New Mexico Mathematics Instructional Scope for Seventh Grade

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3]Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.
[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

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Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text Full text of the standard	**Standard for Mathematical Practices** The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	**Students who demonstrate understanding can:** The cognitive skills students perform to demonstrate to comprehension of a standard. **Depth Of Knowledge:** Correlation of standard to Webb's Depth of Knowledge **Bloom's Taxonomy:** Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities: Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	**Cross Curricular Connections:** Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

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Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

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Ratios & Proportional Relationships

- 7.RP.A

The Number System

- 7.NS.A

Expressions & Equations

- 7.EE.A
- 7.EE.B

Geometry

- 7.G.A
- 7.G.B

Statistics & Probability

- 7.SP.A
- 7.SP.B
- 7.SP.C

7.RP: RATIOS & PROPORTIONAL RELATIONSHIPS

Cluster Statement: A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. ***For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction*** $^{1/2}/_{1/4}$ ***miles per hour, equivalently 2 miles per hour.***	SMP 1: Students make sense of problems and persevere in solving them by examining how a ratio is a relationship between two quantities and a unit rate demonstrates the proportional relationship of two different units in real-world contexts. Students will persevere in demonstrating appropriate representations for these contexts. SMP 6: Students will attend to precision by demonstrating precision in the use of units to accurately represent ratios and unit rates.	• Discover that the structure of computing unit rates with whole numbers is the same concept as unit rates with ratios of fractions. • Compute unit rates in real-world problems that involve complex fractions. • In writing, explain the errors that can be made when computing unit rates with complex fractions. **Webb's Depth of Knowledge:**1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text **7.RP.A.2: Recognize and represent proportional relationships between quantities.** • **7.RP.A.2.A: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.** • **7.RP.A.2.B: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and**	**Standard for Mathematical Practices** SMP 5: Students use appropriate tools strategically by demonstrating their ability to choose appropriate tools to best represent proportional relationships such as graphs and/or the coordinate plane by a variety of methods (paper/pencil, software, etc.) to show the constant of proportionality. SMP 7: Students will look for and make use of structure by seeking patterns and structures in ratio tables in order to make connections between the constant	**Students who demonstrate understanding can:** • Sort real-world examples from non-examples. • Create own examples to demonstrate they understand the concept of proportional relationships. • Communicate (orally/writing) that a proportion is a statement of two equivalent ratios. • Model proportional relationships- concrete, visual, abstract (verbal [sentence], table, graph, equation). • Prove or disprove proportional relationships between two points.

verbal descriptions of proportional relationships. • **7.RP.A.2.C: Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*** • **7.RP.A.2.D: Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.**	of proportionality in a table with the slope of a graph.	• Determine appropriate representation of a proportional relationship. • Fluently assess and solve problems from various representations. • Model proportional relationships in several different ways. • Translate a proportional relationship from verbal, table, graph, equation. • Determine the unit rate from verbal, tables, graphs, equations, diagrams. • Connect that the unit rate is the pattern or numerical coefficient (k or m) of the equation y=kx + b or y = mx + b. • Model proportional relationships in equation form. • Justify in writing the reasoning used to create an equation. • Explain the meaning of a point on a graph in context. • Discover that graphed proportional relationships are straight lines.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply
Standard Text **7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.***	**Standard for Mathematical Practices** SMP 3: Students construct viable arguments and critique the reasoning of others by engaging in mathematical discourse through the use of arguments by using a variety of models that demonstrate proportional relationships in multistep contexts and critique the reasonableness of others in real-world contexts.	**Students who demonstrate understanding can:** • Explore and connect vocabulary terms with real world examples. • Explain how they are used in each situation. • Solve problems proportional problems using cross-multiplication. • Solve percent error and percent increase/decrease problems

	SMP 8: Students look for and express regularity in repeated reasoning formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.	• Explain how formulas for percent error and increase/decrease are similar. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Previous Learning Connections • This cluster connects student learning from 6th grade with ratios. Students learned to understand, represent, compare, and reason with ratios. These skills will be necessary as students analyze proportional relationships.	**Current Learning Connections** • Students connect their understanding of rational numbers to solve for unit rates, proportional reasoning and percent problems throughout grade 7.	**Future Learning Connections** • Students will continue to connect their understandings of units as a way to understand problems and find the solution in a multi-step problem. Students choose and interpret units consistently in formulas, choose and interpret the scale and origin in graphs and data displays.

Clarification Statement:
Students will continue their work with ratios to analyze proportions and proportional relationships. Students expand their knowledge of unit rates to include computations with complex fractions. They recognize and represent proportional relationships in equations, in tables, and on graphs. Students use proportional reasoning to solve multi-step ratio and percent problems involving real world scenarios (percent change, sales tax, simple interest, etc.)

Common Misconceptions
- Direct Versus Proportional Division: Mistakes occur when direct instead of proportional division is used. For example, if it takes 2 people 4 hours to do a certain task, students may mistakenly think that it would take 1 person 2 hours rather than 8 hours. (ASCD Source)
- When using a graph and locating the unit rate, students have difficult identifying which variable the x, or y (x,y) is the unit rate. Using an example such as 1 orange for $0.35, 1 is X and cost is Y.
- Common vocabulary words such as sale, discount, and tax. Student will come in with a variety of background knowledge with a concept of the meaning of this vocabulary.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- In Grade 6, students learned to reason about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double-number-line diagrams. . They also were introduced to special type of ratio called a rate. Provide opportunities to review terms, and methods for solving fraction division.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.RP.A.2 This standard provides a foundation for work with analyzing proportional relationships and using them to solve real-world and mathematical problems because teachers can help students develop the concept of unit rates. Its purpose is to help students see that when you have a context that can be modeled with a ratio and associated unit rate, there is almost always another ratio with its associated unit rate (the only exception is when one of the quantities is zero), and to encourage students to flexibly choose either unit rate depending on the question at hand. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as (text or spoken) for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because using different visual information will allow students to use their learning style to access information such as reading the concept, listening to concept, or having visual animation that allow students to see and using physical manipulative to touch the concept.

Build:

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because this will give students specific information on what you expect them to be able to accomplish and gives them a place to look for information that can help them to relook at the concepts.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because ratios is about comparing where they learned in fourth grade to determine equivalence. Ratios is also a multiplicative comparison therefore looking at fifth grade where they learned to interpret a fraction will activate students prior understanding and reasoning about ratios.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.)because there are three ways to look at a ratio and understanding that no matter the form used the outcome is the same.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with analyzing proportional relationships and using them to solve real-world and mathematical problems benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need to practice ratio understanding as to not confuse it with fractions problems. The more the students have the opportunity to review and practice comparing ratios the increase reasoning skills and higher order thinking skills.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on analyzing proportional relationships and using them to solve real-world and mathematical problems by revisiting student thinking through a short mini-lesson because reviewing equivalent ratios and unit rates reminds students that they can find equivalent ratios using multiplication or division.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas For example, some students may benefit from intensive extra time during and after a unit analyzing proportional relationships and using them to solve real-world and mathematical problems by offering opportunities to understand and explore different strategies because and make sure students understand the difference between rate and unit rate. Connect that unit rate is one of many representations of equivalent ratios they can find.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning: some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying analyzing proportional relationships and using them to solve real-world and mathematical problems because it advances students by challenging them to find unit rates using complex fractions and converting them to decimals.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying how to analyze proportional relationships and use them to solve real-world and mathematical problems the types of mathematical tasks are critical because students come to our classrooms with *Informal Knowledge/Funds of Knowledge*.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://tasks.illustrativemathematics.org/content-standards/tasks/1602
DRILL RIG
A water well drilling rig has dug to a height of –60 feet after one full day of continuous use.
Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?
If the rig has been running constantly and is currently at a height of –143.6 feet, for how long has the rig been running?

- This type of assessment question requires students to, when provided with a context for multiplying and dividing signed rational numbers, provide a means for understanding why the signs behave the way they do when finding products. It is possible to solve this problem with or without negative numbers, depending on how the numbers are interpreted. If depths below the earth are interpreted as negative numbers (in other words, as negative height above the earth's surface), then this problem provides a good context for multiplying and dividing negative numbers. If the teacher wishes for students to use negative numbers, students can be encouraged to model the problem with a number line: the most natural way to do this is to put 0 at the surface of the earth and represent depths below

<table>
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the earth with negative numbers. This has been incorporated into the statement of the problem in order to encourage this approach.
This task complements the work students do with proportional relationships in grade 7 because the problem can be solved by reasoning with a proportional relationship, as shown in the first solution. For the first part of the task, students also need to make a conversion between days and hours. Because of the rate context (and signed numbers in the second solution) the teacher may wish to focus on setting up and understanding the problem, rather than on the arithmetic itself. In this case, use of calculators may be appropriate for this problem.
</td></tr>
<tr><td>Relevance to families and communities:

How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?
During a unit focused on how to analyze proportional relationships and use them to solve real-world and mathematical problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, by having students examine proportional relationships in different recipes. Having students make their favorite recipe that requires them to double or triple the ingredients based on the number of servings the recipe yields vs. the number of servings needed.</td>
<td>Cross-Curricular Connections:

Science: Evaluate design solutions for maintaining biodiversity and probability of surviving and reproducing in specific environment.</td></tr>
</table>

<table>
<tr><th colspan="3">7.NS: THE NUMBER SYSTEM</th></tr>
<tr><td colspan="3">Cluster Statement: A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

• 7.NS.A.1.A: Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
• 7.NS.A.1.B: Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
• 7.NS.A.1.C: Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 4: Students can use tools strategically by choosing between number lines and other manipulatives to demonstrate the meaning of the operations.

