $150. Students create an equation, a table of values and graphical representations of the situation. From the context, equation, or graph, students determine which variable is in a dependent relationship and independent relationship.

Cross-Curricular Connections

Science: Students can create expressions to anticipate the real-world events that happen. Students must understand that events that occur at one scale, may not occur at a larger/smaller scale. Students will be able to create expressions about the scale of cells and molecules as well as create a visual representation of the phenomena that occur within these smaller structures. They will analyze the independent and dependent variables in these situations.
https://www.nextgenscience.org/pe/ms-ls1-1-molecules-organisms-structures-and-processes

English:

- RST.6.8.3- following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- RST.6.8.4- demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- RST.6.8.7- distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- SL.6.1- engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly.

6.G: GEOMETRY

Cluster Statement: A: Solve real-world and mathematical problems involving area, surface area, and volume.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Find the area of triangles and special quadrilaterals. • Decompose and compose shapes into right triangles, triangles and quadrilaterals. • Apply understanding of finding area of triangles and quadrilaterals to finding area of irregular shapes that are made up of these shapes. • Solve real world and mathematical problems by applying these techniques. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	SMP 2: Students reason abstractly and quantitatively to explain why it is useful to compose and decompose shapes in finding the area of polygons with irregular shapes.	
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Find volume of a rectangular prism using formula (V=lwh and V=bh) and explain how this is the same as packing with unit cubes to find volume. • Apply this to using lengths that are fractional. • Solve real-world problems for volume involving fractional lengths of rectangular prisms. **Webb's Depth of Knowledge:** 1-2
6.G.A.2: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	SMP 2: Students reason abstractly and quantitatively by making sense of quantities and their relationships in the problem situation. Students solve for volume of a rectangular prism by finding the number of cubes that fit into the figure. SMP 3: Students construct viable arguments and critique the reasoning of others by making conjectures about the form and meaning of the solution attempt. SMP 6: Students attend to precision when specifying units of measure to clarify the correspondence with	

	quantities in the context of the problem.	**Bloom's Taxonomy:** Apply
Standard Text 6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**Standard for Mathematical Practices** SMP 1: Students make sense of problems and persevere in solving them by accessing their relevant knowledge and experiences and make appropriate use of them in working through the task. Teachers may ask "How might you use your previous knowledge to help you begin?" or "What do you notice about the shape compared to the coordinate points?".[1] SMP 6: Students attend to precision when specifying units of measure to clarify the correspondence with quantities in the context of the problem.	**Students who demonstrate understanding can:** • Draw polygons on the coordinate plane when given coordinates for vertices. • Find the side lengths of the polygons using coordinates. • Solve real-world problems by applying the use of drawing coordinates.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Apply
Standard Text 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**Standard for Mathematical Practices** SMP 3: Students construct viable arguments and critique the reasoning of other by making Mathematically proficient conjectures about the form and meaning of their representations. SMP 4: Students model with mathematics by creating three-dimensional figures using nets made up of rectangles and triangles. Students then analyze their models to draw conclusions and solve real-world problems.	**Students who demonstrate understanding can:** • Create a net (using triangles and rectangles) to represent three-dimensional figures. • Use nets to find surface area of three-dimensional figures. • Solve real-world problems by applying the use of nets of three-dimensional figures to find surface area.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Apply

[1] http://mathpractices.edc.org/pdf/Finding_Parallelogram_Vertices.pdf

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Learners build on their knowledge of area from Grade 3 where they count the area of a rectangle and connect it to their understanding of multiplication in Grade 4. Learners understand how to find the volume of right rectangular prisms with whole numbers in Grade 5. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. (4.MD.3)	• Learners are flexible using the terms base and length when solving for the area of a two or three-dimensional shape. Develop the concept of surface area. Learners understand how to find the volume of right rectangular prisms using fractions in the length of the edges. Connects to lessons on negative integers (6.NS.8) and graphing points in all quadrants. (6.RP.3.a) Find distance on coordinate plane by counting the units on the coordinate plane (no formula). Create polygons in quadrants I, II, III, and IV so learners can apply their knowledge of absolute value. (6.NS.7)	• In Grade 7, learners will continue to draw, construct, and describe geometrical figures and discover relationships between them (without nets). Calculate and compare the volume of cones, cylinders, and spheres. (8.G.C.9) Prepare for grade 8 work with transformations by working with polygons in coordinate plane. Learners will further their knowledge on distance in 8th grade when they start to find the lengths of diagonal lines. Learners will use their knowledge of the Pythagorean Theorem to find distance on the coordinate plane and later use the distance formula. In high school, learners will apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). In high school, learners will give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. In high school, students will use the idea of nets to identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Clarification Statement:

