New Mexico Mathematics Instructional Scope for Eighth Grade

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Overview

This mathematics instructional scope was created by a cohort of New Mexico educators and the New Mexico Public Education Department. This document is organized into three sections. Section 1 describes how to use this document to support equitable and excellent mathematics instruction. Section 2 contains planning support for each cluster of mathematics standards within the grade level or course. Section 3 provides additional resources, references, and glossary.

The intention of this document is to act as companion during the planning process alongside High Quality Instructional Materials (HQIM). A sample template is presented to show a quick snapshot of planning supports provided within each cluster of standards in section 2.

During the creation of this document, we leveraged the work of other states, organizations, and educators from across country and the world. This work would not have been possible without all that came before it and we wish to express our sincerest gratitude for everyone that contributed to the resources listed within our references. This document is a work in progress and in some circumstances, our team of New Mexico educators may have embedded content from resources that have yet to be cited, as these elements are discovered in the use of this tool the references in section 3 will be updated.

Section 1: New Mexico Instructional Scope for Supporting Equitable and Excellent Mathematics Instruction

To better understand the planning supports provided in section 2, for each cluster of standards, this section provides a brief description of each planning support including: *what* support is provided; *why* the planning support is critical for equitable and excellent mathematics instruction; and, *how* to use the planning support with HQIM.

Cluster Statement

What: The New Mexico Mathematics Standards are grouped by Domains with somewhere between 4 to 10 domains per grade level. Within each domain the standards are arranged around clusters. Cluster statements summarize groups of related standards. The cluster statement planning support also indicates if the clusters is major, supporting, or additional work of the grade.

Why: The New Mexico Mathematics Standards require a stronger *focus*[1] on the way time and energy are spent in the mathematics classroom. Students should spend the large majority of their time (65-85%) on the major clusters of the grade/course. Supporting clusters and, where appropriate, additional clusters should be connected to and engage students in the major work of the grade.

How: When planning with your HQIM consider the time being devoted to major versus additional or supporting clusters. Major Work of each grade should be designed to provide students with strong foundations for future mathematical work which will require more time than additional or supporting clusters. Consider also the ways the

[1] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

HQIM makes explicit for students the connections between additional and supporting clusters and the major work of the grade.

Standard Text

What: Each cluster level support document contains the text of each standard within the cluster.

Why: The cluster statement and standards are meant to be read together to understand the structure of the standards. By grouping the standards within the cluster the connectedness of the standards is reinforced.

How: The text of the standards should always ground all planning with HQIM. Reading the standards within a cluster intentionally focuses on the connections within and among the standards.

Standards for Mathematical Practice

What: The Standards for Mathematical Practice describe the varieties of expertise and habits of mind that mathematics educators at all levels should seek to develop in their students.

Why: Equitable and excellent mathematics instruction supports students in becoming confident and competent mathematicians. By engaging with the standards for mathematical practice students are engaging in the practice of doing mathematics and development of mathematical habits of mind—the ability to think mathematically, analyze situations, understand relationships, and adapt what they know to solve a wide range of problems, including problems they may not look like any they have encountered before.[2]

How: When planning with HQIM it is critical to consider the connections between the content standards and the standards for mathematical practice. The planning supports highlight a few practices in which students could engage when learning the content of the standard. Note it is not necessary or even appropriate to engage in all of the practices every day, rather choosing a few and spending time intentionally supporting students in learning both the what (content standards) and the how (standards for mathematical practice) will create a stronger foundation for ongoing learning.

Students Who Demonstrate Understanding Can (Webb's Depth of Knowledge and Bloom's Taxonomy)

What: The New Mexico Mathematics Standards include each aspect of mathematical rigor: conceptual understanding, procedural skill and fluency, and application to the real world.[3] This planning support considers which aspect(s) of rigor are within each standard and then identifies academics skills students need to demonstrate comprehension of the standard and associated mathematical practices. The statements also highlight both the receptive (listening and reading) and expressive (speaking and writing) parts of language by considering the types of mathematical representations (verbal, visual, symbolic, contextual, physical) within the standard and what students need to do with them. The planning supports also provide information about two common classifications on cognitive complexity, Webb's Depth of Knowledge and Bloom's Taxonomy.

Why: Analyzing standards alongside the standards for mathematical practice provide a fuller picture of the mathematical competencies demanded in the standard.

How: When planning for a cluster of standards with your HQIM a critical first step is to analyze the content and language demands of the standards and standards for mathematical practice. The analysis can be used to inform

[2] Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD. (P. 13)

[3] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics

formative assessment, or it can be used to plan/design appropriate formative assessment.[4] The planning supports provide a possible break-down of the standard that can serve as the basis for this sort analysis.

Connections

What: The New Mexico Mathematics Standards are designed around coherent progressions of learning. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.[5] The connections to previous, current and future learning make this coherence visible.

Why: Students build stronger foundations for learning when they see mathematics as an inter-connected discipline of relationships rather than discrete skills and knowledge. The intentional inclusion of connections to previous, current, and future learning can support a more inter-connected understanding of mathematics.

How: When planning with HQIM use the connection planning supports to find ways to support students in making explicit connections within their study of mathematics.

Clarification Statement

What: The clarification statement provides greater clarity for teachers in understanding the purpose of the standards within a cluster.

Why: The New Mexico Mathematics Standards illustrate how progressions support student learning within each major domain of mathematics. The clarification statement provides additional context about the ways each cluster of standards supports student learning of the larger learning progression.

How: When planning with HQIM use the clarification statement to support an understanding of how the materials use specific types of representations or change the learning sequence from instructional approaches not grounded in progressions of learning.

Common Misconceptions

What: This planning support identifies some of the common misconceptions students develop about a mathematical topic.

Why: Students create misconceptions based on an over generalization of patterns they notice or an over reliance on rules rather than underlying mathematics. Rules in mathematics expire[6] over time (e.g., you can't subtract 1-3) as students expand their knowledge of mathematics (e.g., from whole numbers to rational numbers). It is critical to understand some of the common misconceptions students can develop so we can address them directly with students and continue to build a strong foundation for their mathematical learning.

How: When planning with your HQIM look for ways to directly address with students some common misconceptions. The planning supports in this document provide some possible misconceptions and your HQIM might include additional ones. The goal is not to avoid misconceptions, they are a natural part of the learning process, but we want to support students in exploring the misconception and modifying incorrect or partial understandings.

Multi-Layered System of Supports/Suggested Instructional Strategies

What: The section on Multi-Layered Systems of Supports(MLSS)/Suggested Instructional Strategies is designed to support teachers in planning for the needs of all students. Each section includes options for pre-teaching, reteaching, extensions and core instructional supports for students. Targeted pre-teaching and reteaching support student's acquisition of the knowledge and skills identified in the New Mexico Mathematics Standards to support student success with high-quality differentiated instruction. Intensive supports may be provided for a longer duration, more

[4] English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands
[5] Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
[6] Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

frequently, smaller groups, or otherwise be more intensive than targeted supports. Progress monitoring should occur to assess students' responses to additional supports, see Standards Aligned Instructionally Embedded Formative Assessment Resources.

Why: MLSS is a holistic framework that guides educators, those closest to the student, to intervene quickly when students need additional supports. The framework moves away from the "wait to fail" model and empowers teachers to use their professional judgement to make data-informed decisions regarding the students in their classrooms to ensure academic success with the grade level expectations of the New Mexico Mathematics Standards.

How: When planning with your HQIM use the suggestions for pre-teaching as a starting point to determine if some or all of the students in your classroom may need targeted or intensive pre-teaching at the start of unit to ensure they can access the grade level material with the unit. The core-instruction and reteach sections work together to support planning within a unit, look for the ways the materials are supporting greater access for all students and providing options to revisit materials based on formative assessments. The planning supports for each cluster are grounded in the Universal Design Learning (UDL) Framework, additional planning supports based on this framework can be found in Section 3 of this document in the part titled, Planning Guidance for Multi-Layered Systems of Support: Core Instruction.

Culturally and Linguistically Responsive Instruction

What: Culturally and Linguistically Responsive Instruction (CLRI), or the practice of situational appropriateness, requires educators to contribute to a positive school climate by validating and affirming students' home languages and cultures. Validation is making the home culture and language legitimate, while affirmation is affirming or making clear that the home culture and language are positive assets. It is also the intentional effort to reverse negative stereotypes of non-dominant cultures and languages and must be intentional and purposeful, consistent and authentic, and proactive and reactive. Building and bridging is the extension of validation and affirmation. By building and bridging students learning to toggle between home culture and linguistic behaviors and expectations and the school culture and linguistic behaviors and expectations. The building component focuses on creating connections between the home culture and language and the expectations of school culture and language for success in school. The bridging component focuses on creating opportunities to practice situational appropriateness or utilizing appropriate cultural and linguistic behaviors.[7]

Why: The mathematical identities of students are shaped by the messages they receive about their ability to do mathematics and the power of mathematics in their lives outside of school.[8] Mathematics educators must intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages. In addition, create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to supports students in creating mathematical identities as capable mathematicians within school and society.

How: When planning instruction is critical to consider ways to validate/affirm and build/bridge from your students cultural and linguistic assets. The planning supports for each cluster provide an example of how to support equity-based teaching practices. Look for additional ways within your HQIM to ensure all students develop strong mathematical identities.

Standards Aligned Instructionally Embedded Formative Assessment Resources

What: Formative Assessment is the planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of the outcomes and support students to become directed learners. All New Mexico educators have access to standards aligned instructionally embedded formative assessments: iStation at K-2; Cognia at 3-8, and the SAT Suite Question

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[7] Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

[8] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics. (P. 14)

Bank at 9-12. These are intended to be used during instruction for each at each grade alongside assessments within your HQIM.

Why: When student thinking is made visible the teacher can examine the progression of learning towards the goals of the standards and adjust instruction as necessary. By including students in the assessment and analysis process students become strategic and goal-directed with their learning.

How: The planning supports at each cluster provide an example of a task that addresses one more aspect of the cluster of standards. This example can be used to discuss possible responses by students and next steps for instruction. A similar process can then be used to identify additional items from one of the formative assessment resources provided by NM PED and your HQIM.

Relevance to Families and Communities

What: Relevance to families and communities requires finding the relevance of mathematics outside of the classroom by connecting to families and communities and learning about varied and often unexpected ways they use mathematics.

Why: When school mathematics is connected to the mathematics outside of school students can build a bridge between their ways of thinking about quantities outside and inside school created a bridge between home and school.

How: When planning at the year and unit level with you HQIM find ways to intentionally learn from your families and communities the cultural and linguistic ways they use mathematics outside of school.

Cross-Curricular Connections

What: New Mexico defines cross-curricular connections as connections between two or more areas of study made by teachers or students within the structure of a subject.

Why: The purpose of planning cross-curricular connections in an instructional sequence is to ensure that students build connections and recognize the relevance of mathematics beyond the mathematics classroom.

How: When planning with HQIM look for opportunities to make explicit connections to other content areas such as the examples provided for each cluster.

Template of the New Mexico Cluster Level Planning Support for the New Mexico Mathematics Standards

<GRADE/COURSE/DOMAIN ABBREVIATION: DOMAIN NAME>

Cluster Statement: Statement from New Mexico Mathematics Standards summarize a group of related standards.

Major/Additional/Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)
Identifies if the cluster is major, additional or supporting work of the grade.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
Full text of the standard	The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	The cognitive skills students perform to demonstrate to comprehension of a standard.
		Depth Of Knowledge: Correlation of standard to Webb's Depth of Knowledge
		Bloom's Taxonomy: Correlation of standard to Bloom's Taxonomy
Connections to Previous Learning: Supports student connections to learning from previous grade levels.	**Connections to Current Learning** Supports student connections to learning within the grade level.	**Connections to Future Learning** Supports student connections to learning in a future grade.