SMP 8: Students look for and express regularity in repeated reasoning by formulating rules for integer operations by creating and exploring several examples of models.</td>
<td>Students who demonstrate understanding can:
• Solve numerical addition and subtraction equations by using the properties of operations
• Define and apply the commutative, associative, and additive identity properties to rational numbers
• Formulate rules for integer operations
• Expressively (orally and in writing) express understanding of “positive”, “negative”, “additive inverse”, and “zero”
• Model combining positive and negative numbers and provide a rationale for their solutions
• Apply mathematics to real-world examples of positive and negative numbers</td></tr>
<tr><td>Webb’s Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom’s Taxonomy:
Understand</td></tr>
</table>

• **7.NS.A.1.D: Apply properties of operations as strategies to add and subtract rational numbers.**		
Standard Text **7.NS.A.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.** • **7.NS.A.2.A: Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.** • **7.NS.A.2.B: Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts.** • **7.NS.A.2.C: Apply properties of operations as strategies to multiply and divide rational numbers.** • **7.NS.A.2.D: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.**	**Standard for Mathematical Practices** SMP 5: Students use appropriate tools strategically by demonstrating their ability to select and use the most appropriate tool (paper/pencil, manipulatives, and calculators) while solving problems with rational numbers. SMP 6: Students attend to precision by using correct terminology and symbols and labeling units correctly. Students use precision in calculation by checking the reasonableness of their answers and adjusting accordingly.	**Students who demonstrate understanding can:** • Discover the rules for multiplying rational numbers Refer to the negative (-) sign correctly as "negative" or 'the opposite of" to make sense of real-world context. • Conclude that properties of the operations for multiplication are still applicable to rational numbers. • Use reasoning to determine that division by zero is undefined. • Discover that division as the inverse of multiplication still applies to rational numbers. • Generalize rules for division with signed numbers from examples. Use and articulate notations interchangeably $p \div (-q)$ is the same as $p/-q$. Interpret a rational quotient • Clarify their own understanding of the relationship between multiplications and division of rational numbers through writing. • Develop fluency through practice with multiplication and division of rational numbers. • Use properties of the operations to explain the solutions to real world problems. • Clarify and explain their understanding of properties of operations using mathematical discourse. • Use math vocabulary appropriately.

		• Use long division to convert rational numbers in fraction form to decimal form • Explain why and how they know a long division quotient will repeat. • Sort the decimal form of a rational numbers into two types: terminating or repeating.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply
Standard Text **7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)**	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively by representing and solving real world situations using visuals, numbers, and symbols. They demonstrate abstract reasoning by translating numerical sentences into real world situations. SMP 3: Students construct viable arguments and critique the reasoning of others by discussing rules for operations with rational numbers using appropriate terminology and tools/visuals. Students apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position. SMP 4: Students model with mathematics by modeling their understanding of rational number operations using tools such as algebra tiles, counters, visuals, and number lines and connect these models to solve problems involving real-world situations.	**Students who demonstrate understanding can:** • Apply operations with rational numbers to problems that involve the order of operations • Solve mathematical problems that use the four operations with rational numbers • Compute with complex fractions **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• In grade 6, learners understand that positive and negative numbers are used together to describe quantities having opposite directions or values. In grade 6, learners solve problems involving fractions by fractions. In grade 6, learners use order of operations to solve problems.	• In grade 7, learners apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. In grade 7, learners solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form. In grade 7, learners use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.	• In grade 8, learners understand that there are numbers that are not rational and approximate them by rational numbers. In grade 8, learners use square root and cube root symbols to represent solutions to equations of the form x6 2 = p and x 3 = p, where p is a positive rational number.

Clarification Statement:
Students learn to add and subtract rational numbers. As students begin this work visual representations are critical; they become less necessary as students become more fluent with these operations. In sixth grade, students found the distance of horizontal and vertical segments on the coordinate plane. In seventh grade, students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line. This standard allows for adding and subtracting of negative fractions and decimals and interpreting solutions in given context. Students should learn to use the terms "rational numbers", "additive inverse", and "integers" with increasing precision.

Common Misconceptions
The major misconceptions in this cluster are around the conceptualization of integer operations and the properties of subtraction. Students may struggle with using the number line to understand positive and negative numbers as distances from zero. It is important to use models to allow students to visualize rational numbers. When moving into operations, subtraction can lead to misconceptions when students must recall that subtraction is not commutative. They also may find difficulties with conceptualization when expressing that subtraction is the same as adding the inverse.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying application and extension of previous understandings of operations with fractions to add, subtract, multiply, and divide with rational numbers because students learn best when concepts are connected and they can "see" the connection across and within grade levels. This allows a familiarity and comfort level to approach new or different tasks using what they already know.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.NS.C.5 This standard provides a foundation for work with applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers because in Grade 6, the number line is extended to include negative numbers. Students initially encounter negative numbers in contexts where it is natural to describe both the magnitude of the quantity, e.g. vertical distance from sea level in meters, and the direction of the quantity (above or below sea level). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with applying and extending previous understandings of addition and subtraction to add and subtract rational numbers of multiplication and division and of fractions to multiply and divide rational numbers, and representing addition and subtraction on a horizontal or vertical number line diagram benefit when learning experiences include ways to recruit interest such as utilizing classroom instructional routines to involve all students because these routines, done regularly, can benefit all students, though they are particularly supportive of English Language Learners or those struggling with the linguistic components of math. It allows for clearer understanding and application of concepts by spotlighting any misconceptions and reinforcing accurate computation. For example, using MLR-1(stronger and clearer each time), a discussion can be opened for students to talk about positive and negative numbers in real world situations and pre-assessment of prior knowledge can be determined. Then that prior knowledge can be corrected, clarified, or built upon.

Build:

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with applying and extending previous understandings of addition and subtraction to add and subtract rational numbers, of multiplication and division and of fractions to multiply and divide rational numbers, and representing addition and subtraction on a horizontal or vertical number line diagram benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as displaying the learning goals in multiple way because then students become active members of the learning process. They are invested in the process and outcome becomes relevant and applicable. It increases ownership and generates attainable benchmarks.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with applying and extending previous understandings of addition and subtraction to add and subtract rational numbers, of multiplication and division and of fractions to multiply and divide rational numbers, and representing addition and subtraction on a horizontal or vertical number line diagram benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because this opportunity accommodates for various learning styles and is leveled for each learners abilities to communicate. Students are encouraged to represent their thinking at their comfort level while seeing a variety of representations. Moves scholars from concrete to visual to abstract while acknowledging their understanding.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with applying and extending previous understandings of addition and subtraction to add and subtract rational numbers, of multiplication and division and of fractions to multiply and divide rational numbers, and representing addition and subtraction on a horizontal or vertical number line diagram benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because directing and facilitating classroom activities and learning tasks which elicit evidence of learning, activating learners as instructional resources for one another, and activating learners as owners of their own learning. In all three cases, by actively engaging students in the doing of mathematics, manipulatives provide a foundation which encourages discussion and student ownership of their work.

Internalize:

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with applying and extending previous understandings of addition and subtraction to add and subtract rational numbers of multiplication and division and of fractions to multiply and divide rational numbers, and representing addition and subtraction on a horizontal or vertical number line diagram benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) because information becomes accessible and likely to be assimilated by learners when it is presented in a way that primes, activates, or provides any pre-requisite knowledge. Barriers and inequities exist when some learners lack the background knowledge that is critical to assimilating or using new information. However, there are also barriers for learners who have the necessary background knowledge but might not know it is relevant. Those barriers can be reduced when options are available that supply or activate relevant prior knowledge, or link to the prerequisite information elsewhere.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers by clarifying mathematical ideas and/or concepts through a short mini-lesson because it identifies and corrects misconceptions, allows for quick formative checks for understanding to move learning forward.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas For example, some students may benefit from intensive extra time during and after a unit applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers by confronting student misconceptions because it allows response to instruction as well as to response to intervention.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning : For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying application and extension of previous understandings of operations with fractions to add, subtract, multiply, and divide with rational numbers because extension and/or enrichments using rich mathematical tasks can be essential for engaging learners and creating dynamic classrooms. Rich modeling tasks are often the missing piece in problem-solving experiences in the classroom.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying application and extension of previous understandings of operations with fractions to add, subtract, multiply, and divide with rational numbers, goal setting is critical because it provides students opportunities to use mathematics to understand and investigate meaningful situations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://tasks.illustrativemathematics.org/content-standards/7/NS/A/3/tasks/298
Sharing Prize Money
The three seventh grade classes at Sunview Middle School collected the most box tops for a school fundraiser, and so they won a $600 prize to share among them. Mr. Aceves' class collected 3,760 box tops, Mrs. Baca's class collected 2,301, and Mr. Canyon's class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

This type of assessment question requires students to be able to reason abstractly about fraction multiplication as it would not be realistic for them to solve it using a visual fraction model. Even though the numbers are too messy to draw out an exact picture, this task still provides opportunities for students to reason about their computations to see if they make sense. To introduce and scaffold, prompts such as, "Which class should get the

most prize money? Should Mr. Aceves' class get more or less than half of the money? Mr. Aceves' class collected about twice as many box tops as Mr. Canyon's class - does that mean that Mr. Aceves' class will get about twice as much prize money as Mr. Canyon's class?" This task also represents an opportunity for students to engage in Standard for Mathematical Practice 5 Use appropriate tools strategically.	
Relevance to families and communities: During a unit focused on apply and extend previous understanding of operations with fractions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, fractions are used for cooking, the amount of ingredients may need to increase or decrease based on the members of the family. Being able to convert fractions is essential.	**Cross-Curricular Connections:**[1] *Science and Technology:* Science and math are intimately connected, particularly in fields such as chemistry, astronomy and physics. Students who can't master basic arithmetic skills will struggle to read scientific charts and graphs. More complex math, such as geometry, algebra and calculus, can help students solve chemistry problems, understand the movements of the planets and analyze scientific studies. Math is also important in practical sciences, such as engineering and computer science. Students may have to solve equations when writing computer programs and figuring out algorithms. Nursing majors may have great bedside manner. but they also need to know how to precisely calculate dosages to pass their courses. *Social Studies:* Social studies classes, such as history, often require students to review charts and graphs that provide historical data or information on ethnic groups. In geography classes, students might need to understand how the elevation of an area affects its population or chart the extent to which different populations have different average life spans. Knowledge of basic mathematical terms and formulas makes statistical information accessible *Literature and Writing:* Literature might seem like a far cry from math but mastering basic arithmetic can enable students to better understand poetry. The meter of poetry, the number of words to include in a line and the effect that certain rhythms have on the reader are all products of mathematical calculations. At a more mundane level, math can help students plan reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. The linear, logical thinking used in mathematical problems can also help students write more clearly and logically. *Art/Music:* Students interested in pursuing careers in theater, music, dance or art can benefit from basic

[1] *Thompson, Van. (2020, June 24). How Is Mathematics Used in Other Subjects?. sciencing.com. Retrieved from https://sciencing.com/how-is-mathematics-used-in-other-subjects-9861185.html*

	mathematical knowledge. Musical rhythm often follows complex mathematical series, and math can help students learn the basic rhythms of dances used in ballet and theater performances. Art thrives on geometry, and students who understand basic geometric formulas can craft impressive art pieces. Photographers use math to calculate shutter speed, focal length, lighting angles and exposure time.