This cluster builds on previous understanding of area and volume to deepen the understanding of volume and develop the concept of surface area. Students use knowledge and skills to solve real-world and mathematical problems and apply the concepts by manipulating nets, cubes, and other real-world materials.

Common Misconceptions

- To find the area of shapes, students may believe that every shape has a unique formula when in reality area can always be found by decomposing the shape into non-overlapping areas.
- Students may also believe that two triangles with the same area may look exactly alike, when it is possible to have two triangles with the same area that are not congruent triangles.
- The vocabulary term "unit cube" may be difficult for students to understand as the unit cube is 1 unit. The focus with the unit cube should be on developing students understanding that each smaller cube represents a

fraction of the unit cube. In addition, once this understanding is developed, students can use these smaller parts and apply them to rectangular prisms. This application may provide difficult if students are unsure about multiplying fractions.

- Students may confuse the slant height and not recognize it for the height of the triangles in the net. Being that these are nets, students may only find one area and not the area of each individual part of the net and add them together. The concept of nets may be difficult for students to understand, specifically the translation from the 3-D figure to the net and how they coincide. This may need to be reinforced as to how a pyramid and a rectangular prism coincide to their nets.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying 6.G.A because the 6th grade standard builds upon 5.MD.C.3 & 5.MD.C.5 where they went from building arrays to using arrays to find area and volume. 6.G.A utilizes their previous understanding on shape composition and decomposition to understand and develop the formulas necessary for area, surface area and volume.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.MD.C.3, 5.MD.C.5 This standard provides a foundation for work with 6.G.A because they move from building to applying the concept of which the 6th grade standard then extends. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with solving real-world and mathematical problems involving area, surface area and volume benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation. Such as varying methods for response and navigation by providing alternatives to requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard. A hands-on approach using physical manipulatives such as pouring water to find volume, or the actual wrapping of a box in paper to find surface area are examples. Using technology to add marbles to water also helps to solidify the concept for students.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving real-world and mathematical problems involving area, surface area and volume benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities

because they learn from each other. For example, if using a box to find an area, the group has a physical box that they need to measure and calculate the area. To have equal access to learning the information, students who have clear goals, and specific roles and responsibilities give equal access to all students to learn.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with solving real-world and mathematical problems involving area, surface area and volume benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because distinguishing between area, surface area and volume using different forms will give equal access to all students to learn the key vocabulary concepts needed to be successful. For example, just word definitions of area and volume will not hold as much meaning to students and using illustrations, animation and physical objects to demonstrate area(2d), surface area(to cover outside of object) and volume(3d) in understanding the differences. This will help students remember which they need to use when given problems or tasks.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with solving real-world and mathematical problems involving area, surface area and volume benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using providing sentence starters or sentence strips because it allows students to demonstrate their understanding in several ways such as their native language, thought pictures, through verbal explanations etc.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with solving real-world and mathematical problems involving area, surface area and volume benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as embedding prompts to "show and explain your work" (e.g., portfolio review, art critiques) because students who can explain their work will be able to apply their knowledge in different concepts. This will help students to not just give a single number answer but to express their thought process. This can also clear up misconceptions that a student might have.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on 6.G.A by providing specific feedback to students on their work through a short mini

lesson because this can clear any misconceptions of incorrect formula usage and concept misunderstandings.