Clarification Statement: Clarifies the language of the standard.

Common Misconceptions: Guidance on where a student misconception or misunderstanding could potentially occur.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach
Pre-teach (targeted): Guidance for how to activate students' knowledge to support their learning.
Pre-teach (intensive): Guidance for how to use earlier grade standards to build a strong foundational understanding upon which to build grade level concepts.

Core Instruction
Access: Guidance for optimizing universal access to learning experiences.
Build: Guidance for supporting students build their understanding of the cluster.
Internalize: Guidance for ensuring student internalization of the learning goal.

Re-teach
Re-teach (targeted): Guidance for adjusting instruction during a unit by using formative assessment data.
Re-teach (intensive): Guidance for analyzing assessment data to identify content that would benefit from more intensive reteaching.
Extension Ideas: Suggestions that offer additional challenges to 'broaden' students' knowledge of the mathematics within the cluster.

Culturally and Linguistically Responsive Instruction: Provides equity based instructional suggestions aligned to the cluster of standards

Standards Aligned Instructionally Embedded Formative Assessment Resources: Includes reference to high-quality formative assessment resources, including examples from New Mexico's formative assessment banks.

Relevance to Families and Communities:	**Cross Curricular Connections:**
Connecting with families and communities to create relevant connections between mathematics inside and outside of school.	Includes examples of how the cluster provides opportunities to connect to other disciplines such as literacy, science, social studies, and the arts.

Section 2: Cluster Level Planning Support for the New Mexico Mathematics Standards

TABLE OF CONTENTS:

The Number System

- 8.NS.A

Expressions & Equations

- 8.EE.A
- 8.EE.B
- 8.EE.C

Functions

- 8.F.A
- 8.F.B

Geometry

- 8.G.A
- 8.G.B
- 8.G.C

Statistics & Probability

- 8.SP.A

<table>
<tr><th colspan="3">8.NS. THE NUMBER SYSTEM</th></tr>
<tr><td colspan="3">Cluster Statement: A: Know that there are numbers that are not rational and approximate them by rational numbers.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

8.NS.A.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.</td><td rowspan="3">Standard for Mathematical Practices

SMP 2: Students Reason abstractly and quantitatively as they explain how to get more precise approximations of irrational numbers.

SMP 6: Students attend to precision by using rational approximations of irrational numbers to compare and locate them on a number line.

SMP 7: Students will learn specific characteristics of rational and irrational numbers and use their specific structure to classify them.

SMP 8: Students explain how to get more precise rational approximations of irrational numbers.</td><td>Students who demonstrate understanding can:
Classify numbers as rational or irrational.
Understand that every number has a decimal expansion.
Explain that an irrational number is a decimal that does not terminate or repeat, it cannot be written in the form a/b, where b cannot be equal to zero.
Identify and explain that a rational number of repeats or terminates.
Explain what a rational number is and give examples.
Explain what an irrational number is and give examples.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:
Understand</td></tr>
<tr><td rowspan="2">Standard Text

8.NS.A.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</td><td rowspan="2">Standard for Mathematical Practices

SMP 2: Students Reason abstractly and quantitatively as they explain how to get more precise approximations of irrational numbers.

SMP 6: Students attend to precision by using rational approximations of irrational numbers to compare and locate them on a number line.

SMP 7: Students will learn specific characteristics of rational and</td><td>Students who demonstrate understanding can:
Approximate square roots
Plot square roots on the number line.
Express thinking in writing about how to approximate values and locations on a number line.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
</table>

	irrational numbers and use their specific structure to classify them. SMP 8: Students explain how to get more precise rational approximations of irrational numbers.	**Bloom's Taxonomy**: Understand, Apply
Previous Learning Connections • In 5th grade, students learned to round decimals to any place value. In 6th grade, students placed rational numbers on a number line and converted rational numbers to decimals using long division. These skills are needed when understanding irrational numbers.	**Current Learning Connections** • During 8th grade, students will use square root and cube root symbols to encounter irrational numbers.	**Future Learning Connections** • In high school students will extend their knowledge of irrational numbers to complex numbers. They will also use rational exponents.

Clarification Statement:
Expand knowledge of numbers to include irrational numbers. Convert decimals to rational numbers. Use a number line to approximate, compare, and order rational and irrational numbers.

Common Misconceptions

- Students struggle with understanding relationships of the subsets of the Real Number System.
- Some students may think some rational numbers in decimal form repeat three or more digits and students mislabel them as irrational because they do not divide far enough to see the pattern or repeating digits.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers because some students have difficulty understanding what irrational numbers are, how they compare to rational numbers, and where they fit in the Real Number system. Providing additional time for students to make sense of the concepts and procedures in multiple ways can help them clarify misconceptions, develop a better understanding, and have fluency when solving problems with irrational numbers.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7NSA2, Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. This standard provides a foundation for work with 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers because students need a firm foundation in operations with rational numbers in order to apply those operational properties to the irrational numbers. They previously extended their knowledge of fractions to include fractions whose numerator or denominator could be an integer and learned to convert fractions to decimals and vice versa, both skills, which are necessary for students to be able to understand the concept of irrational numbers, identify them, and approximate them in order to solve problems. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers, benefits students when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features because students will benefit from creating their own visuals in their INBs as well as viewing videos and referring to anchor charts created during class that include color, font size and language that help clarify concepts and vocabulary.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers, benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is frequent, timely, and specific because when students are learning about irrational numbers and trying to connect them to what they are familiar with, it is important that they receive constant feedback about misconceptions and errors as well as validation of what they are understanding and applying correctly. Students also need to be able to ask questions that clarify their understanding. The feedback students receive and the questions we ask them need to be specific to prevent miscommunications that lead to other misunderstandings.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers, benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as providing graphic symbols with alternative text descriptions because the symbolism in this content is specific, but can be complicated for students. Anchor charts, notes, and other visuals that include the correct symbols and numeric examples along with verbal descriptions can support students with connecting the mathematical language to their everyday language and experiences. Students would also benefit from working with peers to use language, diagrams, numbers and other representations to help them make sense of the connections between the language and vocabulary, the concepts and the symbolic representations. For example, discussing a rational number, what it is explained using decimals, and a numerical example can help students grasp the concept.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers, benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, graph paper, or number lines because using these tools help students explore irrational numbers and their approximations and compare them to rational numbers.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers, benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because the concept of irrational numbers is hard to grasp initially. Students will benefit from opportunities to explore values such as the square root of 2 as well as practice identifying irrational numbers and approximating them using number lines and other tools.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers by providing specific feedback to students on their work through a short mini-lesson because providing students specific feedback about what is correct thinking and incorrect thinking based on exit tickets, bell ringer, classwork, etc. will help confirm what they know and provide them feedback and support for areas of struggle.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers by addressing conceptual understanding because some students struggle to understand the subsets of the Real Number System displayed in a Venn Diagram and may need to use manipulatives such as boxes that fit inside one another to represent the subsets. Adding examples of numbers in the subsets to the boxes can further help with the concept.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying 8NSA, knowing that there are numbers that are not rational, and approximating them by rational numbers because asking them, for example, how many irrational numbers they think are

	between 1.4 and 1.5 causes them to apply their new learning to something abstract and think more deeply about the concepts in order to find and explain their solution.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying to know that there are numbers that are not rational, and approximate them by rational numbers, goal setting is critical because when students know that the expectation for them to learn this standard that it will connect to their future and encourage them to look forward to higher math classes.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source:
http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/657/original/public_task_334.pdf?1462388138

8.NS.A.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.

- Learning Target: I can identify rational and irrational numbers.
- Webb's Depth of Knowledge: 1
- The task assumes that students are able to express a given repeating decimal as a fraction.

Task

Decide whether each of the following numbers is rational or irrational. If it is rational, explain how you know.

a. $0.33\overline{3}$

b. $\sqrt{4}$

c. $\sqrt{2} = 1.414213\ldots$

d. 1.414213

e. $\pi = 3.141592\ldots$

f. 11

g. $\frac{1}{7} = 0.\overline{142857}$

h. $12.3456565\overline{656}$

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on rational and rational numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the mathematics used within the different careers of your family and community can provide a strong connections between school and careers.	Science: Students can represent their collected data in different forms of rational and irrational numbers.

<table>
<tr><th colspan="3">8.EE: EXPRESSIONS & EQUATIONS</th></tr>
<tr><td colspan="3">Cluster Statement: A: Expressions and Equations Work with radicals and integer exponents.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade).</td></tr>
<tr><td rowspan="3">Standard Text

8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</td><td rowspan="3">Standard for Mathematical Practices

SMP 2: Students reason abstractly and quantitatively when expressing size comparisons of numbers written in scientific notation.

SMP 5: Students use appropriate tools strategically when learning to read and use scientific notation.

SMP 6: Students attend to precision by calculating accurately and efficiently, and accurately express number in scientific notation.</td><td>Students who demonstrate understanding can:
Calculate integer exponents by understanding their properties.
Generate equivalent expressions using the single properties of integer exponents and combinations of the properties.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1</td></tr>
<tr><td>Bloom's Taxonomy:
Apply</td></tr>
<tr><td rowspan="3">Standard Text

8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.</td><td rowspan="3">Standard for Mathematical Practices

SMP 8: Students look for and express regularity in repeated reasoning by using square root and cube root symbols to represent solutions to equations of the form x 2 (x squared)= p and x 3 (x cubed) = p, where p is a positive rational number. Students valuate square roots of small perfect squares and cube roots of small perfect cubes. Students now that √2 is irrational.</td><td>Students who demonstrate understanding can:
Calculate a square root of a perfect squared number or cube root of a perfect cube root number.
Use the square root and cube root symbol in an equation x2(x squared) =p or x3(x cubed) =p.
Explain square root of 2 is an irrational number.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-2</td></tr>
<tr><td>Bloom's Taxonomy:
Application</td></tr>
</table>

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Standard Text 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.*	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively when expressing size comparisons of numbers written in scientific notation. SMP 5: Students use appropriate tools strategically when learning to read and use scientific notation. Common Core Mathematics Companion: The Standards Decoded, Ruth Harbin Miles and Lois A Williams SMP 6: Students attend to precision by calculating accurately and efficiently, and accurately express number in scientific notation.	**Students who demonstrate understanding can:** • Explain the benefits of scientific notation. • Write very small or very big numbers in 'scientific notation. • Understand that some numbers written in scientific notation are estimates. • Compare very small or very big numbers written in scientific notation to determine which is larger or smaller and by how much. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Application, Analysis
Standard Text 8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	**Standard for Mathematical Practices** SMP2: Students use numbers and exponents flexibly and interpret their results in context (MP2). SMP4: Students practice modeling skills, such as identifying essential features of a problem and gathering the required information (MP4). SMP.6 Students will calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Students who demonstrate understanding can**: • Add, subtract, multiply or divide numbers written in scientific notation. • Assess the appropriate size for measurement written in scientific notation. **Webb's Depth of Knowledge:** 1,3 **Bloom's Taxonomy:** evaluation, application
<u>Previous Learning Connections</u> • In 5th grade, students began to develop and understand the powers of 10 and the placement of the decimal when multiplying or dividing by powers of 10. In 6th grade,	**<u>Current Learning Connections</u>** • In 8th grade, students will connect the properties learned in this cluster to use square and square roots, cube and cube roots, when working with irrational numbers (NS	**<u>Future Learning Connections</u>** • In high school, learners will use properties of exponents to rewrite expressions and extend their knowledge of integer exponents to rational exponents.

students continued to write and evaluate numerical expressions involving whole-number exponents.	standards) and volume (geometry standards).	

Clarification Statement:
In this cluster, students explore the properties of exponents, radicals, and scientific notation.