7.EE: EXPRESSIONS & EQUATIONS

Cluster Statement: A: Use properties of operations to generate equivalent expressions.

Major Cluster (Students should spend much of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

<table>
<tr>
<td rowspan="3">Standard Text

7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 2: Students reason abstractly and quantitatively by using expressions in different forms to understand how quantities in an equation are related. For example, students reflect upon each step when solving and identify properties they are using. Students demonstrate quantitative reasoning by representing and solving real world situations using visuals, equations, inequalities and linear relationships into real world situations.

SMP4 Students model with mathematics by writing expressions and equations to model contextual problems. Students will model an understanding of expressions, equations, inequalities, and graphs using tools such as algebra tiles/blocks, counters, protractors, compasses, and visuals to represent real world situations.

SMP 6: Students attend to precision when communicating their reasoning using precise mathematical vocabulary. Students demonstrate precision by correctly using numbers, variables and symbols to represent expressions, equations and linear relationships, and correctly label units. Students use precision in calculation by checking the reasonableness of their answers and adjusting accordingly. Students will use appropriate algebraic language to describe the</td>
<td>Students who demonstrate understanding can:
• Identify properties of operations (Associative, Commutative, and Distributive).
• Use properties of operations to create equivalent expressions.
• Write expressions in standard or expanded form.</td>
</tr>
<tr>
<td>Webb's Depth of Knowledge: 1-2</td>
</tr>
<tr>
<td>Bloom's Taxonomy:
Remember, Understand</td>
</tr>
</table>

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	steps in rewriting expressions and solving equations.	
Standard Text **7.EE.A.2: Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."***	**Standard for Mathematical Practices** SMP 4: Students model with mathematics by writing expressions and equations to model contextual problems. Students will model an understanding of expressions, equations, inequalities, and graphs using tools such as algebra tiles/blocks, counters, protractors, compasses, and visuals to represent real world situations. SMP 7: Students look for and make use of structure by routinely seeking patterns or structures to model and solve problems. Students apply properties to generate equivalent expressions (i.e. 6 + 2x = 2 (3 + x) by distributive property) and solve equations (i.e. 2c + 3 = 15, 2c = 12 by subtraction property of equality; c=6 by division property of equality).	**Students who demonstrate understanding can:** • Use properties to create equivalent expressions. • Rewrite an expression in different forms. • Demonstrate how quantities in an equation are related. • Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide. • Solve real-life and mathematical problems. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand
<u>Previous Learning Connections</u> • In 6th grade, learners extend their knowledge of creating equivalent expressions to include situations in which a knowledge of the rules of integers are needed. In 6th grade, learners extend their understanding of repeated addition as multiplication (representing 3 + 3 + 3 + 3 as 4 x 3), to simplifying variable expressions (j+ j+ j+ j) be written as 4j. In 6th grade, using order of operations, learners broaden their work solving equations and inequalities to include those with more than one step, as well as those with negative coefficients.	**<u>Current Learning Connections</u>** • In 7th grade, learners will develop an understanding of operations with rational numbers when working with expressions and linear equations. In 7th grade, learners will apply knowledge of working with expressions and equations to solve problems involving scale drawings and informal geometric constructions, and work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume. In 7th grade, learners will use vertical angles, adjacent angles, angles on a line, and angles at a point in a multi-step problem to write and solve equations for an unknown angle in a figure.	**<u>Future Learning Connections</u>** • In 8th grade, learners will solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms In 8th grade, learners will use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

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Clarification Statement:
Students apply properties of operations to add, subtract, factor and expand linear equations with rational coefficients. Students then become able to rewrite expressions in different forms to solve a multi-step problem, explain the quantities and graph a solution.

Common Misconceptions:
When an expression has several steps, sometimes students forget to follow the order of operation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies:

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- In grade 6, students learned to read and interpret parts of an expression by using mathematical terms and viewing expressions as single entities. Review definition of expression contrasted to equations. Identify parts of an expression. Review and practice Order of Operations
- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying writing, reading, evaluating algebraic expressions and identifying/generating equivalent expressions because this cluster requires the acquisition of a considerable amount of new vocabulary. The terms that are used to identify the parts and types of expressions will support students in becoming proficient in explaining and discussing many new concepts encompassed in expressions, equations, and inequalities. This is the first experience students have with things such as variables, coefficients, constants, and they will also be learning how to extend previous learning of exponents, order of operations, sums, differences products, quotients, equivalent, like and unlike terms, etc.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 3.OA.B.5: This standard provides a foundation for work with using properties of operations to generate equivalent expressions because this standard lays the foundation for using properties as strategies to multiply and divide. At this level students do not have to know the name of the properties, but they are using them to develop commutative and associative properties of multiplication with whole numbers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with using properties of operations to generate equivalent expressions benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because the contextualized problems will allow students to generate equivalent expressions by understanding the relationship between the quantities while still providing relevance to their learning. For example, the concept of doubling the cost of dinner and a movie is the same as applying the distributive property to the cost of dinner and the movies.

Build:

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using properties of operations to generate equivalent expressions benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because equivalent expressions can help students deepen their understanding of the connections between the quantities. For example, generating equivalent expressions can allow students to examine the inverse operations in an expression.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using properties of operations to generate equivalent expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for unfamiliar references within the text because properties of operations can be easily confused or forgotten and that shouldn't stop learners from generating equivalent expressions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using properties of operations to generate equivalent expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives because students can physically generate expressions to test their equivalence.

Internalize:

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using properties of operations to generate equivalent expressions benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using multiple examples and non-examples to emphasize critical features because understanding why an expression isn't equivalent can help students deepen their understanding of equivalence.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on using properties of operations to generate equivalent expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because combining like terms, factoring and expanding linear equations are examples of using properties of operations. Having an explicit mini lesson on the distributive property as a method for expanding linear equations will support students in understanding the connection between the properties and generative equivalent expressions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit using properties of operations to generate equivalent expressions by helping students move from specific answers to generalizations for certain types of problems because properties of operations are generalized statements to help students understand the structure and pattern of expressions. Taking time to allow students to make the generalization from specific examples will help students deepen their understanding of using the properties to generate equivalent expressions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying using properties of operations to generate equivalent expressions because the properties of operations are applied to find structure and patterns for students in math. Other disciplines have their own concepts that support students when applied. Understanding the concept of going from generalizations to specific examples and then from specific examples to generalizations can help students deepen their understanding of the need for properties. For example, the classification system in Science.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying, using properties of operations to generate equivalent expressions eliciting and using student thinking is critical because when generating equivalent expressions students will be applying different strategies and skills such as factoring, expanding and combining like terms. Students may not feel that they have the academic vocabulary to explain their thought process, but they can show their work through acting it out or simplifying the expressions which will provide evidence of their thinking.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source:
http://s3.amazonaws.com/illustrativemathematics/attachments/000/009/275/original/public_task_1450.pdf?1462395838)

Maria is at an amusement park. She bought 14 tickets, and each ride requires 2 tickets.

a. Write an expression that gives the number of tickets Malia has left in terms of *x*, the number of rides she has already gone on. Find at least one other expression that is equivalent to it.
b. $14-2x$ represents the number of tickets Malia has left after she has gone on *x* rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?
    1. 14
    2. -2

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<table>
<tr><td colspan="2">
3. 2x
c. 2(7−x) also represents the number of tickets Malia has left after she has gone on *x* rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?
4. 7
5. (7−x)
6. 2

This type of assessment question requires students to illustrate how different, but equivalent, algebraic expressions can reveal different information about a situation represented by those expressions. This task can be used to motivate working with equivalent expressions, which is an important skill for solving linear equations and interpreting them in contexts. The task also helps lay the foundation for students' understanding of the different forms of linear equations they will encounter in 8th grade. In part (b), the task asks students to interpret pieces of the expression that arise by parsing the expression from different algebraic perspectives. It requires students to think about the difference between interpreting −2x as −2 times x vs. subtracting 2x from 14. Note that the meaning of the 2 in the expression 2(7−x) is slightly different than the meaning given in the problem statement because of the role it plays in the expression. The class will probably need to have a whole-group conversation to grasp this subtlety.

This task helps illustrate Mathematical Practice 7, Look for and make use of structure. As students work with equivalent expressions in this task, they interpret what these different numbers and expressions mean in terms of the context. For example, 14 – 2x and 2 (7 –x) are equivalent expressions but in terms of the problem, these expressions reveal very different information about tickets and rides. Students are engaged in connecting the real-life context to the structure of the mathematics.
</td></tr>
<tr><td>Relevance to families and communities:
During a unit focused on using properties of operations to generate equivalent expressions , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, writing expressions that represent situations your family and community might experience. Students should make sure they know what the real world meaning each part of the expression represents (term, operation, variable, etc.) Then students can create an equivalent expression and discuss what the new parts of the expression mean in reference to your family or community and the original expression.</td>
<td>Cross-Curricular Connections:

Science: Students can write number sentences for conservation of energy of a system.</td></tr>
</table>

NEW MEXICO
Public Education Department

7.EE: EXPRESSIONS & EQUATIONS		
Cluster Statement: B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **Major Cluster** (Students should spend much of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text: **7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.** ***For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $^{1}/_{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 $^{3}/_{4}$ inches long in the center of a door that is 27 $^{1}/_{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.***	**Standard for Mathematical Practices** SMP 5: Students use appropriate tools strategically by demonstrating their ability to select and use the most appropriate tool (pencil/paper, manipulatives, calculators, protractors, etc.) while rewriting/evaluating/analyzing expressions, solving and representing and analyzing linear relationships. SMP 8: Students use repeated reasoning to understand algorithms and generalize about patterns. During multiple opportunities to solve and model problems, they may notice that a/b ÷ c/d = ad/bc and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers.	**Students who demonstrate understanding can:** • Solve multi-step real life and mathematical problems that include positive and negative rational numbers • Convert between fractions, decimals, and percentages • Use properties of operations as needed to solve the problems. • Justify the reasonableness of their answers using estimation **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Remember, Understand

Standard Text: **7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.** • **7.EE.B.4.A: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.** ***For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*** • **7.EE.B.4.B: Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.** ***For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.***	**Standard for Mathematical Practices:** SMP 1: Students solve real world problems through the application of algebraic concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". SMP 3: Students construct viable arguments and critique the reasoning of others when students discuss the differences among expressions, equations and inequalities using appropriate terminology and tools/visuals. Students will apply their knowledge of equations and inequalities to support their arguments and critique the reasoning of others while supporting their own position.	**Students who demonstrate understanding can:** • Write equations in the appropriate form. • Solve and graph inequalities • Apply the inequality and the solution in the context of the problem. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand
Previous Learning Connections • In 6th grade, students use variables to represent numbers and write expressions when solving a real-world or mathematical problem with equations or expressions. This connects directly to this cluster as students build upon this	**Current Learning Connections** • In 7th grade, students will develop an understanding of operations with rational numbers when working with expressions and linear equations. They will use these skills later in 7th grade when applying these skills to scale	**Future Learning Connections** • In 8th grade students solve linear equations (including rational number coefficients) in one variable with one solution, infinitely many solutions, or no solutions. In 8th grade, learners analyze and solve pairs of simultaneous linear

skill with multiple step problems and the inclusion of rational numbers.	drawings, geometric constructions, area, and volume.	equations (in one and two variables).