Re-teach (intensive) : *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit 6.G.A by confronting student misconceptions because students need to understand which formula is used during which time and that area is 2D volume is 3D. Therefore, it is important to make sure students have a solid foundational understanding of the vocabulary, formulas and concepts associated with this standard.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying to solve real-world and mathematical problems involving area, surface area, and volume because it is a skill that students need to improve upon. Making connections and generalizations between the area, surface area and volume of an object can help them deepen their understanding of the measurements and the formulas.

Culturally and Linguistically Responsive Instruction:

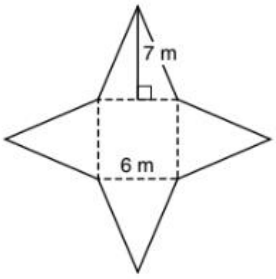
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying 6th grade geometry of solving real-world and mathematical problems involving area, surface area, and volume eliciting and using student thinking is critical because student thinking supports peer learning from different views and validating student thinking creates culture for students who contribute.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: Cognia Testlet for Grade 6 Geometry

1. A net of a solid figure is shown.



a. What solid figure is represented by the net?

b. What is the area, in square meters, of each triangle in the net? Show your work or explain how you know.

c. What is the surface area, in square meters, of the solid figure? Show your work or explain how you know.

6.G.01.04: Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

- Learning Target: I can use a net to name a figure and find the surface area of that solid figure.
- Webb's Depth of Knowledge: 2
- This type of assessment question requires students to calculate the area and surface area of a figure represented by a net. This task will allow a teacher to determine multiple levels of a student's understanding. First if they can identify a figure based on a net. Second, can they determine the are of the square. This requires a student to recall attributes of a square since only one dimension is shown. Last the student will need to calculate the surface area of the figure which requires a student to also know the area formula for a triangle and how to determine the base using the missing dimension from part B. In each part a student is expected to explain their answer which allows a teacher to assess their understanding at a deeper level.

Relevance to families and communities:

During a unit focused on solving real-world and mathematical problems involving area, surface area, and volume , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the mathematics used within the different careers of your family and community can provide a strong connections between school and careers. For example, how geometry of area, surface area and volume is used in one of New Mexico's economy in the oil field.

Cross-Curricular Connections:

Science & English:

- RST.6.8.3 Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
  RST.6.8.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 6-8 texts and topics*.
- RST.6.8.7 Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
- SL.6.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

NEW MEXICO
Public Education Department

6.SP: STATISTICS & PROBABILITY

Cluster Statement: A: Develop understanding of statistical variability.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.SP.A.1: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	SMP 1: Students make sense of problems and persevere in solving them by explain the meaning of statistical questions**.** SMP 2: Students reason abstractly and quantitatively to create a logical representation of the problem. "How does a statistical question anticipate variability?"	• Students will understand that their question promotes an investigation. • Students will understand the difference in quantitative (numerical) data to qualitative(categorical) data. • Students will develop a question that promotes variability in the data. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.SP.A.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	SMP 1: Students make sense of problems and persevere in solving them by explain the meaning of statistical questions**.** SMP 2: Students reason abstractly and quantitatively to create a logical representation of the problem. "How does a statistical question anticipate variability?" SMP 8: Students look for and express regularity in repeating reasoning and repeats the process of statistical reasoning in a variety of contexts.	• Be able to collect data by asking a statistical question. • Understand that a collected data set from a statistical question can be described using center, spread and overall shape. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand

<table>
<tr><td>Standard Text

6.SP.A.3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</td><td>Standard for Mathematical Practices

SMP 1: Students make sense of problems and persevere in solving them by describing the spread and center of the data based on the visual characteristics of the representations.[1]

SMP 2: Students reason abstractly and quantitatively by using quantitative reasoning that entails coherent representation of the single number, not just how to compute them.

SMP 6: Students attend to precision by accurately finding the measure of center and variability with a level of precision appropriate for the given context.</td><td>Students who demonstrate understanding can:
• Find and understand that measures of center (mean/median) summarize a set of data with a single number.
• Find and understand that measures of variation (range/MAD) describe a set of data's variability with a single number.