Common Misconceptions

- Students may confuse the rules, which usually occurs when they are taught to memorize them rather than understand them. Students may also think that finding a power of a power involves adding exponents.
- Students may confuse the relationship between division and negative exponents or forget about the order or operations.
- Some students may confuse square roots and cubes. Some might divide by 2 or 3 instead of finding the square root or cube respectively Some students fail to recognize the relationship between square numbers and area or between cube numbers and volume.
- Some students may forget that correct scientific notation requires that the first factor be written with only one digit to the left of the decimal. Some may struggle to understand which number should be divided when expressing how many times as much one number is than the other. Students may struggle if they add exponents that should be subtracted. Students can confuse the direction to move the decimal point when the exponent is negative.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying expressions and equations that include radicals and exponents because connections can be formed to prior knowledge of writing and evaluating numerical expressions as students struggle, for example, to determine what a number to the 0 power might be based on what they previously know about powers. Or students may use prior knowledge to determine what 6^3* 6^4might be and explain their thinking. This can be a great lead into showing, in expanded form, WHY the answer is 6^7. This can work as a lead-in to division of powers or into discovery of the rules of exponents rather than just giving students the exponent rules. It is necessary for students to have a grasp of how to write and evaluate powers in order to move on to understanding the concepts behind multiplying and dividing by powers, negative powers, and powers of zero.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.A.1 This standard provides a foundation for work with applying and extending previous understanding of arithmetic to algebraic expressions because students must understand how to write and evaluate numerical expressions using exponents. They must understand the difference between multiplying 3 * 4 and evaluating 3^4 in order to write equivalent expressions and evaluate expressions and equations involving powers. Students also need to understand the concept of repeated multiplication applied to powers for that knowledge to be transferred to repeated division being written as powers with negative exponents or as fractions with a numerator of 1 and a power in the denominator. These understandings of repeated multiplication and division and the use of structure in the repeated patterns can help students understand the concept of all numbers to the power of 0 equaling to 1.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with how expressions and equations work with numbers involving radicals and integer exponents benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as by displaying really small or really large objects in flexible formats to vary perceptual features by providing multiple representations of the information and relationships in tables, graphs, diagrams, videos, and through the use of physical exploration (stacking or counting pennies and using smaller knowns of height or weight to explore relationships with much larger amounts) because students may not grasp the concept with one mode of representation, but may understand another. Further understanding can be reached by forming relationships between and showing connections to other representations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with how expressions and equations work with numbers involving radicals and integer exponents benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because comprehending and working with very large or small numbers is difficult. Students need constant feedback from peers and teachers. Through collaboration, students will develop a deeper understanding by sharing and analyzing strategies, making connections between errors and correct thinking, will receive validation of thinking and strategies, and increase their overall skills and understanding of how to read and write these numbers, why the various notations are helpful, and how to solve problems using them.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with how expressions and equations work with numbers involving radicals and integer exponents benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative)because making connections between familiar values and new forms of numerical notation is difficult but necessary. Also, making connections between familiar calculations involving familiar number notations and calculations with values in radical or exponential notation will support students' grasp of the concept and skills involved in working with equations and expressions with these types of numbers. Anchor charts, diagrams, media sources, et al that explicitly show the connections between familiar numerical forms, the new notation, verbal descriptions, and vocabulary terms will support students development of the concepts and their ability to apply familiar computations when using exponential and radical notations.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with Expressions and Equations Work with radicals and integer exponents benefit when learning experiences attend to the multiple ways' students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because some students need more explicit feedback while others do not. Each individual student progresses differently and by showing them that they are individuals, they are more likely to take the feedback as encouragement than criticism.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with Expressions and Equations Work with radicals and integer exponents benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") because self-talk is the most powerful downer and upper for any individual. This will boost the students' self-esteem and will have the can-do attitude and persevere not just in Math but also in other content areas.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on expressions and equations that include radicals and exponents by critiquing student approaches/solutions to make connections through a short mini-lesson because through analysis and sharing of student work on a bell ringer or exit ticket, misconceptions and common errors can be identified, corrected and valued in a safe and meaningful way that strengthens student learning, students will receive validation of their thinking , and students may gain a better understanding through the language and visuals provided by their peers. The mini lesson can be followed up with practice specifically geared toward correction of the misunderstanding or skill.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on expressions and equations that include radicals and exponents by offering opportunities to understand and explore different strategies because working with radicals, cubed and square roots, numbers in scientific notation and powers can be complicated. Students need opportunities to compare numbers written differently to gain an understanding of relative sizes written in forms they are unfamiliar with. Students need opportunities to talk about their strategies and thinking and to understand the notations and conversions and how they apply their understanding to solve problems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying expressions and equations work with radical and integer exponents because different exposure to this concept will lead to better appreciation and understanding of the mathematics.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying expression and equations work with radicals and integer exponents eliciting and using student thinking is critical because this standard is foundational to their future math concepts and other core subjects. When students are given an opportunity to present their process/solution with their own way of solving, empowers the students to take risks in the future and move forward with their learning.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Formative Item Set for Grade 8 Expressions and Equations

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.
For example, 32 × 3-5 = 3-3 = 1/33 = 1/27.

- Learning Target: I can generate equivalent expressions from whole number and fractions using integer exponents.
- Webb's Depth of Knowledge: 2

Which of the following is equivalent to the expression $\frac{1}{9} \cdot 27$?

Ⓐ $3^{-2} \cdot 3^{-3}$
● $3^{-2} \cdot 3^{3}$
Ⓒ $3^{2} \cdot 3^{-3}$
Ⓓ $3^{2} \cdot 3^{3}$

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on working with radicals and integer exponents, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students by having students learn about the mathematics used within the different careers of their	Science - Distance of planets from the sun

family members and community. Students can also research careers, mathematicians, or people influential in their culture and the ways they use math or have contributed to the field.	

<table>
<tr><td colspan="3">

8.EE: EXPRESSIONS & EQUATIONS

</td></tr>
<tr><td colspan="3">Cluster Statement: B: Understand the connections between proportional relationships, lines, and linear equations.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr><td rowspan="3">Standard Text

8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</td><td rowspan="3">Standard for Mathematical Practices

SMP 2: Students reason abstractly and quantitatively by explaining what the slope and y-intercept represent on a graph and in context with the proportional relationship.

SMP 3: Students construct viable arguments and critique the reasoning of others by justifying that similar right triangles provide the same slope for the same non-vertical line.

SMP 4: Students model with mathematics by modeling a contextual proportional relationship by graphing and writing equations.

SMP 5: Students use appropriate tools strategically by utilizing the coordinate plane (graph paper) to graph lines and analyzing graphs modeled by calculators.</td><td>Students who demonstrate understanding can:
Graph proportional relationships.
Interpret the unit rate as the slope of the graph.
Compare two proportional relationships whether it is table, graph or equation.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1, 2</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply</td></tr>
<tr><td>Standard Text

8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.</td><td>Standard for Mathematical Practices

SMP 2: Students reason abstractly and quantitatively by explaining what the slope and y-intercept represent on a graph and in context with the proportional relationship.

SMP 3: Students construct viable arguments and critique the reasoning of others by justifying that similar right triangles provide the same slope for the same non-vertical line.

SMP 4: Students model with mathematics by modeling a contextual proportional relationship by graphing and writing equations.</td><td>Students who demonstrate understanding can:
Identify the Y-intercept of the graph and understand the meaning of the y-intercept in a real-world problem situation.
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.
Graph a line from an equation in the form of y=mx+b, understand what m is (slope) and the b (y-intercept).</td></tr>
</table>

	SMP 5: Students use appropriate tools strategically by utilizing the coordinate plane (graph paper) to graph lines and analyzing graphs modeled by calculators.	• Discover the equation y = mx for a line through the origin (proportional) and the equation y = mx + b for a line intercepting the vertical axis at b. **Webb's Depth of Knowledge:** 3 **Bloom's Taxonomy:** Analyze, Evaluate
Previous Learning Connections • In 6th grade, students used ratio, rate reasoning, and unit rate. In 7th grade, students made connections to the 6th grade skills to compute unit rates and recognize and represent proportional relationships	**Current Learning Connections** • In 8th grade, learners will use these skills to compare properties of functions given a table, a graph, or an equation.	**Future Learning Connections** • In future courses, students will understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Clarification Statement:
Students connect slope to unit rates, tables, lines, and equations. Students will also connect similar triangles to slope.

Common Misconceptions

- Students may make errors if they estimate unit rate from a graph instead of calculating the rate from data or an equation. Errors may occur if they find a single unit rate instead of comparing unit rates, compare unit rates from one relationship with the unit rate in the other relationship or forget to divide to calculate the unit rate
- Some errors may occur if students divide the differences between x-coordinates by the difference between y-coordinates. If students apply the slope formula incorrectly errors will arise. Students will make errors if they confuse the x-axis and the y-axis.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language in connection with prior learning when studying the connections between proportional relationships, lines, and linear equations because the language of the 8th grade cluster is completely new, but the skills needed for success began in 6th grade. Previously to 8th grade, slope is referred to as rate, unit rate, and constant of proportionality. Constant proportionality is structured in the y=kx form, so shifting students from y=kx to y=bx or even y=mx+b will take a shift in language and terminology, yet the skills of finding slope have already been developed.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.RP.A.2: This standard provides a foundation for work with understanding ratio relationships and calculating rate which would support finding slope because it introduces a relationship between two values. This standard also introduces ratios written as a fraction which supports dividing values to produce a rate. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with creating connections between proportional relationships, lines, and linear equations will benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students will need to interpret values from graphs, lines, tables, and equations that represent real world variables given the context of a problem, and, in order to show mastery of this cluster, explain the relationship of these values to one another, to the context of the problem, and to mathematical practice. Choosing applicable, contextualized tasks related to students' lives, for example, comparing a distance-time graph in the context of running the mile in P.E. class to a distance-time equation with realistic values, can generate interest, engagement, and access for students who may have learning gaps in their mathematical understanding

Build

Effort and Persistence: *How will the learning for students provide options for sustaining and persistence?*

- For example, learners engaging with creating connections between proportional relationships, lines, and linear equations will benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because the most common student misconception in this cluster takes place when students reverse X and Y values when analyzing lines, graphs, tables, and ordered pairs. Using effective error analysis, for example providing students with a word problem and graph drawn with an incorrect slope (possible error: unit rate- x divided by y instead of y divided by x) and asking students to examine both the graph and the word problem for an error can clarify the misconception, connect visual associations to positive and negative slope and aid learner self-reflection to promote future success.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with creating connections between proportional relationships, lines, and linear equations will benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because vocabulary such as slope, y intercept, equation of a line, hypotenuse, etc. are all brand new to 8th grade content. Students have worked with x and y variables and graph reading to find unit rate or constant of proportionality in 6th and 7th grade standards. although the skills are deeply connected and related, the language does not overlap. For example, connecting the language, showing the shift in language, not shift in skill, between $y = kx$ (constant of proportionality equation) and $y = mx$ (slope equation) to the previously learned structure in 6th and 7th grade will access foundational skills and understanding within these new concepts.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with creating connections between proportional relationships, lines, and linear equations will benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) solving problems using a variety of strategies because integrated technology can help solidify the connections between lines, equations, graphs and tables by creating visual illustrations with labels, graphing animations that demonstrate a line ascending or descending at a constant rate, and/or animations of similar triangles illustrating same slope concept when deriving the $y= mx$ and $y = mx +b$ formulas.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with creating connections between proportional relationships, lines, and linear equations will benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as embedding new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) because the concept of dependent and independent relationships is relatively understood by students when considering context. For example, students understand the connection between speed and time when placed in the context of driving- i.e. the fast your drive, the sooner you arrive at your destination. However, the mathematical vocabulary, terminology and equation portion of this cluster clouds student's ability to draw the connections, and ultimately, that is the goal of this standard. Using familiar ideas and contexts allows students to logically draw connections between variables and then use mathematical language to formalize their understanding versus using unfamiliar context where the whole process is brand new to student understanding.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding the connections between proportional relationships, lines, and linear equations by critiquing student approaches/solutions to make connections through a short mini-lesson because there are several components and representations of information in this cluster. Students will be presented with tables, graphs, ordered pairs, equations, and triangles. Students may be able to recognize a relationship between values when presented in a table, but struggle with reading graphs. Taking the time to critique approaches/ make connections with the way other students arrived at an answer will model successful ways to approach a task or problem.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit understanding the connections between proportional relationships, lines, and linear equations by helping students move from specific answers to generalizations for certain types of problems because when placed in real world context, students can often draw the correct connections between variables based on experience and not mathematical computation. For students struggling with the mathematical recognition, it may be valuable to focus on generalizations involving proportional relationships to boost confidence and understanding before addressing misconceptions within the process of finding slope or using similar triangles to show slope is the same.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying the connections between proportional relationships, lines, and linear equations because value relationships cross over into many disciplines. For example, students could develop an equation and model to determine cost/profit of items in a school store to guarantee enough funding for a field trip/class party.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

