

7.RP: RATIOS & PROPORTIONAL RELATIONSHIPS

Cluster Statement: A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<p>Standard Text</p> <p>7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 1: Students make sense of problems and persevere in solving them by examining how a ratio is a relationship between two quantities and a unit rate demonstrates the proportional relationship of two different units in real-world contexts. Students will persevere in demonstrating appropriate representations for these contexts.</p> <p>SMP 6: Students will attend to precision by demonstrating precision in the use of units to accurately represent ratios and unit rates.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Discover that the structure of computing unit rates with whole numbers is the same concept as unit rates with ratios of fractions. Compute unit rates in real-world problems that involve complex fractions. In writing, explain the errors that can be made when computing unit rates with complex fractions. <p>Webb's Depth of Knowledge:1-2</p> <p>Bloom's Taxonomy: Understand, Apply</p>
<p>Standard Text</p> <p>7.RP.A.2: Recognize and represent proportional relationships between quantities.</p> <ul style="list-style-type: none"> 7.RP.A.2.A: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. 7.RP.A.2.B: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and 	<p>Standard for Mathematical Practices</p> <p>SMP 5: Students use appropriate tools strategically by demonstrating their ability to choose appropriate tools to best represent proportional relationships such as graphs and/or the coordinate plane by a variety of methods (paper/pencil, software, etc.) to show the constant of proportionality.</p> <p>SMP 7: Students will look for and make use of structure by seeking patterns and structures in ratio tables in order to make connections between the constant</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Sort real-world examples from non-examples. Create own examples to demonstrate they understand the concept of proportional relationships. Communicate (orally/writing) that a proportion is a statement of two equivalent ratios. Model proportional relationships- concrete, visual, abstract (verbal [sentence], table, graph, equation). Prove or disprove proportional relationships between two points.

<p>verbal descriptions of proportional relationships.</p> <ul style="list-style-type: none"> • 7.RP.A.2.C: Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$. • 7.RP.A.2.D: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. 	<p>of proportionality in a table with the slope of a graph.</p>	<ul style="list-style-type: none"> • Determine appropriate representation of a proportional relationship. • Fluently assess and solve problems from various representations. • Model proportional relationships in several different ways. • Translate a proportional relationship from verbal, table, graph, equation. • Determine the unit rate from verbal, tables, graphs, equations, diagrams. • Connect that the unit rate is the pattern or numerical coefficient (k or m) of the equation $y=kx + b$ or $y = mx + b$. • Model proportional relationships in equation form. • Justify in writing the reasoning used to create an equation. • Explain the meaning of a point on a graph in context. • Discover that graphed proportional relationships are straight lines.
<p>Standard Text</p> <p>7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</p>	<p>Standard for Mathematical Practices</p> <p>SMP 3: Students construct viable arguments and critique the reasoning of others by engaging in mathematical discourse through the use of arguments by using a variety of models that demonstrate proportional relationships in multistep contexts and critique the reasonableness of others in real-world contexts.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> • Explore and connect vocabulary terms with real world examples. • Explain how they are used in each situation. • Solve problems proportional problems using cross-multiplication. • Solve percent error and percent increase/decrease problems
		<p>Webb's Depth of Knowledge: 1-2</p>
		<p>Bloom's Taxonomy: Understand, Apply</p>

	<p>SMP 8: Students look for and express regularity in repeated reasoning formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.</p>	<ul style="list-style-type: none"> Explain how formulas for percent error and increase/decrease are similar.
		<p>Webb's Depth of Knowledge: 1-2</p>
		<p>Bloom's Taxonomy: Understand, Apply</p>
<p><u>Previous Learning Connections</u></p> <ul style="list-style-type: none"> This cluster connects student learning from 6th grade with ratios. Students learned to understand, represent, compare, and reason with ratios. These skills will be necessary as students analyze proportional relationships. 	<p><u>Current Learning Connections</u></p> <ul style="list-style-type: none"> Students connect their understanding of rational numbers to solve for unit rates, proportional reasoning and percent problems throughout grade 7. 	<p><u>Future Learning Connections</u></p> <ul style="list-style-type: none"> Students will continue to connect their understandings of units as a way to understand problems and find the solution in a multi-step problem. Students choose and interpret units consistently in formulas, choose and interpret the scale and origin in graphs and data displays.
<p>Clarification Statement: Students will continue their work with ratios to analyze proportions and proportional relationships. Students expand their knowledge of unit rates to include computations with complex fractions. They recognize and represent proportional relationships in equations, in tables, and on graphs. Students use proportional reasoning to solve multi-step ratio and percent problems involving real world scenarios (percent change, sales tax, simple interest, etc.)</p>		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> Direct Versus Proportional Division: Mistakes occur when direct instead of proportional division is used. For example, if it takes 2 people 4 hours to do a certain task, students may mistakenly think that it would take 1 person 2 hours rather than 8 hours. (ASCD Source) When using a graph and locating the unit rate, students have difficulty identifying which variable the x, or y (x,y) is the unit rate. Using an example such as 1 orange for \$0.35, 1 is X and cost is Y. Common vocabulary words such as sale, discount, and tax. Student will come in with a variety of background knowledge with a concept of the meaning of this vocabulary. 		