Webb's Depth of Knowledge: 1-2

Bloom's Taxonomy:
Understand, Analyze</td></tr>
<tr><td><u>Previous Learning Connections</u>
• In Grade 5, learners made line plots to display a data set of measures in fractions of a unit.</td><td><u>Current Learning Connections</u>
• Mean, median, mode and range are new concepts to 6th grade students. Students will create dot plots, histograms and box plots. They will draw inferences and make comparisons between them. Mastery includes finding mean, median, mode and interquartile range.</td><td><u>Future Learning Connections</u>
• In Grade 7, learners build on their understanding of interpreting information about a population by using population samples. In Grade 7, learners begin to look at two separate data sets to make comparisons. In high school, learners interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</td></tr>
<tr><td colspan="3">Clarification Statement:
Students will develop an understanding of statistical thinking. They will learn how to write statistical questions used to survey and collect data. They will study measures of center and variability with newly learned knowledge of mean, median, mode, and range. Students will discover that different ways to measure center produce different values and that interpreting measures of center for the same data develops the understanding of how each measure can change how the data gets interpreted.</td></tr>
<tr><td colspan="3">Common Misconceptions
• Students must shift their thinking from asking their question only about themselves to asking it in a larger population. Students are looking for a question that produces variability in the data. Students may try to ask a question of themselves such as "How big is my shoe size?" instead of asking the question of a larger</td></tr>
</table>

[1] http://mathpractices.edc.org/pdf/Creating_Data_Sets_from_Statistical_Measures.pdf

population as a class or school (What are the shoe sizes in my class? What are the shoe sizes in my school?). In addition, students may try to ask a question that does not produce variability in the data by asking a yes or no question (do you like playing football?) or that provides categorical data (Do you like cats or dogs?). Students may assume that asking someone what zip code they live in is numerical data. This would actually be classified as categorical.

- Students may have issues with the vocabulary word symmetrical. They may have trouble describing data when it is not a traditional visual representation they have studied before (dot plot, histograms, etc.). In addition, students may mix up mean and median and what their purpose is for representing the data set.
- The concept of center, spread, and shape may provide difficult vocabulary for students. As students begin to analyze variability, they may not understand the connection between range, spread, and variability are all the same concept. Students may have trouble calculating mean and median given a histogram or dot plot.
- Students may have trouble connecting that mean is the average, as it has previously been described this way.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying developing the understanding of statistical variability because students will need to become familiar with ideas around statistical data, measures of central tendency, variability and other new concepts.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.MD.B.2: This standard provides a foundation for work with developing the understanding of statistical variability because it focuses on 5th grade work that students have done using line plots to organize data and then fraction operations to interpret and solve problems with the data. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with developing understanding of statistical variability benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because as students work to recognize and understand statistical questions, including using measures of center and variability to describe them, this can easily be done by creating questions and data sets that are socially relevant to them therefore creating interest and connections with the content.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with developing understanding of statistical variability benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because as students identify with their community this will foster collaboration and connection with other students.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).*

- For example, learners engaging with developing understanding of statistical variability benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams because students can represent statistical data in many different ways. For example, calculating measures of center and variability, displaying data in number plots and graphs, and summarizing data in both words and verbally to form a convincing argument.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with developing understanding of statistical variability benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because students are able to describe statistical data in several ways and this allows for students to make decisions about which strategies to use to achieve their goal. They can also use relevant and interesting data for their statistical analysis as this easily lends itself to student choice and multiple representations.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with developing understanding of statistical variability benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) because as students recognize and understand what statistical data is and how it can be modeled, situations can be created to allow students to analyze relevant data and decide how to appropriately model it using measures of center and variability.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on developing the understanding of statistical variability by clarifying mathematical ideas and/or concepts through a short mini-lesson because as students use statistical data to solve problems, they will need practice and clarification on using measures of central tendency and variability to decide how to most effectively describe the data. This could be done in small groups using protocols to examine data and present appropriate data to answer a question.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit developing the understanding of statistical variability by confronting student misconceptions because there are so many new concepts in this cluster that looking at common misconceptions could help students avoid confusion. For example, students need to be clear on the difference between statistical data and categorical data and understand that only statistical data allows the use of measures of central tendency and variability to describe the data.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying developing the understanding of statistical variability because this cluster lends itself to using data from many sources. Students do not have to gather the data as they will in 7th grade but could use data from a current science or social studies concept to develop these skills.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