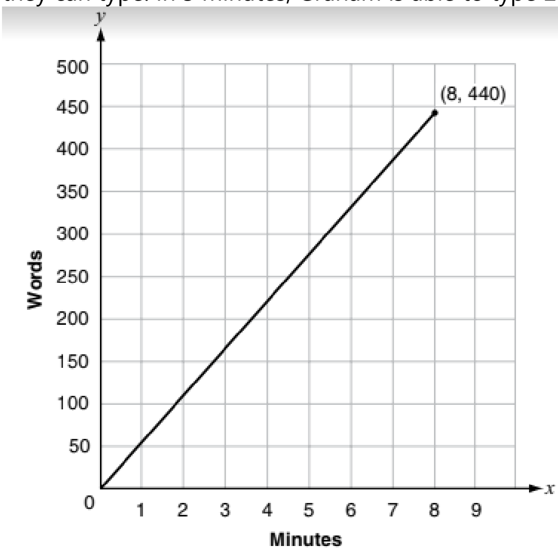
Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying the connections between proportional relationships, lines, and linear equations the pattern of questions within the classroom is critical because it allows students to communicate mathematically. It allows them to answer questions about rate of change, linear and proportional relationships. They can communicate their method of understanding the difference between linear and proportional relationships, while making a connection between them. This allows the teacher to formatively assess them while checking for understanding. The questions can be oral, on paper (exit tickets) or group questions that allow students to discuss different strategies in a safe classroom environment (It is important that the teacher create an environment where students feel safe to share).

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Formative Item Set for Grade 8 Expressions and Equations

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

- Learning Target: I can compare the unit rate given in a statement and slope from a graph.
- Webb's Depth of Knowledge: 3

As part of applying for a job, Graham and Claire each completes a keyboarding skills test to determine how fast they can type. In 5 minutes, Graham is able to type 260 words. Claire's results are shown in the graph.



a) How much faster, in words per minute, is Claire than Graham? Show your work or explain your reasoning.
b) Claire estimates that it will take her about 36 minutes to complete the document. What error did Claire make and what's the actual time it will take her to complete the document? Show your work or explain your reasoning.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on the connections between proportional relationships, lines, and linear equations , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, by having students examine proportional relationships in different recipes. Having students make their favorite recipe that requires them to double or triple the ingredients based on the number of servings the recipe yields vs. the number of servings needed.	Science: Compare rates and relationships in scientific data.

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8.EE: EXPRESSIONS & EQUATIONS		
Cluster Statement: C: Analyze and solve linear equations and pairs of simultaneous linear equations. **Major Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)		
Standard Text **8.EE.C.7: Solve linear equations in one variable.** • **8.EE.C.7.A: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers).** • **8.EE.C.7.B:Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.**	**Standard for Mathematical Practices** SMP 1: Students make sense of problems and persevere in solving them by solving real-world systems of linear equations and interpreting the results. SMP 2: Students reason abstractly and quantitatively by representing a real-world situation by writing a linear equation (or system of linear equations), then interpreting the meaning of the solution. Students explain the type of solution a system of linear equations will produce (one solution, no solution, or infinitely many solutions) by looking at the graph or system of equations. SMP 3: Students construct viable arguments and critique the reasoning of others by justifying the most efficient solution method of a system of linear equations. SMP 4: Students model with mathematics by writing and solving a system of linear equations to determine the solution of a real-world problem. SMP 5: Students use appropriate tools strategically by using a graphing calculator and/or graph paper for a coordinate grid to write and solve linear equations and/or a system of linear equations.	**Students who demonstrate understanding can:** • Combine like terms. • Expand an equation using the distributive property. • Solve one step equations, two step equations and multi-step (including equations where you must combine like terms and expand using the distributive property). • Use inverse operations to solve equations. • Determine whether an equation will have one solution (x=a), no solution (a=b) or infinite solutions (a=a) by simplifying the equation. (a and b are numbers). **Webb's Depth of Knowledge:** 2-3 **Bloom's Taxonomy:** Apply, Evaluate

	SMP 6: Students attend to precision by appropriately labeling the solution of a linear equation or a system of linear equations. SMP 7: Students look for and make use of structure by applying inverse operations and algebraic reasoning to solve a linear equation or a system of linear equations.	
Standard Text **8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.** • **8.EE.C.8.A: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.** • **8.EE.C.8.B: Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection.** ***For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*** • **8.EE.C.8.C: Solve real-world and mathematical problems leading to two linear equations in two variables.** ***For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.***	**Standard for Mathematical Practices** SMP 1: Students make sense of problems and persevere in solving them by solving real-world systems of linear equations and interpreting the results. SMP 2: Students reason abstractly and quantitatively by representing a real-world situation by writing a linear equation (or system of linear equations), then interpreting the meaning of the solution. Students explain the type of solution a system of linear equations will produce (one solution, no solution, or infinitely many solutions) by looking at the graph or system of equations. SMP 3: Students construct viable arguments and critique the reasoning of others by justifying the most efficient solution method of a system of linear equations. SMP 4: Students model with mathematics by writing and solving a system of linear equations to determine the solution of a real-world problem. SMP 5: Students use appropriate tools strategically by using a graphing calculator and/or graph paper for a coordinate grid to write and solve linear equations and/or a system of linear equations.	**Students who demonstrate understanding can:** • Calculate two linear equations with two variables in a real-world problem. • Calculate the value of two variables from two linear equations either algebraically or graphically. • Graph two equations and estimate solutions. • Analyze and solve systems of two linear equations with two variables in real-world problems. • Solve systems of two linear equations in two variables algebraically and/or graphically. • Estimate solutions by graphing the equations. • Solve simple cases by inspection. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply, analyze

	SMP 6: Students attend to precision by appropriately labeling the solution of a linear equation or a system of linear equations. SMP 7: Students look for and make use of structure by applying inverse operations and algebraic reasoning to solve a linear equation or a system of linear equations.	
Previous Learning Connections: • In 6th and 7th grade, students use variables to write expressions and equations and apply the properties of operations to generate equivalent expressions. Students also solve equations, including those that involve real-world problems.	**Current Learning Connections** • In 8th grade, students use the equation of a linear model to solve problems in the context of bivariate (two variables) measurement data, interpreting the slope and intercept. Students will use these equations to graph linear and proportional relationships.	**Future Learning Connections** • In high school, students will create, solve, and rewrite equations, inequalities, and systems of equations (include equations arising from linear, exponential, and quadratic functions) They will make connections to this content to construct a viable argument to justify a solution method.

Clarification Statement:
Students analyze, solve, and interpret linear equations and systems of linear equations.

Common Misconceptions
- Students may make errors if they substitute incorrectly or confuse the variable terms and the constant.
- Students may confuse the slope and y-intercept. Some students might forget that the way two lines intersect or do not intersect shows the number of solutions for a system of equations. They may make the error of solving an equation by substituting in only one equation in the system, try to use elimination without eliminating a variable, or become confused as to where to include given information into an equation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying analyzing and solving linear equations and pairs of simultaneous linear equations because 8th grade is the first time students will be connecting equation with lines and graphs and interpreting graphs to find solutions for linear equations. Students' previous work with linear equations stopped at isolating and solving for single variables by using inverse operations. Students will need additional time to connect the values in an equation to graphing and drawing conclusions from graphed lines.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5 AND 7.EE.B.4: These standards provide a foundation for work with analyzing and solving linear equations and pairs of simultaneous linear equations because it is critical that students understand that an equation can be simplified and solved by using a specific process. Students must understand the context of the variable, and its real-world implication. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with analyzing and solving linear equations and pairs of simultaneous linear equations benefit when learning experiences include ways to recruit interest such as creating an accepting and supportive classroom climate because < errors and mistakes will be made when students are learning to master this cluster. Student's work in previous grade levels asked students to solve for single variables with rational coefficients, stressing inverse operations. This cluster includes multiple variables, multiple solutions, ordered pairs, connecting graphing lines to equations and solutions to points of intersection, all of which is brand new learning to 8th grade students. Creating an accepting and sporting classroom climate, where mistakes are common and accepted, may additionally support students in the learning process and aid in their persistence to keep trying.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with analyzing and solving linear equations and pairs of simultaneous linear equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because previous learning in this concept prepares students for using distributive property, combining like terms, and isolating a single variable, yet these skills are steps to mastery in the 8th grade cluster. Affirming success with these skills and using the as scaffolds to reach mastery in this cluster will aid students in reaching the desired outcome.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with analyzing and solving linear equations and pairs of simultaneous linear equations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because success in this standard is when students can draw connections between equations, solutions, and graphic representations of the linear equations, and graphed solutions contained within the lines drawn. breaking down the complex components of linear equations, especially with rational coefficients, and simplifying each piece with words and symbols and begin to create connections between what is written, what is graphed, and what certain points on the graph represent when it comes to solutions.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with analyzing and solving linear equations and pairs of simultaneous linear equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as <providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because <precision in graphing ordered pairs and solutions is essential when understanding how a values of ordered pairs on a graph relates to an the values written in an equation. Although this cluster includes rational coefficients, manipulating rational coefficients is not the heart of the cluster, and therefore providing calculator aides will create equal access for students of all computation skills to communicate what they understand about linear equations.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with analyzing and solving linear equations and pairs of simultaneous linear equations benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing checklists and project planning templates for understanding the problem, setting up prioritization, sequences, and schedules of steps because this cluster represents a culmination of skills. Students will need to appropriately use the four operations with rational coefficients adhering to order of operations, use the distributive property, combine like terms, isolate variables using inverse operations, identify graphed values in ordered pairs, connect slope-intercept form to graph representations, and analyze how ordered pairs would fall on a line based upon the equation and the slope of the line. Creating processes, steps, and sequences will create reliable systems that students can depend on while navigating skills they may be comfortable with and brand-new skills.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on analyzing and solving linear equations and pairs of simultaneous linear equations by clarifying mathematical ideas and/or concepts through a short mini-lesson because students may struggle procedurally to solve linear equations for solutions, especially when working with rational coefficients, and without this crucial understanding, students will struggle with the entirety of the cluster.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit in analyzing and solving linear equations and pairs of simultaneous linear equations by confronting student misconceptions because many misconceptions in this unit reflect low procedure and fluency skills when it comes to manipulating values using the four operations or reading a graph and understanding x values, y values. Confronting misconceptions about appropriately and correctly using the four operations and recognizing key components of general graphs can improve student learning as these skills are extended with linear equations and graph reading.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying analyzing and solving linear equations and pairs of simultaneous linear equations because the recognition of solutions satisfying simultaneous equations is clear and non-examples are also clear when looking at a graph, however justifying an example from a non-example becomes abstract when placed in context. Students needing extension could be given graphs of examples and non-examples and be asked to create real-world mathematical word problems that could be represented by each graph.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying how to analyze and solve linear equations and pairs of simultaneous linear equations the types of mathematical tasks are critical because students should understand where to look on a graph to find the solution. They should be able to analyze and interpret the solution when the lines intersect, when they are parallel or when they coincide.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source:
http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/757/original/public_task_472.pdf?1462392465

8.EE.C.8 Analyze and solve linear equations and pairs of simultaneous linear equations.