Clarification Statement:
Students apply properties of operations to add, subtract, factor and expand linear equations with rational coefficients. Students then become able to rewrite expressions in different forms to solve a multi-step problem, explain the quantities and graph a solution.

Common Misconceptions
- Students may have difficulty with representing numbers in different forms such as moving from a percentage to a fraction.
- Students may need support scaffolding multi-step problems that require steps that build upon each other.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*
- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying solving real life and mathematical problems using numerical and algebraic expressions and equations because this cluster focuses on solving two step equations/inequalities and the previous 6th grade cluster focused on one-step equations. Providing time for students to struggle and to determine how to apply their previous knowledge from one step-equations can help clear up misconceptions because students will have had time to develop their thought process instead of just going through steps.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*
- 6.EE.B.7 This standard provides a foundation for work with solving real life and mathematical problems using numerical and algebraic expressions and equations because in this standard students are expected to solve real world and mathematical problems in the form of $x + p = q$ and $px=q$, which are one step equations with positive rational numbers. In the 7th grade cluster, students are introduced to two step equations & inequalities with positive and negative rational numbers. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:
Interest: *How will the learning for students provide multiple options for recruiting student interest?*
- For example, learners engage with solving real life and mathematical problems using numerical and algebraic expressions and equations benefit when learning experiences include ways to recruit interest such as supporting culturally relevant connections because students will be invested in understanding and engaging with the mathematics.

Build:

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving real life and mathematical problems using numerical and algebraic expressions and equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because students need to see that numeric and algebraic equations and expressions are relevant to their life. Students will be more engaged and persist in their learning if they can see the cultural connections.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving real life and mathematical problems using numerical and algebraic expressions and equations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf because real life and mathematical problems tend to rely heavily on textual information that can hinder a student's ability to solve problems. Providing the text in a student's language will allow the teacher to determine if the student's struggles are mathematical.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving real life and mathematical problems using numerical and algebraic expressions and equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem because depending on a student's strengths and backgrounds they will approach problems in a variety of different ways using a variety of different strategies.

Internalize:

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using properties of operations to generate equivalent expressions benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using multiple examples and non-examples to emphasize critical features because understanding why an expression isn't equivalent can help students deepen their understanding of equivalence.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on solving real life and mathematical problems using numerical and algebraic expressions and equations by examining tasks from a different perspective through a short mini-lesson because students often struggle with the concept of an inequality versus an equation, even though solving both is very similar. By looking at a task through the perspective of needing one answer versus a number set students may be able to deepen their understanding of solving an equation/inequality.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas For example, some students may benefit from intensive extra time during and after a unit solving real life and mathematical problems using numerical and algebraic expressions and equations by addressing conceptual understanding because in this cluster students are solving two-step equations/ inequalities. Students might forget to keep the equation/inequality in balance when solving. Teachers can check this by having them use algebra tiles when solving equations/inequalities.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning about ... For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying solving real life and mathematical problems using numerical and algebraic expressions and equations because of the link between expressions, equations and inequalities. What is similar, different, what generalizations about each can be made? What do we know about the solutions for each?

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying to solve real life and mathematical problems using numerical and algebraic expressions and equations facilitating meaningful mathematical discourse is critical because these real life and mathematical problems tend to have multiple entry points for students in order to solve the problem. Students should be able to enter the problem at their level

and then take the task to a higher level through connections to previous learning or to additional strategies. Allowing students to discuss the mathematical strategy they used to solve the problem provides them a voice and an opportunity to share their thinking with the group in a way that is okay to be different.	
Standards Aligned Instructionally Embedded Formative Assessment Resources: *Source:* http://tasks.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643 A coach buys a uniform and a basketball for each of the 15 players on the team. Each basketball costs \$9. The coach spends a total of \$420 for uniforms and basketballs. Enter the cost, in dollars, of 1 uniform. This type of assessment question requires students to illustrate an uptick from students work in grade 6 (6.EE.B.7) where they were required to write and solve simple equations of the form x + p = q and px = q to the grade 7 work of writing and solving real-world problems leading to equations of the form px + q= r, where *p, q* and *r* are specific rational numbers. Items assessing this standard could also involve coefficients that aren't whole numbers and aren't positive numbers. This item assesses the modeling aspects of 7. EE.B.4a, not the part of the standard about fluency with solving algebraic equations.	
Relevance to families and communities: During a unit focused on solving real life and mathematical problems using numerical and algebraic expressions and equations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about calculating the cost of bills within a budget for a family. Students could write an expression or equation for each bill for the month. Students could even create an inequality with the amount of money set aside for bills so they could determine the amount of discretionary money left after paying the bills.	**Cross-Curricular Connections:** Science: • Collaborate with peers to define or describe an issue in society and how to evaluate solutions. • Run tests of solutions and change designs as needed. • Construct scientific arguments for how uneven distributions of Earth's Mineral, energy, groundwater resources are the result of past and current geoscience processes. Examples: Metal ores, volcanic activity, soil weathering, rock deposits, mining by humans.

7.G: GEOMETRY

Cluster Statement: A: Draw construct and describe geometrical figures and describe the relationships between them.
Additional Cluster (Students should spend much of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	SMP 2: Students reason abstractly and quantitatively as they are expected to think about the relationships between the numbers, not just compute them. SMP 4: Students model with mathematics by using opportunities to represent the problem, situation and/or their solution symbolically, graphically, and/or pictorially. Students can apply the geometry concepts they know to solve problems arising in everyday life, society and the workplace. This may include applying area and surface of 2-dimensional figures to solve interior design problems or surface area and volume of 3-dimensional figures to solve architectural problems.	• Solve problems involving scale drawings. • Calculate length and area from scale drawings. • Reproduce a scale drawing at a different scale. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
7.G.A.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	SMP 5: Students consider available tools that might include concrete models, a ruler, a protractor, or dynamic geometry software such as virtual manipulatives and simulations. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.	• Draw a geometric figure with given conditions. • Explain why a set of given conditions does (or does not) produce the desired figure. • Measure side lengths and angle measures with given tools. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand

<table>
<tr>
<td rowspan="3">Standard Text

7.G.A.3: Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 6: Students attend to precision by refining their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students determine quantities of side lengths represented with variables, specify units of measure, and label geometric figures accurately. Students use appropriate terminology when referring to geometric figures.

SMP 8: Students look for and express regularity in repeated reasoning by noticing if calculations are repeated and looking both for general methods and for shortcuts.</td>
<td>Students who demonstrate understanding can:
Identify the two-dimensional cross-sections that are formed by slicing three-dimensional figures.
Describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids.</td>
</tr>
<tr>
<td>Webb’s Depth of Knowledge: 1-3</td>
</tr>
<tr>
<td>Bloom’s Taxonomy:
Understand, Apply, Analyze</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
In 5th grade, learners classify two-dimensional figures in a hierarchy based on properties. In 6th grade, learners understand and solve ratios and rates, generate equivalent ratios, and use ratios and rates to solve problems. In 6th grade, learners calculate perimeter & area of two-dimensional figures and find volume of 3D figures. In 6th grade, students explore the characteristics of a right rectangular prism and rectangular pyramid.</td>
<td><u>Current Learning Connections</u>
In 7th grade, learners can expand their work with expressions and equations as they write and solve equations related to similar figures, scale drawings, and the missing angle measures of triangles. In 7th grade, learners’ work with similar figures supplements the concepts they have already learned (or will be learning) when studying direct variation and proportional reasoning.</td>
<td><u>Future Learning Connections</u>
In 8th grade, learners connect their previous understanding of similar figures with the properties of translations, rotations, reflections and dilations. In 8th grade, learners build on their experimentation with triangles and start to make informal arguments about their properties, such as angle sum, exterior angles of a triangle, and angles created when parallel lines are cut by a transversal line. In 8th grade, learners build on knowledge of triangle side lengths which leads to the investigation of the Pythagorean Theorem and its converse.</td>
</tr>
<tr>
<td colspan="3">Clarification Statement:
Students work to draw and construct geometric shapes, particularly triangles from given angle and side measurements. Students find relationships and connections between a 3D figure and slicing it into a plane figure. Students use scale drawings to find the actual lengths from scale drawing or redrawing a scale drawing to another scale.</td>
</tr>
</table>

Common Misconceptions:

7.G.A- To minimize errors, have students use graph paper to make their scale drawings. Students without a solid grasp of measurement units such as those for area, will have difficulty with this standard, as will students who need more help with proportional reasoning. Use the opportunity to measure the classroom or other hands on measurements to reinforce measurement units for those students.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies:

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- During a unit focused on drawing, constructing, and describing geometrical figures and describe the relationships between them, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about geometric shapes are used in cultural art and design connects the students' home connections to the mathematical principles they are learning at school.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.NF.B.4: This standard provides a foundation for work with solving problems involving scale drawings of geometric figures because students will do best if they have procedural fluency with their use of fractions with operations. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:

- For example, learners engaging with drawing, constructing and describing geometrical figures and describing the relationships between them benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as drawing freehand with ruler and protractor, using physical manipulatives, and technologies because students may struggle with limited modalities of engaging with the geometrical figures, a selection of navigational tools across ability groups will allow students to develop conceptual understanding of the construction and relationship between geometrical figures.