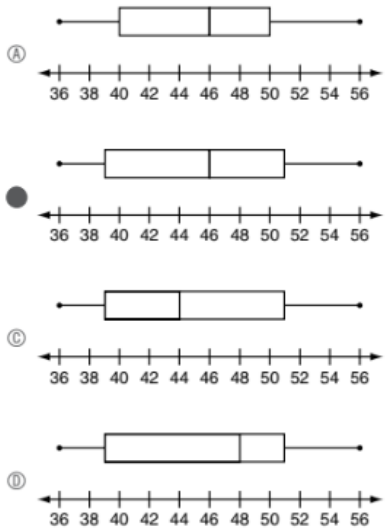
Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying the development in the understanding of statistical variability the types of mathematical tasks are critical because students can use data that is relevant to their home, school or social culture when working with statistical variability. The power in connecting mathematics to student's personal experiences and culture can easily be accessed through choosing (or even better, allowing students to choose) topics and statistical questions that are relevant and meaningful on a personal level. As mathematics becomes more personal, students can begin to identify as a mathematician.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 6 Statistics & Probability

. Look at this data set.

40, 36, 48, 52, 50, 44, 38, 56

Which box plot represents the data?



Relevance to families and communities:

During a unit focused on the development in the understanding of statistical variability, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can collect or use statistical data that answers relevant questions related to their family and community culture.

Cross-Curricular Connections:

Science:
Students can answer a question regarding their experiment by collecting data. This data can be displayed in different ways in addition to students finding the measures of center (mean and median) and describing the shape of the data. (MS-LS1-4, From Molecules to Organisms: Structures and Processes)
https://www.nextgenscience.org/pe/ms-ls1-4-molecules-organisms-structures-and-processes

English:

- RST.6.8.3- following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
- RST.6.8.4- demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.
- RST.6.8.7- distinguish among facts, reasoned judgment based on research findings, and speculations in a text.
- SL.6.1- engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners

	on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly. Social Studies: • RH.6-8.1-Students can apply their knowledge of center and shape of the data to answer questions regarding different graphs that are presented. This data that is collected and analyzed can help to answer a larger question surrounding the data set.

NEW MEXICO
Public Education Department

6.SP: STATISTICS & PROBABILITY

Cluster Statement: B: Summarize and describe distributions.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.SP.B.4: Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	SMP 1: Students make sense of problems and persevere in solving them by understanding how measures of center and measures of variability are represented by graphical displays and which displays reveal specific information (actual data values, number of data, mean, median, minimum, maximum, shape – symmetrical or skewed, etc.) relating to the data. SMP 2: Students reason abstractly and quantitatively by comparing different types of representations for a given data set noting the advantages and disadvantages for using particular representations.	• Understand and be able to calculate measure of center, and the quartile ranges. • Understand the when it is appropriate to use a dot plot, histogram and box plot. For example, a dot plot will show exact values for each piece of data, but a histogram will show how many pieces of data fell within a specific range. • Create and display data on number lines using dot plots, histograms and box plots.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
6.SP.B.5: Summarize numerical data sets in relation to their context, such as by: • 6.SP.B.5.A: Reporting the number of observations. • 6.SP.B.5.B: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	SMP 1: Students make sense of problems and persevere in solving them by understanding how measures of center and measures of variability are represented by graphical displays and which displays reveal specific information (actual data values, number of data, mean, median, minimum, maximum, shape – symmetrical or skewed, etc.) relating to the data.	• Correlate the number of observations to the sample size. • Express how sample size is represented in a dot plot vs histogram, vs box plot. • Identify the initial survey question as numerical vs categorical (quantitative vs qualitative) data. • Describe the data by reading the graph's labels (units used) • Use the correct context, describe the overall pattern including any