- Learning Target: I can solve linear equations and pairs of simultaneous linear equations.
- Webb's Depth of Knowledge*: 3*
- This task can be used to both assess student understanding of systems of linear equations or to promote discussion and student thinking that would allow for a stronger solidification of these concepts. The solution can be determined in multiple ways, including either a graphical or algebraic approach.

Ivan's furnace has quit working during the coldest part of the year, and he is eager to get it fixed. He decides to call some mechanics and furnace specialists to see what it might cost him to have the furnace fixed. Since he is unsure of the parts he needs, he decides to compare the costs based only on service fees and labor costs. Shown below are the price estimates for labor that were given to him by three different companies. Each company has given the same time estimate for fixing the furnace.

- Company A charges $35 per hour to its customers.
- Company B charges a $20 service fee for coming out to the house and then $25 per hour for each additional hour.
- Company C charges a $45 service fee for coming out to the house and then $20 per hour for each additional hour.

For which time intervals should Ivan choose Company A, Company B, Company C? Support your decision with sound reasoning and representations. Consider including equations, tables, and graphs.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on how to analyze and solve linear equations and pairs of simultaneous linear equations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, provide students with real-world examples that are in context and relative to their cultural and linguistic background. Provide questions that help them make connections to concepts and their cultural understanding of math.	Science: Compare linear relationships and systems of equations in scientific data.

NEW MEXICO
Public Education Department

8.F: FUNCTIONS		
Cluster Statement: A: Define, evaluate, and compare functions. **Major Cluster** (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade).		
Standard Text **8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.**	**Standard for Mathematical Practices** SMP 2: Students can reason abstractly and quantitatively by determining if a relationship is a function.	**Students who demonstrate understanding can:** • Know and flexibly use the terms function, input, and output. • Analyze tables and graphs by interpreting their relationships as functions. • Understand that a function is a rule that states each input has exactly one output, not just how to recognize them. • Understand that each function produces a graph. • Formulate and defend opinion on whether a table or graph is a function or not with use of counterexamples. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text **8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function**	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by identifying important quantities such as rate of change and y-intercept in a real-world situation. SMP 5: Students can use tools by utilizing the coordinate plane (graph paper) to graph relationships	**Students who demonstrate understanding can:** • Determine the slope and the y-intercept from an equation, a table, a graph, and a verbal description. • Explain orally and in writing that slope represents rate of change and y-intercept represents initial value or starting value. • Understand how to generate additional ordered pairs for a function. • Compare the properties of a graph, an equation, a table, and verbal descriptions given a real-world linear situation.

has the greater rate of change		**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Analyze, Evaluate
Standard Text **8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s2 (s squared) giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line**.	**Standard for Mathematical Practices** SMP 1: Students can make sense of problems and persevere in solving them by explaining correspondences between equations, verbal descriptions, tables, and graphs of important features such as rate of change and y-intercepts. SMP 7: Students can look for and make use of structure by using $y = mx + b$ as the equation for a linear function	**Students who demonstrate understanding can:** • Understand a linear function has a constant rate of change called slope and will produce a line on a graph. • Understand a nonlinear function does not have a constant rate of change and will not produce a line on a graph. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
<u>Previous Learning Connections</u> • In 7th grade, learners analyze proportional relationships and use them to solve real-world and mathematical problems. They solve real-world and mathematical problems using numerical and algebraic expressions and equations These previously learned skills help build connections between expressions and linear equations in Grade 7 to linear relationships of functions in Grade 8.	**<u>Current Learning Connections</u>** • This learning connects to the learning that will occur later in 8th grade when students begin analyzing graphs of functional relationship and construct functions to model relationships between two quantities.	**<u>Future Learning Connections</u>** • In high school, student make connections to the learning done within this cluster when they interpret functions that arise in application in terms of the context.

Clarification Statement:
Students understand that a **function** is a rule that takes an **input** and produces only one **output**; therefore, functions occur when there is exactly one y-value associated with any x-value. Students identify functions from equations, graphs, and tables/ordered pairs and are not expected to use the function notation f(x) at this level. This standard requires students to clarify the definitions of key terms including **function**, **input, output, y-value,** and **x-value**.

Common Misconceptions

It is vital to ensure students have a common understanding of the vocabulary terms in this unit as common errors are often correlated to the confusion of the terms function, input, output, x-value, and y-value. Students can confuse the relationship between an input and output and the idea that each input only has one output. When making connections to graphs and tables, this same confusion is found when using the terms x-value and y-value. The confusion around the relationship between the input and output is heightened when students realize that more than one input can give the same output.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying how to define, evaluate and compare functions because this allows the learner to become engaged with the content in real world situations. This helps to build interest as well as expose the learner to new content. Students can also begin to form basic definitions of a function by being exposed to new contexts.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.RP.A.2 : This standard provides a foundation for work with how to define, evaluate and compare functions because students must first develop an understanding of proportional relationships and how they correspond among a table, graph and equation. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with define, evaluate, and compare functions benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because in order to promote interest in the content that is being learned, the learner needs to feel that the information is relevant and meaningful. In a task where students may be asked to look at tables to determine if it represents a function, the tables should represent information and relationships that are relevant.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with define, evaluate, and compare functions benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because engagement with peers can significantly increase sustained effort in learning the content. A task where students must look at tables and determine whether they represent functions might seem daunting or difficult for some students and therefore students may not want to persist in the task. If learners are interacting with peers in flexible groups where each learner has a role, the content can be engaging and accessible.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds.*

- For example, learners engaging with define, evaluate, and compare functions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because access to information should not be a hindrance to the learning. Displaying the task on an anchor chart and the graphs where students can easily see them will allow better capacity for processing the information and not decoding the prompt, task or symbols.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with define, evaluate, and compare functions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of novel solutions to authentic problems because providing students with multiple opportunities to express their learning will help to develop independence when identifying functions from graphs. Students should be given graphs that represent different situations that students can express their ideas, connections, and thinking about.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with define, evaluate, and compare functions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because learners need to be able to deal with frustration and avoid anxiety when confronting a difficult task. When examining a graph that doesn't make sense to the learner, it would be helpful to offer options for what may be giving the student issues in understanding whether the graph represents a function. If learners feel that we can move past frustration, then they can focus on internalizing the ideas presented in the task.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on how to define, evaluate and compare functions by providing specific feedback to students on their work through a short mini-lesson because if a student is struggling with understanding that a function is a rule that assigns exactly one output to one input. These misunderstandings can go back throughout elementary and relate to a misunderstanding of proportionality. This can be a quick exit ticket of defining a function of a table. Then feedback should be given promptly.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit how to define, evaluate and compare functions by addressing conceptual understanding because students should be able to apply their understanding of defining, evaluating and comparing functions in different contexts. The teacher should be able to see that the students can define functions based on different representations and contexts.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying how to define, evaluate and compare functions because students can transfer their understanding into different scenarios where a function is an appropriate representation of the data. The burden of proving that this is a function relationship will depend on their ability to display and make connections between functions and how to evaluate if the example is really a function. Students can connect the verbal rule of the function with a representation in a table, and then a graph and an expression.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

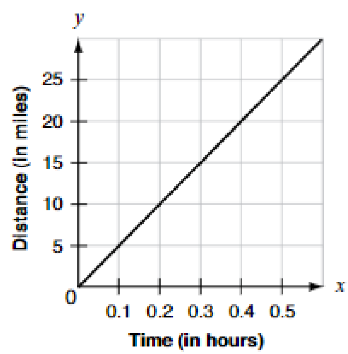
Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying how to define, evaluate and compare functions facilitating meaningful mathematical discourse is critical because it is important to motivate students to define the meaning of a function as they understand it. Allowing students to communicate their different definitions of a function by analyzing different relationships. In this way students are led to not only share their ideas but also to listen to others in a positive way. Students may be asked or challenged to defend their ideas, and this gives way building discourse as a mathematical community of learners.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Formative Item Set for Grade 8 Functions

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

- Learning Target: I can compare two rates given in a statement and a graph.
- Webb's Depth of Knowledge: 2

A zebra runs at a rate of 40 miles per hour. The graph shows the rate at which a gazelle runs.



What is the difference in their running rates in miles per hour?

- Ⓐ 10
- Ⓑ 15
- Ⓒ 40
- Ⓓ 50

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on how to define, evaluate and compare functions , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students may provide ideas for what kids of relationships can be shown using a function. These relationships or situations can be unique to the learner's home culture or surrounding community. This is an opportunity for students to connect real word scenarios to mathematics.	Science: Physical Science constant speed/average speed Social Studies: Geography/History of Travel looking at distance/time Art: Mixing Paint adjusting paint parts to create a certain shade/quantity Gym: Keeping score in a game (For every touchdown, you get x amount of points)

8.F: FUNCTIONS

Cluster Statement: B: Use functions to model relationships between quantities.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade).

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	SMP 4: Students model with mathematics as they construct functions to model linear relationships. SMP 7: Students make use of qualitative features found in verbal descriptions of functions and sketch the functions.	• Write the function for a linear relationship between two quantities. • Identify the rate of change • Identify the slope of the function from two points (x,y), from a graph and a table. • Interpret the rate of change (slope) and initial value of a linear function from a table, graph, equation or verbal description. • Calculate the slope of a line using the rise over run ratio. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	SMP 3: Students construct viable arguments and critique the reasoning of others as they describe the relationship between two quantities. SMP 4: Students model with mathematics when creating tables or sketching graphs of functional relationships.	• Interpret linear and nonlinear graphs. • Describe the relationships between two quantities (linear, nonlinear, increasing or decreasing). • Sketch graphs of linear and nonlinear functions. • Analyze the sketches of linear and nonlinear functions. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Analyze, Create

Previous Learning Connection	Current Learning Connections	Future Learning Connections
• In 7th grade, students analyze proportional relationships and use them to solve real-world and mathematical problems. Students solved real-world and mathematical problems using numerical and algebraic expressions and equations.	• In 8th grade, students graph proportional relationships, interpreting the unit rate as the slope of the graph. Students are working to interpret the equation y = mx + b as defining a linear function and understand that a function is a rule that assigns to each input exactly one output. By the end of 8th grade, students will compare properties of two functions each represented in a different way.	• In high school, students begin to apply the concept of a function with use of function notation. Students will interpret functions that arise in application in terms of the context.

Clarification Statement:
Students determine and interpret the rate of change and the initial value to construct a linear model. They use a real-world situation to sketch a graph and use a graph to write a verbal description of a real-world situation.