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- In Grade 6, students learned to reason about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double-number-line diagrams. They also were introduced to special type of ratio called a rate. Provide opportunities to review terms, and methods for solving fraction division.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.RP.A.2 This standard provides a foundation for work with analyzing proportional relationships and using them to solve real-world and mathematical problems because teachers can help students develop the concept of unit rates. Its purpose is to help students see that when you have a context that can be modeled with a ratio and associated unit rate, there is almost always another ratio with its associated unit rate (the only exception is when one of the quantities is zero), and to encourage students to flexibly choose either unit rate depending on the question at hand. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because using different visual information will allow students to use their learning style to access information such as reading the concept, listening to concept, or having visual animation that allow students to see and using physical manipulative to touch the concept.

Build:

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because this will give students specific information on what you expect them to be able to accomplish and gives them a place to look for information that can help them to relook at the concepts.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because ratios is about comparing where they learned in fourth grade to determine equivalence. Ratios is also a multiplicative comparison therefore looking at fifth grade where they learned to interpret a fraction will activate students prior understanding and reasoning about ratios.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.)because there are three ways to look at a ratio and understanding that no matter the form used the outcome is the same.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need to practice ratio understanding as to not confuse it with fractions problems. The more the students have the opportunity to review and practice comparing ratios the increase reasoning skills and higher order thinking skills.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on analyzing proportional relationships and using them to solve real-world and mathematical problems by revisiting student thinking through a short mini-lesson because reviewing equivalent ratios and unit rates reminds students that they can find equivalent ratios using multiplication or division.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas. For example, some students may benefit from intensive extra time during and after a unit analyzing proportional relationships and using them to solve real-world and mathematical problems by offering opportunities to understand and explore different strategies because and make sure students understand the difference between rate and unit rate. Connect that unit rate is one of many representations of equivalent ratios they can find.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning: some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying analyzing proportional relationships and using them to solve real-world and mathematical problems because it advances students by challenging them to find unit rates using complex fractions and converting them to decimals.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying how to analyze proportional relationships and use them to solve real-world and mathematical problems the types of mathematical tasks are critical because students come to our classrooms with *Informal Knowledge/Funds of Knowledge*.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <https://tasks.illustrativemathematics.org/content-standards/tasks/1602>

DRILL RIG

A water well drilling rig has dug to a height of -60 feet after one full day of continuous use.

Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

If the rig has been running constantly and is currently at a height of -143.6 feet, for how long has the rig been running?

- This type of assessment question requires students to, when provided with a context for multiplying and dividing signed rational numbers, provide a means for understanding why the signs behave the way they do when finding products. It is possible to solve this problem with or without negative numbers, depending on how the numbers are interpreted. If depths below the earth are interpreted as negative numbers (in other words, as negative height above the earth's surface), then this problem provides a good context for multiplying and dividing negative numbers. If the teacher wishes for students to use negative numbers, students can be encouraged to model the problem with a number line: the most natural way to do this is to put 0 at the surface of the earth and represent depths below

<p>the earth with negative numbers. This has been incorporated into the statement of the problem in order to encourage this approach.</p> <ul style="list-style-type: none"> • This task complements the work students do with proportional relationships in grade 7 because the problem can be solved by reasoning with a proportional relationship, as shown in the first solution. For the first part of the task, students also need to make a conversion between days and hours. Because of the rate context (and signed numbers in the second solution) the teacher may wish to focus on setting up and understanding the problem, rather than on the arithmetic itself. In this case, use of calculators may be appropriate for this problem. 	
<p>Relevance to families and communities:</p> <p>How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?</p> <p>During a unit focused on how to analyze proportional relationships and use them to solve real-world and mathematical problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, by having students examine proportional relationships in different recipes. Having students make their favorite recipe that requires them to double or triple the ingredients based on the number of servings the recipe yields vs. the number of servings needed.</p>	<p>Cross-Curricular Connections:</p> <p>Science: Evaluate design solutions for maintaining biodiversity and probability of surviving and reproducing in specific environment.</p>