• 6.SP.B.5.C: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. • 6.SP.B.5.D: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	SMP 2: Students reason abstractly and quantitatively by comparing different types of representations for a given data set noting the advantages and disadvantages for using particular representations.	striking deviations such as outliers. • Compute the measures of center: median and/or mean. • Compute the measures of variability: interquartile range and/or mean absolute deviation. • Express how measures of center and variability change the shapes of distribution. **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Understand, Apply
<u>Previous Learning Connections</u> • In Grade 5, learners made line plots to display a data set of measures in fractions of a unit. They will build upon this skill in 6th grade by summarizing increasingly complex data sets in different contexts.	**<u>Current Learning Connections</u>** • Students will create dot plots, histograms and box plots. They will draw inferences and make comparisons between them. Students will also learn mean, median, mode and interquartile range which will connect in this cluster.	**<u>Future Learning Connections</u>** • In Grade 7, students build on their understanding of interpreting information about a population by using population samples. In Grade 7, students begin to look at two separate data sets to make comparisons. In the high school standards, learners interpret differences in shape, center, and spread in the context of the data sets,

Clarification Statement:
Students will develop an understanding of statistical thinking. They will use dot plots, histograms and box plots to draw inferences and make comparisons between data sets. Students should recognize that data distribution may not have a definite center and that interpreting those different measures of center can change how data gets interpreted.

Common Misconceptions
- Students may confuse the different visual representations (dot plot, histogram, number line, and box plots).
- When creating a box plot, students may have difficulty in correctly identifying the lower and upper quartile. Since this is median, it may need to be reinforced that data sets with even values will need to find the mean between the middle two numbers.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying how to summarize and describe distributions because it

allows refresher of prerequisite skills needed to be successful in understanding the whole concept like graphing on number lines.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.MD.B.2 *Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally*. This standard provides a foundation for work with summarizing and analyzing distributions because students are expected to skillfully plot data in fractions on a number line. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with summarizing and analyzing distributions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as descriptions (text or spoken) for dot plots, histograms, box plots in graphs, videos, or animations; auditory cues for key concepts and transitions in visual information; and vocabulary word wall for key terms because this cluster is loaded with vocabulary words that students may not have a clear comprehension of their meanings. By exposing students with easy access to this information will help students put the pieces together through seeing with visual graphics, hearing with its descriptions, and reading its definitions.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with summarizing and analyzing distributions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing prompts that guide learners in when and how to ask peers and/or teachers for help because this is a great way to keep the students from interrupting other students and teacher. A class routine on how to ask peers and or/teacher must have been clearly established and practiced in order to support student engagement. The Ask-Three-Before-Me strategy published by John Hopkins University found in: http://olms.cte.jhu.edu/olms2/data/ck/sites/273/files/18047_PT_CoopLearnHB%2025(1).pdf is one way to establish this routine.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling*

to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds).

- For example, learners engaging with summarizing and analyzing distributions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because it provides opportunities for students to be familiar in modeling the correct usage of statistical vocabulary and terms, thereby helping students correctly integrate these new concepts into their current understanding of numbers.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with summarizing and analyzing distributions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) because it allows students' expression of their own thinking and creativity.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with summarizing and analyzing distributions benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing graphic organizers and templates for data collection and organizing information because students will have an opportunity to always go back to the graphic organizers and templates that will help them internalize the information as they keep coming back.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on summarizing and analyzing distributions by providing specific feedback to students on their work through a short mini lesson because it enhances students' learning and achievement. The authentic, immediate feedback to students' work when provided at the real-time is as powerful as catching misconception or misunderstanding that needs fixed.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on summarizing and organizing distributions by confronting student misconceptions because it allows teachers to start by asking students what they think, acknowledge the process and confront them with facts. Students be clarified when confused with the concept of "mean absolute deviation" and "mean"; not be clear about the differences between bar graphs and histograms.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying how to summarize and analyze distributions because it provides the students with opportunities to explore and present their individual creativity. For instance, the open-ended task could be an example from Illustrative mathematics found in: http://tasks.illustrativemathematics.org/content-standards/6/SP/B/4/tasks/2047.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