Build: Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with drawing, constructing and describing geometrical figures and describing the relationships between them benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports (e.g., peer-tutors) because students will deepen their conceptual understanding through the process of explaining how their geometrical figures meet certain conditions with their peers. This allows students to make connections to what they already know to what they are learning as they make sense of their understanding with each other.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with drawing, constructing and describing geometrical figures and describing the relationships between them benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.) because students' understanding is supported when math vocabulary and symbols are processed with students in multiple ways. Providing charts/materials with students creates shared language around terms and symbols and provides for clarification and deeper understanding to allow for access for all students.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with drawing, constructing and describing geometrical figures and describing the relationships between them benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips because students will be able to describe geometrical figures and the relationships between them when given the opportunity to negotiate their understanding through dialogue with peers. When educators provide sentence starters/sentence strips, students are supported in practicing the language structures and functions expected to be used with the mathematical content.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with drawing, constructing and describing geometrical figures and describing the relationships between them benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because student learning deepens with given opportunities to practice and review concepts. Students make connections from prior learning to new information when allowed time to manipulate the new information through multiple exposures. Students develop a variety of strategies when able to engage with concepts over time.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on drawing, constructing, and describing geometrical figures and describing the relationships between them by clarifying mathematical ideas and/or concepts through a short mini-lesson because when students explain their thinking and have time to process their learning misconceptions or gaps in learning can be addressed.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas. For example, some students may benefit from intensive extra time during and after a unit to drawing, constructing, and describing geometrical figures and describing the relationships between them by helping students move from specific answers to generalizations for certain types of problems because at times students can become too focused on the specific area within the cluster without stepping back to see the connection across the cluster, such as, students using a tool to measure the angles of triangles and are missing the larger connections of geometric principles across shapes for determining geometric conditions.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning about drawing, constructing, and describing geometrical figures and describing the relationships between them, some learners may benefit from an extension such as open ended tasks linking multiple disciplines because students benefit when math understandings are applied to other areas of content and real-world application such as architecture, art, reconstructive surgery, etc.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying drawing, constructing, and describing geometrical figures and describing the relationships between them facilitating meaningful mathematical discourse is critical because the standards are asking students to solve, create and describe geometric figures which lends itself to providing opportunities to purposely plan discourse for students to share their ideas and methods. In that discourse utilizing protocols that validate students' contributions connected with home culture and home language will lower students' affective filters and allow them to take risks. The protocols need to be created in a way that removes teacher's, often unknowingly, biases. These protocols should provide students opportunities to rehearse their ideas in a small group or team and then a random process of calling on students to share their thinking to the class. This affirms that all contributions are wanted and needed to build the knowledge of the whole These types of processes affirm students that their home cultures and languages are positive assets as their contributions become a part of the curriculum. How we set up this process

is crucial. We need to have direct conversations about how different cultures talk, show body language, respond, etc. so students develop a class culture that is open and affirming.	
Standards Aligned Instructionally Embedded Formative Assessment Resources: *Source:* https://tasks.illustrativemathematics.org/content-standards/7/G/A/1/tasks/107 **Floor Plan** Mariko has an 80:1 scale-drawing of the floor plan of her house. On the floor plan, the dimensions of her rectangular living room are 1 7/8 inches by 2 ½ inches. What is the area of her real living room in square feet? Answer: 208 1/3 ft^2 This type of assessment question requires students to translate between measurements given in a scale drawing and the corresponding measurements of the object represented by the scale drawing. If used in an instructional setting, it would be good for students to have an opportunity to see other solution methods, perhaps by having students with different approaches explain their strategies to the class. Students who can only solve this by first converting the linear measurements will have a hard time solving problems where only area measures are given; scaffold this by having students work in groups, partners; and/or provide prefilled worksheets.	
Relevance to families and communities: During a unit focused on drawing, constructing, and describing geometrical figures and describe the relationships between them, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about geometric shapes are used in cultural art and design connects the students' home connections to the mathematical principles they are learning at school.	**Cross-Curricular Connections:** Science: Model the Solar System at Scale Art: Geometric Drawings/ Architectural Drawing

<table>
<tr><th colspan="3">7.G: GEOMETRY</th></tr>
<tr><td colspan="3">Cluster Statement: B: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</td><td rowspan="3">Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by solving problems involving geometric principles.

SMP 4: Students can model with mathematics by using geometric models to solve problems.</td><td>Students who demonstrate understanding can:
Explain the relationships between radius and diameter.
Explain that the ratio of circumference to diameter can be expressed as pi.
Apply formulas to determine area, circumference, diameter, and radius of a circle to solve real-world problems.
Solve real world problems involving circumference and area of a circle.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply</td></tr>
<tr><td rowspan="3">Standard Text

7.G.B.5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</td><td rowspan="3">Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by solving problems involving geometric principles.

SMP 4: Students can model with mathematics by using geometric models to solve problems.</td><td>Students who demonstrate understanding can:
Use understandings of angles (supplementary, complementary, vertical, adjacent) and deductive reasoning to write and solve equations.
Write and solve equations based on a diagram of intersecting lines with some known angle measures.
Justify angle measurements using facts about complementary, supplementary, vertical and/or adjacent angles.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply</td></tr>
</table>

<table>
<tr>
<td rowspan="3">Standard Text

7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by solving problems involving geometric principles.

SMP 4: Students can model with mathematics by using geometric models to solve problems.</td>
<td>Students who demonstrate understanding can:
Calculate the area, volume and surface area of two-dimensional and three-dimensional objects.
Explain why the formula works and how the formula relates to the measure (area and volume) and the figure.
Solve real-world problems involving geometry concepts such as area, volume, and surface area.
Justify their solutions to problems involving area, volume, and surface area</td>
</tr>
<tr>
<td>Webb's Depth of Knowledge: 1-2</td>
</tr>
<tr>
<td>Bloom's Taxonomy:
Apply</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
In 4th grade, students learned how to find the area of rectangles, special quadrilaterals, triangles, and polygons. In 6th grade, students began to explore volume, finding the volume of rectangular prisms, find surface area using nets, and find volume of rectangular prism.</td>
<td><u>Current Learning Connections</u>
Throughout 7th grade, students will use their knowledge of angle measurements along with algebra to determine missing information about particular geometric figures.</td>
<td><u>Future Learning Connections</u>
In 8th grade, learners use the formulas from within this cluster to find the volume of cones, cylinders, and spheres.</td>
</tr>
<tr>
<td colspan="3">Clarification Statement:
Students work on geometric problem solving. Students use basic information such as area, surface area, and volume formulas and facts about types of angles (supplementary, complementary, vertical, and adjacent) to solve real-world problems.</td>
</tr>
<tr>
<td colspan="3">Common Misconceptions
7.G.B- The formulas for the area of a circle and the circumference of a circle are often confused by students. Teaching students to memorize these formulas without any understanding of how they relate to a circle increases the chance for confusion. Build the understanding before presenting the formulas.</td>
</tr>
<tr>
<td colspan="3">Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</td>
</tr>
</table>

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., scaled images) when studying Real-Life And Mathematical Problems Involving Angle Measure, Area, Surface Area, And Volume because students can use this skill to solve real-world mathematical problems.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.G.A.1: This standard provides a foundation for work with solving Real-Life and Mathematical Problems Involving Angle Measure, Area, Surface Area, And Volume because solving problems involving areas and volumes provide a context for developing and using equations. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access:
Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with solving real-life and mathematical problems involving angle measure, area, surface area, and volume benefit when learning experiences include ways to recruit interest such as utilizing classroom instructional routines to involve all students because providing instructional routines consistently that support the output of all students' voices send the message that all students are capable and their ideas are valued so in the long run students will persevere in challenging work.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving real-life and mathematical problems involving angle measure, area, surface area, and volume benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because students will gain skills of communication and collaboration in the activities that this cluster lends itself to as students apply their mathematical understandings to solving real-live problems.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving real-life and mathematical problems involving angle measure, area, surface area, and volume benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students have had previous learnings in the area most of the areas of this cluster and explicit instruction that makes connections for students with this content will support students to connect previously learned knowledge to new knowledge.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving real-life and mathematical problems involving angle measure, area, surface area, and volume benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives because students will benefit in giving explanations and descriptions of their created concrete and virtual geometric representations.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with solving real-life and mathematical problems involving angle measure, area, surface area, and volume benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing differentiated models of self-assessment strategies because the ability to self-assess and apply adaptive reasoning supports students to become flexible critical thinkers as these standards are expecting students to apply a geometric concepts to a variety of real-life problems. For example, students can receive feedback from peers and reassess the accuracy of their work.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on solving Real-Life And Mathematical Problems Involving Angle Measure, Area, Surface Area, And Volume by critiquing student approaches/solutions to make connections through a short mini-lesson because they can see multiple ways to solve a problem or reasons why a solution is incorrect or correct.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas some students may benefit from intensive extra time during and after a unit solving Real-Life And Mathematical Problems Involving Angle Measure, Area, Surface Area, And Volume by offering opportunities to understand and explore different strategies because they can see multiple ways to solve a problem or reasons why a solution is incorrect or incorrect and find new ways to approach a problem

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning about For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying solving Real-Life And Mathematical Problems Involving Angle Measure,

<table>
<tr><td></td><td>Area, Surface Area, And Volume because there are many applications of geometry in life that a student might have interest in, for example, building a treehouse</td></tr>
<tr><td colspan="2">Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying Solving real-life and mathematical problems involving angle measure, area, surface area, and volume the use of mathematical representations within the classroom is critical because understanding how to read or interpret geometry drawings and the nomenclature for representations can allow students to draw on their experiences.</td></tr>
<tr><td colspan="2">Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/7/G/A/1/tasks/107
The circumference of a circle is approximately 37.7 centimeters. Enter the radius of the circle, in centimeters. Round your answer to the nearest tenth.
Answer Key: 6.0