Common Misconceptions
Students may use different scales on the axes and then try to compare rates. Point out that in order to compare the constant rate of change visually, the scales and labels on the axes must be the same. Make sure students identify the correct scales on a graph, not all scales increase by 1 or by the same increment.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying use functions to model the relationship between quantities because this allows the learner to become engaged with the content in real world situations. This helps to build interest as well as expose the learner to new content. Students can also begin to form basic definitions of a function by being exposed to new contexts.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.RP.A.2: This standard provides a foundation for work with use functions to model the relationship between quantities because students must first develop an understanding of proportional relationships and how they correspond among a table, graph and equation. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with using functions to model relationships between quantities benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because this will help students to make real world connections to mathematics as well as create a relevant goal. For example, learners can be given the task of completing a home experiment where they measure heart rate in beats per minute as the function of a rate and graphed. They will measure heart rate with and without jumping jacks. This experiment can be conducted at home with family and then shared with their peers.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using functions to model relationships between quantities benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because this will help the learner confront any feeling of discouragement in their work of using functions to model relationships. The feedback given may be about how to construct a more precise graph, or how to determine if their relationship is a function. it must be positive and focused on the learners' process of working through the task and putting effort rather than the outcome produced.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with using functions to model relationships between quantities benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as <allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because the more exposure to symbols and ideas that learners have will help them comprehend and decode effectively. This can be anchor charts with tasks, graph and function tables displayed. This can be student group work and tasks taken home and displayed on chart paper.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with <using functions to model relationships between quantities benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because when students receive feedback not only from teachers but also from peers they can internalize that feedback and improve on their methods for using functions to model relationships. From a student perspective, receiving feedback can be a constructive experience in determining if functions are being displayed and used properly.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using functions to model relationships between quantities benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations)because this helps the learner transfer their learning to new and relevant contexts. For example, students can be asked to think of their relationships or situations that can be represented as a function and modeled. This gives the learner the opportunity to make connections between the mathematics and the real world they live in.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on use functions to model the relationship between quantities by critiquing student approaches/solutions to make connections through a short mini-lesson because students may display small misunderstandings that could hinder their comprehension. Graphs are everywhere in the study of functions, but it is important to distinguish a function from its graph. For example, a linear function does not have a slope, but the graph of a non-vertical line has a slope.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit use functions to model the relationship between quantities by addressing conceptual understanding because students use functions to model relationships between quantities, which makes this cluster one that has a primary focus on application problems. This builds on previous work with algebraic patterns, input/output rules, and ratios and proportional relationships for which students should be able to apply to real world situations. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying use functions to model the relationship between quantities because this leaves opportunity for class discussion that offers students to verbally show their thinking. Students can create an extension activity that can be used with peers that examine connections between (x,y) values and interpret them from a table or graph.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying using functions to model relationships between quantities eliciting and using student thinking is critical because this contributes to the classroom culture of all learners being mathematicians. Allowing students to express ideas as an opportunity to share thinking in different representations such as drawings, graphs, and verbal descriptions fosters confidence in the conclusions they have made. Students who work together and share ideas in cooperative groups benefit from comparing their models of functions and those relationships.

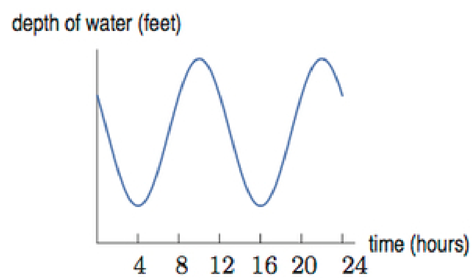
Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/8/F/B/5/tasks/628

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

- Learning target: I can interpret the graph of a function.
- Webb's Depth of Knowledge: 3
- This is a simple task about interpreting the graph of a function in terms of the relationship between quantities that it represents.

Task

The figure below gives the depth of the water at Montauk Point, New York, for a day in November.



a. How many high tides took place on this day?

b. How many low tides took place on this day?

c. How much time elapsed in between high tides?

Relevance to families and communities:
During a unit focused on using functions to model relationships between quantities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can complete an experiment at home with family members where they are measuring heart rate as a function, with and without jumping jacks. Students can display their findings about the increase in beats per minute as a

Cross-Curricular Connections:
Science: Students could examine scientific data and predict the effect of a change in one variable on another

function on a graph. These findings can then be brought and shared in the classroom environment.	

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<table>
<tr><td colspan="3">

8.G: GEOMETRY

</td></tr>
<tr><td colspan="3">Cluster Statement: A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)</td></tr>
<tr>
<td rowspan="3">Standard Text

8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:

• 8.G.A.1.A: Lines are taken to lines, and line segments to line segments of the same length.
• 8.G.A.1.B: Angles are taken to angles of the same measure.
• 8.G.A.1.C: Parallel lines are taken to parallel lines.</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 4: Students model with mathematics. Students model on the coordinate plane to explore congruent and similar figures.

SMP 5: Students use appropriate tools strategically.

SMP 6: Students attend to precision. Students are careful to bring lines to lines and angles to angle when performing transformations.

SMP 7: Students look for and make use of the structure of figures as they transform them.</td>
<td>Students who demonstrate understanding can:
• Construct transformations by using models, transparencies or geometry software, and develop an understanding of the relationship of the original to its image.
• Analyze the relationships between corresponding sides and corresponding angles of the original figure to its image.
• Translate figures, given a set of rules, on the coordinate plane.
• Evaluate and describe transformations.
• Accurately transform figures on the coordinate plane using rotations, translations, reflections, and the correct notation.
• Identify transformations performed to transform an image to the original.</td>
</tr>
<tr><td>Webb's Depth of Knowledge: 3-4</td></tr>
<tr><td>Bloom's Taxonomy:
Analyze, Evaluate, Create</td></tr>
<tr>
<td>Standard Text

8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</td>
<td>Standard for Mathematical Practices

SMP 4: Students model with mathematics. Students model on the coordinate plane to explore congruent and similar figures.

SMP 5: Students use appropriate tools strategically.

SMP 6: Students attend to precision. Students are careful to bring lines to</td>
<td>Students who demonstrate understanding can:
• Identify congruent figures by describing a sequence of rotations, translations or reflections that map one figure onto another.
• Effectively describe the series of transformations verbally or in writing.
• Create congruent figures by applying a series of transformations (use correct notation).</td>
</tr>
</table>

	lines and angles to angle when performing transformations. SMP 7: Students look for and make use of the structure of figures as they transform them.	• Understand that a series of rotations, translations or reflections preserves the size and shape of the figure (congruence). **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Create
Standard Text **8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.**	**Standard for Mathematical Practices** SMP 4: Students model with mathematics. Students model on the coordinate plane to explore congruent and similar figures. SMP 5: Students use appropriate tools strategically. SMP 6: Students attend to precision. Students are careful to bring lines to lines and angles to angle when performing transformations. SMP 7: Students look for and make use of the structure of figures as they transform them.	**Students who demonstrate understanding can:** • Identify the image of a figure on a coordinate grid given a scale factor and center of dilation. • Create a dilation of a polygon on a square grid given a scale factor and center of dilation. • Describe (orally) a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points. • Draw and label a diagram of a line segment rotated 90 degrees clockwise or counterclockwise about a given center. • Generalize (orally and in writing) the process to reflect any point in the coordinate plane. • Identify (orally and in writing) coordinates that represent a transformation of one figure to another. • Determine and describe a series of transformations from a pre-image to an image. • Recognize the relationship between the original coordinates and the coordinates of the image and understand that rotations, reflections and translations follow a specific pattern on the coordinate plane. • Recognize that you can use coordinates to find the scale

		factor of a dilation. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** understand
Standard Text 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**Standard for Mathematical Practices** SMP 4: Students model with mathematics. Students model on the coordinate plane to explore congruent and similar figures. SMP 5: Students use appropriate tools strategically. SMP 6: Students attend to precision. Students are careful to bring lines to lines and angles to angle when performing transformations. SMP 7: Students look for and make use of the structure of figures as they transform them.	**Students who demonstrate understanding can:** • Understand the concept of similar figures. • Conclude that a two-dimensional figure is similar to another by describing a sequence of translations, rotations, reflections and dilations that will map the original figure onto the image (vice-versa). • Express their understanding verbally and in written form. • Create similar figures given a sequence of transformations. **Webb's Depth of Knowledge:** 1-4 **Bloom's Taxonomy:** understand, apply, create
Standard Text 8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**Standard for Mathematical Practices** SMP 3: Students construct viable arguments and critique the reasoning of others when explaining the relationships of angles and how they are used to find missing measurements. SMP 4: Students model with mathematics when using formulas and drawings to show angle sums adding to form a line.	**Students who demonstrate understanding can:** • Use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal. • Apply their knowledge of angle relationships to reason about parallel lines. • Identify exterior and interior angles of triangles. • Apply their knowledge to determine if two triangles are similar. • Use the angle-angle criterion for similarity of triangles. • Determine if two triangles are similar or not and explain how they know. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• In 4th-7th grade, students draw, construct, and describe geometric figures (such as angles and polygons) and their relationships. Students solve real-life and mathematical problems involving angle measure.	• In 8th grade, this cluster does not directly connect to any other cluster.	• In future courses, students develop a more formal understanding of transformations in the plane and prove theorems about triangles, lines, and angles.

Clarification Statement:
Students describe and apply translations, rotations, reflections, and dilations to understand congruent and similar figures. Students explain and understand angle relationships.

Common Misconceptions

- Students may see a reflection as a translation
- Students may think rotation, reflection, or translations change the size or shape of a figure.
- Students may forget that dilations with a scale factor between 0 and 1 result in a smaller image. Students may forget to change signs in coordinates when reflecting over an axis.
- Students will make errors if he/she looks at the wrong transversal. Students may confuse congruent and supplementary angles, apply rules to lines that are not parallel.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying congruence and similarity using physical models, transparencies and geometry software because students will be able to make connections to vocabulary using examples and definitions. Some of this vocabulary could be names of figures and angles and others can be about the topic of congruence and similarity.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.G.A.2: This standard provides a foundation for work with congruence and similarity because when students are asked to sketch, draw, and compose geometric shapes, they are laying the foundation for the practice of geometric deduction that will be used further on throughout their education . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding congruence and similarity using physical models, transparencies or geometry software benefit when learning experiences include ways to recruit interest such as creating socially relevant tasks because students will be more interested in activities or learning goals that are more relevant to their lives. One of the ways teachers can do this is by creating meaningful activities that demonstrate the value of this learning towards the goals of the learner. Provide students with a purpose for the learning that is clear to all learners.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding congruence and similarity using physical models, transparencies or geometry software benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because it is important for learners to be able to cooperate and collaborate with other learners. This can foster important conversations that will keep learners engaged in the goals of a task. For example, one task may be that students are asked to look at a set of figures and determine which ones are the same size and same shape. Students would be asked to explain their reasoning with their peers. In their explanation they may be asked to explain what it means for two figures to be the same size and the same shape. This task can initiate conversations about congruence and prior knowledge from earlier years of what same size and same shape means.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding congruence and similarity using physical models, transparencies or geometry software benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because this will make vocabulary more accessible to all learners. For example, in a task where students are asked to explain whether a set of given figures is the same size and the same shape, they may rely on previous knowledge gained throughout elementary school. The idea of congruence in year 8 is embedded in the standards throughout elementary. Students can use alternate expressions of the meaning for congruence to explain their thinking and make connections to the new word.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding congruence and similarity using physical models, transparencies or geometry software benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing sentence starters or sentence strips because they will support the learner in constructing an explanation or description of their thinking. This can be used during almost any activity to encourage and support discussion among peers. For example, when students are given the task of trying to explain whether figures are the same size and same shape, the student can use the sentence frame of "I notice ____, so…" or "First, I _____ because…".