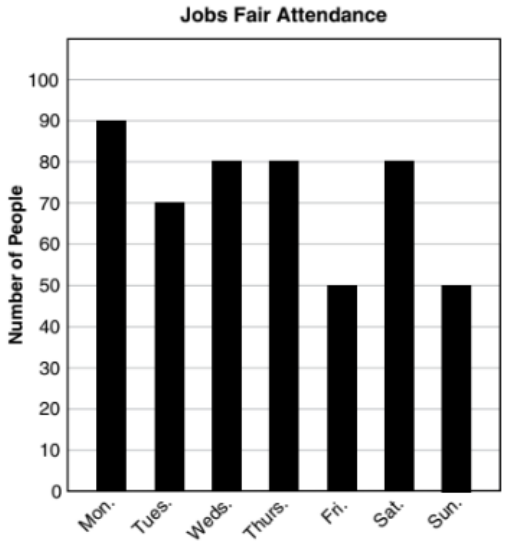
Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying summarize and describe distributions eliciting and using student thinking is critical because it allows for teachers to gather authentic information through formative and summative assessments that can be used to further support students' learning. During aggressive monitoring in the classroom, teachers listen carefully to student thinking and make note of which ideas to bring to the forefront of whole class discussions. It is helpful to create opportunities for students to share their thinking about distributions, their choice of measures of center and variability with their peers directly.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Testlet for Grade 6 Statistics & Probability

6.SP.B.5: Summarize numerical data sets in relation to their context, such as by: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

- Learning Target: I can use a bar graph to answer questions about a data set.
- Webb's Depth of Knowledge: 2
- This type of assessment question requires students to apply their knowledge of histograms and to understand the concept of mean and median to determine the solutions to part a-c. Part A will inform the teacher if a student understands how to read a histogram visually in order to calculate the total number of people who attended the fair. Part B will inform the teacher if the student can read the histogram visually in order to collect the data needed and then if they understand how to find the mean of a data set and can round the total accurately. Part C also requires a student to be able to read a histogram visually and apply that knowledge to calculate the median. A student also needs to know how to determine the median of a data set.

1. A school held its annual Jobs Fair. The number of people that attended each day of the fair is represented in this graph.



a. What is the total number of people that attended the job fair?

b. What is the **mean** number of people per day that attended the job fair, rounded to the nearest whole number? Show your work or explain how you know.

c. What is the **median** number of people per day that attended the job fair? Show your work or explain how you know.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on how to summarize and describe distributions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways data collection is used in the home and community can a be a great way to connect school tasks with home tasks.	Science: • Students will develop a question that they can study in regards to kinetic energy and possibly how temperature changes They will be able to look at data sets to determine the trends in the data. Specifically, students can see different results by the transfer of kinetic energy. Students can analyze the data set finding the mean, median, mean absolute deviation AND describe what these values mean in the context of the situation. (MS-PS3-4, Energy) https://www.nextgenscience.org/pe/ms-ps3-4-energy English: • RST.6.8.3- following precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. • RST.6.8.4- demonstrating the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grade 6-8 texts and topics.

	• RST.6.8.7- distinguish among facts, reasoned judgment based on research findings, and speculations in a text. • SL.6.1- engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 6 topics, texts, and issues building on other's ideas and expressing their own clearly. Social Studies: CCSS.ELA-LITERACY.RH.6-8.6/CCSS.ELA-LITERACY.RH.6-8.7-Students can conduct a question based on a question that relates to math (i.e. what is the average household income?). This data produces quantitative data and the measures of center and shape can be analyzed.

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest:** ? What do you anticipate in the range of student interest for this lesson? □ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ? How will you reduce barriers to perceiving the information presented in this lesson?	**Perception:** ? What do you anticipate about the range in how students will perceive information presented in this lesson? Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ? How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action:** ? What do you anticipate about the range in how students will physically navigate and respond to the learning experience? Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT ? How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence:** ? What do you anticipate about the range in student effort? Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ? How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ? What do you anticipate about the range of student background experience and vocabulary? Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ? How will the learning provide multiple	**Expression & Communication:** ? What do you anticipate about the range in how students will express their thinking in the learning environment? Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal

ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation:** ? What do you anticipate about barriers to student engagement? □ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension:** ? What do you anticipate about barriers to student comprehension? □ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ? How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions:** ? What do you anticipate about barriers to students demonstrating what they know? Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

> **Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
>
> **Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

> **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
>
> **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
>
> **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and $c \neq 0$, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.