This type of assessment question requires students to work with area (begins in grade 3 where they develop an understanding of area as a two-dimensional measurement.) By grade 6, students extend that earlier work and find areas of triangles, special quadrilaterals and other polygons. This item shows the expectation for grade 7 where students are expected to know the formula for the circumference of a circle and use it to solve problems. Some assessment programs provide the circumference formula for items like the one shown, and others use a more literal interpretation of the phrase ''know the formulas'' found in standard 7.G.B.4 and do not provide those formulas for students. Item specifications or other documentation for specific programs can be helpful in determining the approach a particular assessment takes on this standard.</td></tr>
<tr><td>Relevance to families and communities:
During a unit focused on solving real-life and mathematical problems involving angle measure, area, surface area, and volume, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example,</td><td>Cross-Curricular Connections:[1]

Science and Technology: Science and math are intimately connected, particularly in fields such as chemistry, astronomy and physics. Students who can't master basic arithmetic skills will struggle to read scientific charts and graphs. More complex math, such as geometry, algebra and calculus, can help students solve chemistry problems,</td></tr>
</table>

[1] Thompson, Van. (2020, June 24). How Is Mathematics Used in Other Subjects?. *sciencing.com*. Retrieved from https://sciencing.com/how-is-mathematics-used-in-other-subjects-9861185.html

learning about how geometry problems are used in real life to solve an issue like, purchasing a new air conditioner gives students a connection on different ways solving real-life and mathematical problems involving angle measure, area, surface area, and volume are used in the home and community.	understand the movements of the planets and analyze scientific studies. Math is also important in practical sciences, such as engineering and computer science. Students may have to solve equations when writing computer programs and figuring out algorithms. Nursing majors may have great bedside manner. but they also need to know how to precisely calculate dosages to pass their courses. *Social Studies:* Social studies classes, such as history, often require students to review charts and graphs that provide historical data or information on ethnic groups. In geography classes, students might need to understand how the elevation of an area affects its population or chart the extent to which different populations have different average life spans. Knowledge of basic mathematical terms and formulas makes statistical information accessible *Literature and Writing:* Literature might seem like a far cry from math but mastering basic arithmetic can enable students to better understand poetry. The meter of poetry, the number of words to include in a line and the effect that certain rhythms have on the reader are all products of mathematical calculations. At a more mundane level, math can help students plan reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. The linear, logical thinking used in mathematical problems can also help students write more clearly and logically. *Art/Music:* Students interested in pursuing careers in theater, music, dance or art can benefit from basic mathematical knowledge. Musical rhythm often follows complex mathematical series, and math can help students learn the basic rhythms of dances used in ballet and theater performances. Art thrives on geometry, and students who understand basic geometric formulas can craft impressive art pieces. Photographers use math to calculate shutter speed, focal length, lighting angles and exposure time.

7.SP: STATISTICS & PROBABILITY

Cluster Statement: A: Use random sampling to draw inferences about a population.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	SMP 2: Students can reason abstractly and quantitatively by making generalizations and predictions based on random samples. SMP 3: Students can construct viable arguments by using statistical methods as justification for predictions inferences. SMP 4: Students can model with mathematics by developing probability models and use them to find probabilities of events. SMP 5: Students can use tools by using organized lists, tables, tree diagrams, and simulation tools.	• Critique examples of sampling as statistical tools using precise mathematical vocabulary; random sampling, population, and valid generalization. • Design random samplings to collect the data given statistical questions. • Defend the samplings as random. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	SMP 2: Students can reason abstractly and quantitatively by making generalizations and predictions based on random samples. SMP 3: Students can construct viable arguments by using statistical methods as justification for predictions inferences. SMP 4: Students can model with mathematics by developing probability models and use them to find probabilities of events.	• Draw valid inferences and generalizations from random samplings of populations • Justify their inferences and generalizations as valid using appropriate vocabulary • Explain the variability in multiple random samples and gauge how far off an estimate may be. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

	SMP 5: Students can use tools by using organized lists, tables, tree diagrams, and simulation tools.	
Previous Learning Connections • In 6th grade, learners summarize quantitative data using quantitative measures of center and variability.	**Current Learning Connections** • In 7th grade, learners focus on the process of selecting a random sample, and the value of doing so.	**Future Learning Connections** • In high school, students make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Clarification Statement:
Students learn about sampling populations and that a sampling must be representative of the population in order to make valid inferences and generalizations. To measure variation and estimates or predictions about a characteristic, students must conduct multiple samples of the same size from populations with unknown characteristics.

Common Misconceptions
- Use random sampling to draw inferences about a population
- The concept of random is difficult for some students. It may be necessary to physically demonstrate a random vs a non-random sampling to eliminate misconceptions. For example, a non-random sampling would be to ask all girls to stand up to answer a question about video game preferences. A random sample would be to ask every third student to answer the same question. Ask student to compare and contrast answers for each example.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*
- For example, some learners may benefit from targeted pre-teaching that introduces new representations when studying using random sampling to draw inferences about a population because the idea of a random sample is a new concept for students. They need time to understand what a random sample is and what it isn't before they are expected to make inferences based on one.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*
- 6.SP.A.1 This standard provides a foundation for work with using random sampling to draw inferences about a population because this standard is when students are introduced to a statistical question and the variability in data. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access
Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*
- For example, learners engaging with using random sampling to draw inferences about a population benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to

access and comprehend for many others such as displaying information in a flexible format to vary perceptual features because by providing descriptions (text or spoken) for all images, graphics, video, or animations; allowing use of touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; providing physical objects and spatial models to convey perspective or interaction, and/or providing auditory cues for key concepts and transitions in visual information ensures that all learners have equal access to information, in formats that encourages student to seek what is familiar and to explore additional options.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with drawing informal comparative inferences about two populations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as engaging learners in assessment discussions of what constitutes excellence because when students can evaluate their strengths and limitations, they can become more fluent in identifying their shortcomings and understanding the benefits of solid, relevant assessments. Instructions and vested interest are better aligned with intended outcomes.

Internalize
Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with drawing informal comparative inferences about two populations benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as using activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely because one of the key factors in learners losing motivation is their inability to recognize their own progress. It is important, moreover that learners have multiple models and scaffolds of different self-assessment techniques so that they can identify, and choose, ones that are optimal. Rubrics, examples and non-examples, timely feedback, and reflection encourages perseverance, recognition of progress and creates a less competitive environment conducive to further risk taking and exploration.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on using random sampling to draw inferences about a population by providing specific feedback to students on their work through a short mini-lesson because students need to make sure that the random sample is in fact a random representation of a population before any inferences can be made about the population.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas as some students may benefit from intensive extra time during and after a unit using random sampling to draw inferences about a population by confronting student misconceptions because there is variability in estimations and predictions and how to gauge the difference. Also, the need for multiple random samples.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying using random sampling to draw inferences about a population because the concept of random sampling and applying it to make inferences about a population is a large concept. Being able to provide extra time for students to explore the samples, and the variability will help students in other clusters.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying use of random sampling to draw inferences about a population, goal setting is critical because it provides students opportunities to use mathematics to understand and investigate meaningful situations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/7/SP/A/tasks/235
Election Poll, Variation 1
Members of the seventh-grade math group have nominated a member of their group to be class president. Every student in seventh grade will cast a vote. There are 2 candidates in the race and a candidate needs at least 50% of the votes to be elected. The math group wants to conduct a survey to assess their candidate's prospects. There are almost 500 students in the seventh grade at their school. They do not have the resources to interview all seventh graders so they have decided to interview a sample of 40 seventh graders. They will obtain the seventh-grade list of names from their school principal's office and select the sample from this list. They plan to ask each student in the sample whether they plan to vote for their candidate or the other candidate.

a. How should the students select the sample of 40 to have the best chance of obtaining a representative sample? Describe clearly how they could use the random number table provided below to select the sample of 40 students. "Clearly" means that someone other than you could duplicate the sampling process by following your description.

Random Number Table Generated in Excel				
196	14	57	441	219
459	284	356	306	119
358	241	406	122	390
238	98	392	433	256
335	189	24	260	452
468	106	28	294	46
20	385	37	109	4
437	70	464	471	432
454	474	1	280	117
492	390	154	115	336
460	377	101	312	350
115	126	64	333	291
445	297	449	171	234
438	224	357	13	500
288	284	254	86	173
449	340	11	9	387
359	133	494	31	458
217	174	343	3	350
171	195	127	141	276
299	246	394	164	294

b. Suppose that all 40 students selected from the list of seventh graders in the school respond to the survey, and the results showed that 18 students would vote for the math group's candidate. In order to get elected, a candidate must receive at least 50% of the votes. Some members of the math group believe that on the basis of this sample outcome it is unreasonable to think that their candidate can win. Others in the group believe that it is possible that their candidate might win. Based on the initial survey results, should the math group students be discouraged, or is it reasonable to think their candidate might win? Justify your response

- This type of assessment question requires students to discover the fundamental statistical ideas of using data summaries (statistics) from random samples to draw inferences (reasoned conclusions) about population characteristics (parameters). In the task built around an election poll scenario, the population is the entire seventh grade class, the unknown characteristic (parameter) of interest is the proportion of the class members voting for a specific candidate, and the sample summary (statistic) is the observed proportion of voters favoring the candidate in a random sample of class members.
- There are two important goals in this task: seeing the need for random sampling and using randomization to investigate the behavior of a sample statistic. These introduce the basic ideas of statistical inference and can be accomplished with minimal knowledge of probability.
- Random sampling (like mixing names in a hat and drawing out a sample) is not a new idea to most students, although the terminology is likely to be new. Most students readily grasp this as a fair way to select the sample because everyone gets an equal chance of being selected. Standard 1 uses the term "representative," which has no technical definition in statistics and might be interpreted in terms of fairness. Students should understand that most samples, even if randomly selected, would not have exactly the same characteristics as the population from which they came.
- Using simulation to repeatedly select random samples from a population with a specified proportion of successes will be a new idea to most students. Some discussion should revolve around this seemingly backward statistical notion of first specifying a population and then seeing if it could have produced the observed result as a reasonably likely outcome. Specifying the population structure allows the use of probability to determine the likelihood of the observed sample, and that is the basis of drawing statistical conclusions.