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with understanding congruence and similarity using physical models, transparencies or geometry software benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as increasing the length of on-task orientation in the face of distractions because some students may experience a certain amount of frustration in the face of a difficult task. Doing this will help learners to avoid being anxious about tasks and stay focused on being motivated to achieve their learning goal.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding congruence and similarity using physical models, transparencies and geometry software by revisiting student thinking through a short mini-lesson because this will allow the learner to review what their thinking was prior to the lesson and reflect on changes in thinking that have been made. This will also allow the instructor to identify any misconceptions based on the concept of congruence, or a misunderstanding of the process in determining congruence and similarity.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of understanding congruence and similarity using physical models, transparencies and geometry software by confronting student misconceptions because once misconceptions are identified whether based on misunderstanding of congruence or modeling the concept with dilations rotations, reflections and translations, then the teacher can address those misunderstandings on a more specific level. Teacher may also decide whether content vocabulary is an issue for students and re-teach these vocabulary words on a more intensive basis. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying and understanding congruence and similarity using physical models, transparencies and geometry software because this type of task would allow for some integration of other disciplines such as art in order to express understanding. An example of this would be allowing students to create a mosaic using transformations.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying understanding congruence and similarity using physical models, transparencies, and geometry software the types of mathematical tasks are critical because they can allow for multiple, creative solutions. Tasks should be worded to support a wide variety of approaches and solutions. Open ended tasks that elicit a wide range of ideas are better than tasks that prescribe a certain strategy and outcome.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/8/G/A/2/tasks/646

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

- Learning Target: I can prove two figures are congruent.
- Webb's Depth of Knowledge: 2
- Students' first experience with transformations is likely to be with specific shapes like triangles, quadrilaterals, circles, and figures with symmetry. Exhibiting a sequence of transformations that shows that two generic line segments of the same length are congruent is a good way for students to begin thinking about transformations in greater generality.

Task

Line segments AB and CD have the same length. Describe a sequence of reflections that exhibits a congruence between them.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding congruence and similarity using physical models, transparencies, and geometry software, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about connections students can make with vocabulary such as rotation, translation, rotations and dilations, to their home languages can help to build independence and confidence.	Art: Geometric artwork

8.G: GEOMETRY

Cluster Statement: B: Understand and apply the Pythagorean Theorem.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse.	SMP 3: Students construct viable arguments and critique the reasoning of others by explaining a proof of the Pythagorean Theorem. SMP 4: Students use modeling to understand the meaning of the Pythagorean Theorem and prove it.	• Model a proof of the Pythagorean Theorem and verbally or in written form explain the proof. • Understand the converse of the Pythagorean Theorem and be able to apply it to any triangle to prove it is or is not a right triangle. **Webb's Depth of Knowledge:** 2-4 **Bloom's Taxonomy:** Apply, Evaluate
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	SMP 7: Students look for and make use of structure by discovering how the Pythagorean Theorem can be used to solve for any side of a right triangle.	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. • Solve problems where they must apply the Pythagorean Theorem. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

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Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	SMP 7: Students use the structure of the coordinate plane to draw a right triangle, an example of looking for and making use of structure in the coordinate plane. SMP 6: Students attend to precision when substituting the correct values into the Pythagorean Theorem, calculating correctly, and using academic vocabulary correctly in explanations.	• Apply the Pythagorean Theorem to find the distance between two points on a coordinate system. • Recognize the diagonal line is the hypotenuse and the vertical and horizontal legs that connect are the legs. • Solve real-world problems using the Theorem as a strategy. • Explain solution strategies using correct mathematical vocabulary. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply

<u>Previous Learning Connections</u>	**<u>Current Learning Connections</u>**	**<u>Future Learning Connections</u>**
• In 6th grade, students graph points in a coordinate system and find the horizontal or vertical distance between two points in a coordinate system. Students draw polygons in a coordinate system when given vertices. In 7th grade, students expand these skills to find the area of squares.	• In 8th grade, students will use square root symbols to represent solutions and approximate square root values.	• In high school, students prove theorems about triangles. Students use Pythagorean Theorem to solve problems and discover other mathematical relationships.

Clarification Statement:
Students explore the relationships between sides of a right triangle to understand the formula $a^2 + b^2 = c^2$. They solve problems applying the Pythagorean Theorem.

Common Misconceptions
- Some students might calculate the length of the triangle leg instead of the hypotenuse. Confuse the leg for the hypotenuse.
- Students may forget to find the square root.
- Students try to find missing side lengths for triangles that are not right triangles and need experiences reconstructing the proof by drawing squares on the sides of the triangle to see that the areas do not add up.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*
- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time) when studying to understand and apply the Pythagorean Theorem because students are already very

familiar with triangles and to revisit the type of triangles, angles of a triangle and know that this theorem is only applicable to a right triangle. They will also benefit from reviewing exponents, squares and square roots.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.G.B.6 : This standard provides a foundation for work to understand and apply the Pythagorean Theorem- because reviewing what they learned about triangles from the previous year will help them connect to the right triangle. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding and application of the Pythagorean Theorem benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because students can see real-world use of the theorem and make connections. The Pythagorean Theorem is used in football, baseball, in construction and architecture. GPS coordinates use the Pythagorean Theorem as well. When students solve problems that they can relate to, it makes the math more meaningful and relevant.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding and application of the Pythagorean Theorem benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because when students see that the math standard connects to their cultural background, they see it as relevant to them. The Pythagorean Theorem has ancient proofs that span the Earth, over many continents and countries. The ancient uses, by our ancestors are relevant to our understanding the modern uses of the Theorem.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding and application of the Pythagorean Theorem benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.)m because a visual model of the Pythagorean Theorem, with key vocabulary highlighted will help students makes connections to the key components of the theorem (a,b are legs and c is the hypotenuse). Videos can model uses of the Pythagorean Theorem, the components and key vocabulary of the theorem and show students how to apply it to real world problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding and application of the Pythagorean Theorem benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because these math tools will help students work efficiently on the task. Using calculators is an efficient way for students to square numbers and to get the square root of numbers. Using graph paper can help students find the distance between two points in a coordinate system. Geometric software can also be helpful for this standard.

Internalize

Comprehension: *How will the learning for student's support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding and application of the Pythagorean Theorem benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as using outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships because graphic organizers can be powerful visual aids that help students review the key components of the Pythagorean Theorem. They can use them to visualize the proofs, the converse of the theorem and the application of the theorem. They can also include concept mastery routines that emphasize the key concepts (ex. How to solve for a missing leg? How to solve for a missing hypotenuse? How to apply the converse to prove a triangle is a right triangle or not?).

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on how to understand and apply the Pythagorean Theorem by clarifying mathematical ideas and/or concepts through a short mini-lesson because a clear understanding of a right triangle and the part of a right triangle will make the application of the Pythagorean Theorem clearer.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit to understand and apply the Pythagorean Theorem-insert language of cluster by addressing conceptual understanding because application of the theorem is a multi-layer approach and students will have a better learning path if concepts underlying the theorem is clear to them.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying to understand and apply the Pythagorean

	Theorem because students will have a better appreciation of the mathematics around them and know that the presence of mathematics is beyond math class.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

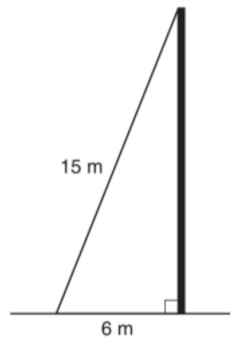
Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying to understand and apply Pythagorean Theorem facilitating meaningful mathematical discourse is critical because when students can articulate what they understand or are confused about helps them validate what they currently know/not know. In some instances, students share what they know about triangles based on their cultural background.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Formative Item Set for Grade 8 Geometry

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

- Learning Target: I can use the Pythagorean Theorem to find the length of one leg of a right triangle.
- Webb's Depth of Knowledge: 2

This diagram shows a 15-meter wire attached to the top of a telephone pole. The wire is attached to the ground at a point 6 meters from the base of the pole.



What is the height of the pole?

Ⓐ $\sqrt{189}$ meters

Ⓑ $\sqrt{219}$ meters

Ⓒ $\sqrt{231}$ meters

Ⓓ $\sqrt{261}$ meters

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on to understand and apply Pythagorean Theorem, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the mathematics used within the different careers of your family and community can provide a strong connections between school and careers.	Language Arts: Students can do research on a famous mathematician that has a known proof of the Pythagorean Theorem and write an essay about the proof.

NEW MEXICO
Public Education Department

8.G: GEOMETRY

Cluster Statement: C: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Major Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade).

<table>
<tr>
<td>Standard Text

8.G.C.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</td>
<td>Standard for Mathematical Practices

SMP 1: Students make sense of problems and persevere in solving them by finding the volume of composite shapes.

SMP 3: Students construct viable arguments and critique the reasoning of others by explaining the relationship between the cylinder, cone, and sphere. Students will discuss and determine the shapes that construct a composite shape.

SMP6: Students attend to precision by labeling volumes with units cubed and areas as units squared. They approximate a precise volume working with pi.</td>
<td>Students who demonstrate understanding can:
• Write formulas from memory for finding the volume of cones, spheres, and cylinders. are special equations that are specific in use.
• Make connections between the 3-D figures and their formulas.
• Use formulas to calculate volumes of cones, cylinders and spheres.
• Explain the relationship in their volumes.
• Apply the formulas to solve real world application problems related to volume.

Webb's Depth of Knowledge: 1-3

Bloom's Taxonomy:
Understand, Apply, Evaluate</td>
</tr>
<tr>
<td><u>Previous Learning Connections</u>
• In 5th and 6th grade, students find the volumes of right rectangular prisms. In 7th grade, students find the area of a circle and solve real-world problems involving area and volume.</td>
<td><u>Current Learning Connections</u>
• In 8th grade, students continue this work using square root and cube root symbols</td>
<td><u>Future Learning Connections</u>
• In high school, students use geometric shapes and their measurements to describe objects and solve design problems.</td>
</tr>
</table>

Clarification Statement:
Students know and apply the volume formulas of a cylinder, cone, and a sphere.

Common Misconceptions
Errors may occur if students do not substitute lengths correctly. Students may confuse the volume solids for different solids. They may forget how height, radius, and diameter relate to volume, confuse diameter and radius, forget the approximate value of pi.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying how to solve real world mathematical problems involving volume of cylinders, cones and spheres because students will be expected to know and understand how to use formulas for finding volume of cylinders, cones and spheres.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.EE.A.2: This standard provides a foundation for work with solving real world mathematical problems involving volume of cylinders, cones and spheres because students must understand how to use square root and cubed root symbols in order to represent solutions to equations . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with solving real world mathematical problems involving cylinders, cones, and spheres benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because the idea of cylinders, cones and spheres can seem abstract until a context is assigned to these figures. For example, students may relate this to the task of shipping a package that has cylinder containers of oats. Students may know the dimensions of the cylinder containers but need to figure out the dimensions for the box to ship them in this provides a real-world context and builds interest for the learner.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving real world mathematical problems involving cylinders, cones, and spheres benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because students will be able to attach their own experiences and prior knowledge to the task. For example, if students are asked to think of shipping items that are either cylinders, cones, or spheres, they might take time to brainstorm what kinds of items come in those shapes. Some students may think of basketballs or cans of soda. This allows the learner to think of a context that they can relate to and therefore help them to stay focused on the task.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or* *puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving real world mathematical problems involving cylinders, cones, and spheres benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) because if students are not able to visualize the figures that are being described by dimensions only or do not have a realistic interpretation of those figures, students may find this concept too abstract and therefore unrelatable. For example, use of geometry manipulatives or illustrative software that can be manipulated to represent vocabulary and terms may be helpful.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving real world mathematical problems involving cylinders, cones, and spheres benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) because this allows learners to have another opportunity to develop a wider range of expression that is familiar to them. Learners may find it hard to express their ideas in words alone. The ability to "show" using composition of their ideas is another form of expressing their learning.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with solving real world mathematical problems involving cylinders, cones, and spheres benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because as students complete tasks for review and practice, they will have multiple opportunities to transfer the information they have learned to new situations. This practice will make the learning more memorable and accessible to the learner.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on solving real world mathematical problems involving volume of cylinders, cones and spheres by providing specific feedback to students on their work through a short mini-lesson because while students are engaged in using formulas to find volume for these figures, errors may occur that are small but will result in a learner not achieving a correct solution. This would be a good time for the instructor to provide immediate feedback to the learner during this process that will then help the learner correct his/her process.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit for solving real world mathematical problems involving volume of cylinders, cones and spheres by addressing conceptual understanding because if a learner is demonstrating an incorrect solution, it can be assumed that the student is either having conceptual misunderstandings or procedural misunderstandings. If students are attending to precision in their work, then it may help to focus on attaching meaning to the concept that is being learned.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying solving real world mathematical problems involving volume of cylinders, cones and spheres because students can use different forms of expression to show what they have learned about volume of cylinders, cones and spheres by working on a project to display or build a silo and demonstrate the volume. They will calculate the volume of a real-world silo and use their model to explain.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying how to solve real world mathematical problems involving volume of cylinders, cones and spheres supporting productive struggle is critical because it will allow students to move past only trying to attain correct solutions, but instead focus on the struggle of working through a difficult problem. Working through a task should help the learner attach meaning to the answers they are getting as well as determine the relationship between the solutions they are getting and the work they are doing. When finding the volume of cylinders, cones and spheres, students can engage in a meaningful task that is relevant and therefore encourages the student to persist.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: http://tasks.illustrativemathematics.org/content-standards/8/G/C/9/tasks/517