Relevance to families and communities:	**Cross-Curricular Connections:**
How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as	Science: Examining biological characteristics of a sample Social Studies: Population Sampling and Data Analysis

capable mathematicians that can use mathematics within school and society? For example, when studying use of random sampling to draw inferences about a population the types of mathematical tasks are critical because students come to our classrooms with Informal Knowledge/Funds of Knowledge.	

NEW MEXICO
Public Education Department

7.SP: STATISTICS & PROBABILITY

Cluster Statement: B: Draw informal comparative inferences about two populations.

Additional Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	SMP 5: Students choose appropriate mathematical and visual representations, including technology-based tools, to represent the data distributions. SMP 8: Students look for and express regularity in repeated reasoning. Students look to make generalized comparisons between situations that involve bias using specific criteria.	• Find measures of center and measures of variation for two or more data sets. • Compare two data sets for variability by comparing graphs. • Make inferences about data sets by comparing their statistical measures. • Model and compare two real-world data sets by measuring the difference between centers and expressing it multiple of a measure of variability **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Evaluate

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	SMP 1: Students make sense of problems and persevere in solving them. Students make sense of information by connecting visual, tabular, and symbolic representations of sample populations in real-life contexts. SMP 6: Students attend to precision to collect accurate measurement information from sample populations and precise language when generating and interpreting data.	• Draw valid comparative inferences about two populations. • Select the appropriate measure(s) of center (mean and median) or variability (MAD and IQR) when comparing two sets of data and justify that selection. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Evaluate

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• In 6th grade, learners develop an understanding of graphs, mean, median, mode, Mean Absolute Deviation (M.A.D.) and interquartile range (IQR). In 6th grade, learners recognize there will be variability in the data of a statistical question and will account for it in the answers. In 6th grade, learners understand a data set has a distribution which can be described by its center, spread, and overall shape and can summarize numerical data sets by reporting the number of observations along with describing the nature of the attribute under investigation and how it was measured and its units.	• This is an additional cluster, so the connections between this cluster and other grade level clusters is limited to 7.SP.A and 7.SP.C that examine different aspects of Statistics & Probability.	• In future courses, learners will represent data with plots on the real number line (dot plots, histograms, and box plots). In future courses, learners will use statistics appropriate to the shape and context of the data distribution to compare center (median, mean) and spread (IQR, standard deviation) of two or more different data sets. In future courses, learners will interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points.

Clarification Statement:
In this cluster students draw valid comparable inferences about two populations using measures of center (mean, median) and measures of variability (mean absolute deviation, interquartile range).

Common Misconceptions
Students may struggle with the key vocabulary utilized within in this cluster. It will be important to emphasize vocabulary acquisition.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying; Drawing informal comparative inferences about two populations because in previous clusters, students worked with one population.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.SP.A.2: This standard provides a foundation for work with Drawing informal comparative inferences about two populations because why and how inferences and generalizations are made helps to justify reasoning. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with using random sampling to draw inferences about a population benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information from a random sample in a flexible format to vary perceptual features such as providing a variety of representations including visuals, contexts, tables, graphs, and symbols because students are analyzing data from a sample to draw overall inferences for an entire population and students will benefit from having the information presented in multiple forms.

Build
Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using random sampling to draw inferences about a population benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as encouraging and supporting opportunities for peer interactions and supports because students can question or convince each other of the inferences they were able to make based on the data from the random sample.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using random sampling to draw inferences about a population benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students need to have an established understanding of a random sample before they can be expected to make inferences.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using random sampling to draw inferences about a population benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips because this will give students examples of inferences they should be able to draw from the random sample.

Internalize
Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using random sampling to draw inferences about a population benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students will be expected to draw their own inferences based on random samples and these explicit practice opportunities will support their learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on drawing informal comparative inferences about two populations by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may not understand why it may be necessary to conduct multiple samples of the same size

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas drawing informal comparative inferences about two populations by offering opportunities to understand and explore different strategies because students can organize by using lists, tables, tree diagrams, and simulations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

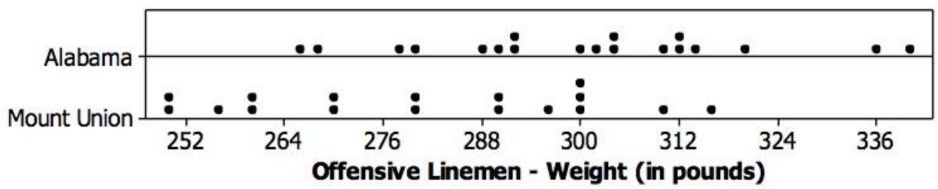
- To extend students learning about an opportunity to explore links between various topics when studying drawing informal comparative inferences about two populations because students can apply probabilities to real-life scenarios that link science disciplines for example, genetics and a Punnett square.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying drawing informal comparative inferences about two populations the types of mathematical tasks are critical because in this cluster of standards many of the ideas are new to students. We need to create learning opportunities that are focused on conceptual understanding as the entry point. We can build connections with students' cultures and languages as we purposefully work with students to use data that is relevant as they explore probability and develop models. This will also allow us to create opportunities for students to practice the situational appropriateness in the use of these mathematical principles in a variety of real-world situations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/7/SP/B/3/tasks/1341
Offensive Linemen
College football teams are grouped with similar teams into "divisions" (and in some cases, "subdivisions") based on many factors such as game attendance, level of competition, athletic department resources, and so on. Schools from the Football Bowl Subdivision (FBS, formerly known as Division 1-A) are typically much larger schools than schools of any other division in terms of enrollment and revenue. "Division III" is a division of schools with typically smaller enrollment and resources.
One particular position on a football team is called "offensive lineman," and it is generally believed that the offensive linemen of FBS schools are heavier on average than the offensive linemen of Division III schools.
For the 2012 season, the University of Mount Union Purple Raiders football team won the Division III National Football Championship while the University of Alabama Crimson Tide football team won the FBS National Championship. Below are the weights of the offensive linemen for both teams from that season.



a. Based on visual inspection of the dotplots, which group appears to have the larger average weight? Does one group seem to have greater variability in its weights than the other, or do the two groups look similar in that regard?

b. Compute the mean and mean absolute deviation (MAD) for each group. Do your measures support your answers in part (a)?

c. Choose from the following to fill in the blank: "The average Alabama offensive lineman's weight is about _________ than the average Mount Union offensive lineman's weight."

   i. 20 pounds lighter

   ii. 15 pounds lighter

   iii. 15 pounds heavier

   iv. 20 pounds heavier

2. "This difference in average weights is approximately ___________ of either team."

   i. About half of the MAD

   ii. Slightly more than 1 MAD

   iii. Twice the MAD

d. The offensive linemen on the Alabama team are not a random sample from all FBS offensive linemen. Similarly, the offensive linemen on the Mount Union Team are not a random sample from all Division III offensive linemen. However, for purposes of this task, suppose that these two groups can be regarded as random samples of offensive linemen from their respective divisions/subdivisions. If these were random samples, would you think that offensive linemen from FBS schools are typically heavier than offensive linemen from Division III schools? Explain your decision using answers to the previous questions and/or additional analysis.

In this task, students are able to conjecture about the differences and similarities in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability. This will reinforce that much can be gleaned simply from visual comparison of appropriate graphs, particularly those of similar scale. Since the two distributions have similar variability and almost identical MADs, students are able to express the difference in mean values with reference to the MAD of either group. As a possible extension, students can investigate if these distributions are in fact similar to the distributions of offensive lineman weights at similar schools (such as schools in the same respective divisions or conferences).

Relevance to families and communities:	**Cross-Curricular Connections:**[1]
How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? During a unit focused on investigating chance processes and developing, using, and evaluating probability models, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the about how probability is connected to games that families enjoy playing. Discussing how probability makes the games more or less interesting.	*Science and Technology:* Science and math are intimately connected, particularly in fields such as chemistry, astronomy and physics. Students who can't master basic arithmetic skills will struggle to read scientific charts and graphs. More complex math, such as geometry, algebra and calculus, can help students solve chemistry problems, understand the movements of the planets and analyze scientific studies. Math is also important in practical sciences, such as engineering and computer science. Students may have to solve equations when writing computer programs and figuring out algorithms. Nursing majors may have great bedside manner. but they also need to know how to precisely calculate dosages to pass their courses. *Social Studies:* Social studies classes, such as history, often require students to review charts and graphs that provide historical data or information on ethnic groups. In geography classes, students might need to understand how the elevation of an area affects its population or chart the extent to which different populations have different average life spans. Knowledge of basic mathematical terms and formulas makes statistical information accessible *Literature and Writing:* Literature might seem like a far cry from math but mastering basic arithmetic can enable students to better understand poetry. The meter of

[1] Thompson, Van. (2020, June 24). How Is Mathematics Used in Other Subjects?. *sciencing.com*. Retrieved from https://sciencing.com/how-is-mathematics-used-in-other-subjects-9861185.html

	poetry, the number of words to include in a line and the effect that certain rhythms have on the reader are all products of mathematical calculations. At a more mundane level, math can help students plan reading assignments in literature classes by discerning their average reading time and estimating how long it will take them to read a particular work. The linear, logical thinking used in mathematical problems can also help students write more clearly and logically. *Art/Music:* Students interested in pursuing careers in theater, music, dance or art can benefit from basic mathematical knowledge. Musical rhythm often follows complex mathematical series, and math can help students learn the basic rhythms of dances used in ballet and theater performances. Art thrives on geometry, and students who understand basic geometric formulas can craft impressive art pieces. Photographers use math to calculate shutter speed, focal length, lighting angles and exposure time.

NEW MEXICO
Public Education Department

<table>
<tr><th colspan="3">7.SP: STATISTICS & PROBABILITY</th></tr>
<tr><td colspan="3">Cluster Statement: C: Investigate chance processes and develop, use, and evaluate probability models.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td><td rowspan="3">Standard for Mathematical Practices

SMP 2: Students reason abstractly and quantitatively about the numerical values used to represent probabilities as values between 0 and 1.

SMP 7: Students look for and make use of structure when recognizing that probability can be represented in tables, visual models, or as a rational number.</td><td>Students who demonstrate understanding can:
• In writing, express the likelihood of a chance event with a probability range from 0 to 1.
• Recognize that the probability of any single event can be expressed with the terms impossible, unlikely, equally likely, likely, or certain.
• Express probability as a fraction, decimal or percent.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1</td></tr>
<tr><td>Bloom's Taxonomy:
Understand</td></tr>
<tr><td rowspan="2">Standard Text

7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</td><td rowspan="2">Standard for Mathematical Practices

SMP 4: Students model with mathematics when applying this standard to a real-world context using mathematical probability representations that are algebraic, tabular or graphic.

SMP 8: Students look for and express regularity in repeated reasoning by using repeated reasoning when approximating probabilities. They refine their approximations based upon experiences with data.</td><td>Students who demonstrate understanding can:
• Collect data on chance events (hands-on events such as spinning a spinner and simulations) and approximate the relative frequency of an event given the probability.
• Students recognize that as the number of trials increase, the relative frequency approaches the probability
• Explain the difference between relative frequency and theoretical probability using appropriate language
• Determine the sample space for a probability model</td></tr>
<tr><td>Webb's Depth of Knowledge: 2-3</td></tr>
</table>

New Mexico Public Education Department

		Bloom's Taxonomy: Understand, Apply
Standard Text **7.SP.C.7:** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. • **7.SP.C.7**. A: Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* • **7.SP.C.8:** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	**Standard for Mathematical Practices** SMP 3: Students construct viable arguments and critique the reasoning of others by approximating probabilities, creating probability models and explaining the reasoning for their approximations. They also question each other about the representations they create to represent probabilities. SMP 6: Students attend to precision by using precise language and calculations to represent probabilities in mathematical and real-world contexts.	**Students who demonstrate understanding can:** • Calculate the probability of a (simple) event as a fraction, decimal, or percent. • Determine the probability of events by developing uniform and non-uniform probability models (theoretical probability). • Compare the models to the observed frequency and explain their reasoning for any discrepancies between the model and the observed frequency using appropriate vocabulary. • Develop their understanding of probability by making predictions, comparing the predictions, replicating experiments, and comparing results.
		Webb's Depth of Knowledge: 2-3
		Bloom's Taxonomy: Apply, Analyze, Evaluate

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
7.SP.C.8: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. • **7.SP.C.8.A:** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. • **7.SP.C.8.B:** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. • **7.SP.C.8.C**: Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	SMP 1: Students make sense of probability situations by creating visual, tabular and symbolic models to represent the situations. They persevere through approximating probabilities and refining approximations based upon data. SMP 6: Students attend to precision by using precise language and calculations to represent probabilities in mathematical and real-world contexts.	• Understand similarities and differences between compound events and simple events. • Find the sample space of a compound event. • Create organized lists, tables, tree diagrams, and simulations to find the probability of a compound event. • Represent the probability of a compound event as a fraction, decimal, or percent. • Design and use a simulation (using a random number table, calculator, dice, cards, or other manipulatives) to generate frequencies of compound events. • Justify their selection of a particular situation and explain how it models a compound event **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
<u>**Previous Learning Connections**</u> • The 6.SP standards connect directly to this standard. In 6th grade, students approximated the probability of a chance event by collecting data on the chance process that produces it and observing	<u>**Current Learning Connections**</u> • In 7th grade, students will recognize and represent proportional relationships between quantities. In 7th grade, students also use proportional relationships to solve multistep ratio and	<u>**Future Learning Connections**</u> • In 8th grade, learners construct and interpret a two-way table summarizing data on two categorical variables collected from the same subject. In high school, learners recognize the purposes of and differences

its long-run relative frequency, and predict the approximate relative frequency given the probability. Students also used ratio and rate reasoning to solve real-world and mathematical problems.	percent problems. These skills continue	among sample surveys, experiments, and observational studies; explain how randomization relates to each. In high school, learners find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

Clarification Statement:
This cluster focuses on probability and is the first-time students encounter this topic formally. Students learn the likelihood of chance events and approximate probabilities. They investigate chance using probability models they develop. The cluster begins with single events and builds up to finding the probability of compound events using tree diagrams, lists, tables, and simulations.

Common Misconceptions

- Relative frequency may be difficult to understand, students may want to express this as probability. Explain how probability helps determine the approximate relative frequency.
- Reviewing and understanding vocabulary words is crucial for this standard. Use words frequently and have students discuss them during class discussion.
- Keeping lines straight when using tree diagrams can be difficult. Encourage students to use graph paper and a ruler in order to keep the outcomes apart from each other. There is a greater chance when students create lists randomly, they will miss one or more outcomes.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying; Investigate chance processes and develop, use, and evaluate probability models because the probability model is first introduced in this grade level and students may get confused more easily.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.RP.A.3: This standard provides a foundation for work with Investigate chance processes and develop, use, and evaluate probability models because understanding ratio concepts and using ration reasoning to solve problems will help in solving probability models. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with investigating chance processes and developing, using, and evaluating probability models benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because of the application aspect of this cluster of standards students will have an increased interest in applying the design of probability to areas of their own interest. Including students in the choice and creation of ways to test their ideas around data to examine probability and chance will support recruiting student interest.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with investigating chance processes and developing, using, and evaluating probability models benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because providing students with clear targets as they are creating and developing probability models will support them moving towards that target.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with investigating chance processes and developing, using, and evaluating probability models benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as making relationships between elements explicit because the terms involved in this cluster, e.g. probability, chance event, frequencies, can be easily mistaken for common words used, e.g. probably, by change, frequently, which can cause students learning content in a second language to struggle with mathematical meaning.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with investigating chance processes and developing, using, and evaluating probability models benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of novel solutions to authentic problems because learners often connect well to how mathematical concepts play out in authentic problems and with this cluster heavily focused on students being able to develop probability models students will benefit from expressing their ideas and understandings from examples.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with investigating chance processes and developing, using, and evaluating probability models benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students are being asked to work in a creative mindset supplying students with metacognitive approaches to frustrations will give them strategies for coping when they make mistakes. Students reflecting on what did work in their thinking and what they can glean from their mistakes will provide an improved outcome to their models.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- Examine assessments for evidence of lingering misconceptions (see common misconceptions). If students exhibit one more of these misconceptions, consider addressing the misconception by re-engaging with content during a unit on investigation of chance processes and developing, using, and evaluating probability models by clarifying mathematical ideas and/or concepts through a short mini-lesson because the probability model is first introduced in this grade level and students may need time to clarify the concept.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- Examine assessments for evidence of students still developing the underlying ideas as some students may benefit from intensive extra time during and after a unit on investigation of chance processes and developing, using, and evaluating probability models by offering opportunities to understand and explore different strategies because students can find probabilities by using organized lists, tables, tree diagrams, and simulations

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- To extend students learning: For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying investigation of chance processes and developing, using, and evaluating probability models because students can apply probabilities to real-life scenarios that link science disciplines for example, genetics and a Punnett square.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying Investigating chance processes and developing, using, and evaluating probability models, the types of mathematical tasks are critical because probability models are encountered in everyday life, especially seen in news reporting or studies that students might see. With a little building of procedural fluency from conceptual understanding, students can understand probability meanings and interpret meanings on their own.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: http://tasks.illustrativemathematics.org/content-standards/7/SP/C/tasks/1581

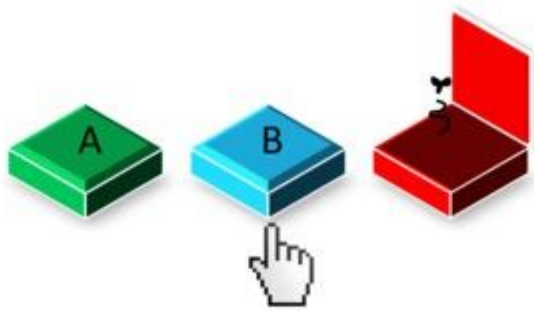
Stay or Switch?

You are playing a video game. At the end of every level, there are three boxes. One contains 10,000 points, and the other two are empty. You can choose one of the boxes, but before the one you choose opens, one of the other boxes always opens to show that it is empty. The game allows you to either (1) stay with your first choice or (2) switch to the other unopened box.

For example, there are three boxes, A, B, and C.

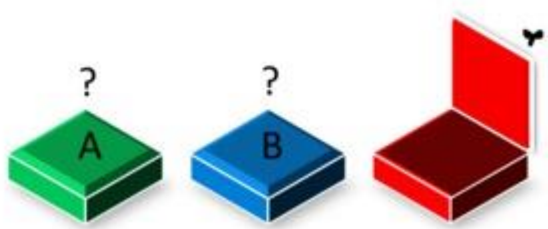
Suppose you choose box B:

Before box B opens, one of the other boxes opens to show that it is empty. Imagine that in this case it is box C:



The game allows you to stay with your first choice or to switch to the other unopened box.

Should you stay or should you switch?



This type of assessment question requires students to find the theoretical probability of an event by systematically recording all the possible outcomes in the sample space and identifying those that correspond to the event. The tricky thing about the problem is that most people assume that it doesn't matter if they stay or switch, thinking that because there are two choices, the chances of getting the points is "50-50."

Because the final result is often counter-intuitive, students will need to actively debate and try to convince one another that these probabilities are correct.	
Relevance to families and communities: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society? During a unit focused on Investigating chance processes and developing, using, and evaluating probability models, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning how to interpret probability or design models can be used in their daily life and connecting these examples to the work within the classroom.	**Cross-Curricular Connections:** Science: • Make an argument on growth and development of organisms: animal reproductions, plant reproduction for specialized features. MS-LS1-4 • Develop a model and identify components. Describe relationships between components. Model data they create. Identify limitations of models. Describe how the data they generate can be used to create designs through testing and modification. Engineering Design Process. MS-ETS1-4 • Model genetic information and sexual reproduction results. Punnett squares. MS-LS3-2 Scatterplots of temperatures of water vs mass of ice added MS-PS3 • Model genetic information and sexual reproduction results. Punnett squares. MS-LS3-2 Use simulations to generate data that can be used to modify a proposed object, tool, or process MS-ETS1

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ? What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ⍰ How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ⍰ What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ⍰ How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ⍰ What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding

ENGAGEMENT ⍰ How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ⍰ What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ? How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols:** ? What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ? How will the learning provide multiple	**Expression & Communication:** ? What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ⍰ How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: ⍰ What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ⍰ How will the learning support transforming accessible information into usable knowledge	**Comprehension**: ⍰ What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ⍰ How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ⍰ What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

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Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

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[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN[2]
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.