8.G.C.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

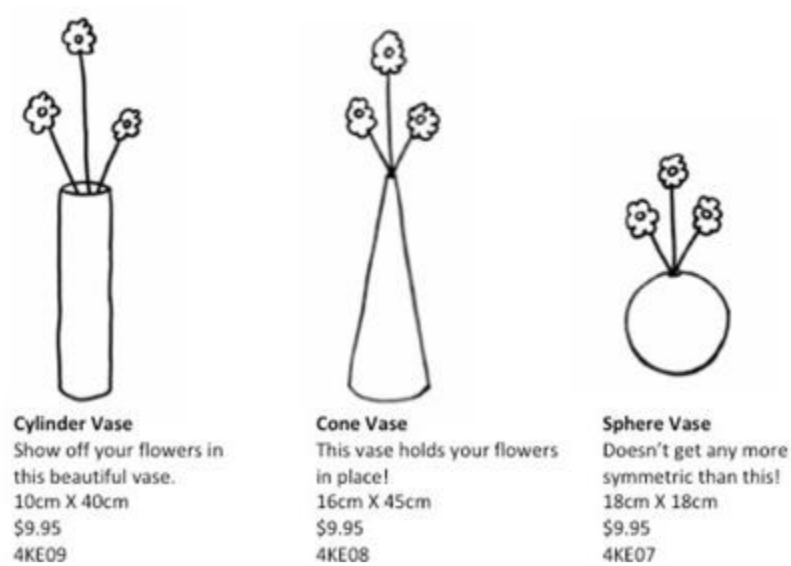
- Learning Target: I can solve volume in the real world, using formulas.
- Webb's Depth of Knowledge: 2

- The purpose of this task is to give students practice working the formulas for the volume of cylinders, cones and spheres, in an engaging context that provides an opportunity to attach meaning to the answers. When used in a classroom setting, the task could be supplemented by questions that ask students to thinking about the relationship between volume and liquid capacity. For example, after part (b), the teacher could ask the students for other ways to determine which vase holds the most water, with the expectation that students might respond with the idea of pouring water from one vase into another.

Task

My sister's birthday is in a few weeks and I would like to buy her a new vase to keep fresh flowers in her house. She often forgets to water her flowers and needs a vase that holds a lot of water. In a catalog there are three vases available and I want to purchase the one that holds the most water. The first vase is a cylinder with diameter 10 cm and height 40 cm. The second vase is a cone with base diameter 16 cm and height 45 cm. The third vase is a sphere with diameter 18 cm.

a. Which vase should I purchase?
b. How much more water does the largest vase hold than the smallest vase?
c. Suppose the diameter of each vase decreases by 2 cm. Which vase would hold the most water?



Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on how to solve real world mathematical problems involving volume of cylinders, cones and spheres , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, students can create their own tasks for finding volume that include spheres, cylinders and cones that they are familiar with in their own home culture. They can take these abstract figures and assign items that they come in contact within other	Art: Students are given a 3-D glass shape to create sand art. They can calculate the amount of sand needed to create their art piece.

situations and develop scenarios in which they would need to find the volume of these items.	

8.SP: STATISTICS & PROBABILITY

Cluster Statement: A: Investigate patterns of association in bivariate data.

Supporting Cluster (Students should spend the large majority of their time (65-85%) on the major work of the grade/course. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.)

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	SMP 2: Students reason abstractly and quantitatively by using a table to make predictions about data that is not included in the table SMP 4: Students model with mathematics by choosing appropriate ways to display data. SMP5: Students use appropriate tools to collect data.	• Construct a Scatter Plot using two sets of quantitative data. • Identify outliers and clusters in a scatter plot. • Determine if there is a linear or nonlinear association in a scatter plot; determine if a linear association is positive or negative. • Explain what the different patterns mean in different contexts. • Describe the patterns and associations they see between two quantities. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply
Standard Text 8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	**Standard for Mathematical Practices** SMP 3: Students construct viable arguments and critique the reasoning of others: Students can make viable arguments based on the relationships and associations determined in the scatterplot. Students discuss what makes some lines a better fit than others	**Students who demonstrate understanding can:** • Construct a trend line and justify its placement among the data. • Model real-world linear relationships on a graph. • Use a trend line to determine whether a set of paired data has a linear association, nonlinear association or no association. • Determine whether the association is positive or negative, strong or weak. • Justify a fit line is a good fit or not. • Explain orally and/or inwriting the meaning of the fit line and

		its properties in terms of the graph's context. • Formulate an equation of the linear model.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Apply
Standard Text 8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	**Standard for Mathematical Practices** SMP 2: Students reason abstractly and quantitatively by making sense of abstract situations as they interpret and use the slope, y-intercept and the equation of linear models to make predictions and solve real-world problems. The numerical value of the slope is interpreted in the context of the problem. SMP 6: Students practice using precise wording to describe the positive or negative association between two variables given scatter plots of data.	**Students who demonstrate understanding can:** • Use linear models to make predictions from data in a scatterplot (trend line) in context. • Interpret the slope and intercept for the context. • Write the linear equation. • Analyze and Interpret the meaning of the slope and y-intercept in a linear model from data in a scatterplot. • Make predictions from the line. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Apply, Analyze
Standard Text 8.SP.A.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not*	**Standard for Mathematical Practices** SMP 1: Students solve problems using a linear model in the context of bivariate data SMP 2: Students explain what different patterns of association mean in specific contexts SMP 3: Students justify, orally and in writing, associations using precise mathematical language SMP 4: Students model with math by using and interpreting two-way frequency tables that represent	**Students who demonstrate understanding can:** • Create two-way frequency tables to display data. • Collect categorical data on two variables • Analyze and Interpret the data in two-way frequency tables. • Calculate relative frequencies and describe possible associations between the variables. **Webb's Depth of Knowledge:** 2-4

they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?	relationships SMP 6: Students solve problems efficiently, accurately, and with the degree of precision appropriate for the context of the problem using precise language. SMP 8: Students understand the broader applications of patterns in bivariate data and see the structure in similar situations.	**Bloom's Taxonomy:** Apply, Create
Previous Learning Connections • In 5th and 6th grade, students learn to plot points in a coordinate grid.	**Current Learning Connections** • In 8th grade, students are able to construct an equation or a function to model a linear relationship and determine/interpret the slope and y-intercept (seen in standards 8.EE.B and 8.F.B)	**Future Learning Connections** • In future courses, students compute and interpret the correlation coefficient and distinguish between correlation and causation. Students will represent two variables on a scatter plot and describe how they are related. They construct, interpret, and summarize data in a two-way table.

Clarification Statement:
Students construct scatter plots and interpret patterns focusing on linear association. They construct two-way tables and interpret relationships using relative frequencies.

Common Misconceptions
- Students may make the error of not reading the plot from left to right; students may interpret a roughly linear relationship will only be will only be shown with data points that fall directly on a line.
- Sometimes when a scatter plot shows no association, students may struggle so they need examples of data that may have no association (length of a person's hair and his or her final grade in mathematics).
- Students may struggle with numbering the axes so that the data is visible, but not misleading.
- Students often think that a line of fit must go through at least some of the data points on the scatter plot.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit when studying patterns of association in bivariate data because students can oftentimes express patterns in data when presented in context versus presented as a scatter plot with an equation. Students can generalize about relationships between categorical data based on experiences and context from their lives prior to introducing the mathematical practices associated with forming these generalizations.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.G.A.2: This standard provides a foundation for work with interpreting values of points in the context of a situation to develop the recognition of patterns between data and scatter plot representations and two way tables because students interpret real word problems and produce a graph based on information gathered from the problem. This learning is essential when it comes to developing awareness of how real-world information is represented visually and how visual representations relate to each other. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with investigations of patterns of association in bivariate data benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because when the bivariate data they collect, create scatterplots, create two-way frequency tables and make connections to a linear model is relevant to them they are able to make connections to the standards. When the concepts are relevant to them, they take interest in their learning.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with investigations of patterns of association in bivariate data benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because together students can collect and analyze bivariate data. When making connections to a linear model, students can discuss and defend their choice of the placement of the line of best fit (trend line). Students can collaborate with their group and choose the best representation of the data.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with investigations of patterns of association in bivariate data benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc.) because when creating the visual of a scatter plot, two-way table, outliers, clusters and a trend line, students can make connections to the key vocabulary and concepts. Students can create a graphic organizer with visual clues for the key concepts.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with investigations of patterns of association in bivariate data benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because looking for patterns in bivariate data may be difficult for some students to internalize. Looking for the best linear model might be difficult for some students to see and apply to making predictions.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with investigations of patterns of association in bivariate data benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as providing graphic organizers and templates for data collection and organizing information because investigations of patterns of association in bivariate data requires students to organize their data in graphs and tables.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on patterns of association in bivariate data by examining tasks from a different perspective through a short mini-lesson because interpretation of data, especially using straight lines to model relationships, leaves room for discussion amongst peers for how a student arrives at a particular conclusion. If students are struggling with drawing conclusions, hearing examples and seeing peers model their thinking may help alleviate misconceptions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of recognizing patterns of association in bivariate data by addressing conceptual understanding because many scaffolded skills produce mastery of this cluster. Students need to be able to construct and interpret a scatter plot, describe relationships using statistical jargon, assess model fit, use and interpret equations and read and construct two-way tables. By addressing conceptual understanding of each of these skills, misconceptions can be revealed.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying patterns of association in bivariate data because data collection is heavily supported in 8th grade Science. Instead of being given a two-way table, students can conduct experiments in connection with NGSS science standards, collect bivariate data, represent that data in a two-way table, and hypothesize correlations between the two variables.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

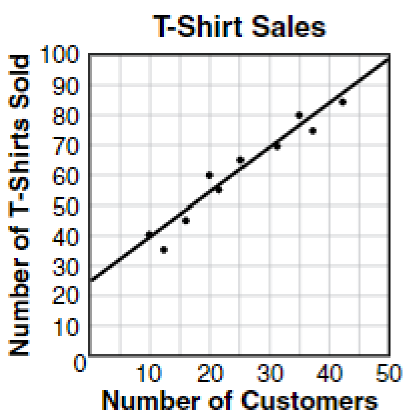
Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when studying patterns of association in bivariate data the types of mathematical tasks are critical because all students need to make connections to mathematics to make it relevant to them. Teachers can build/bridge various cultures and linguistics behaviors by creating tasks where students collect data that is relevant to them. When students display their data in tables and scatter plots, they can analyze the data and study trends that they relate to their personal lives.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Cognia Formative Item Set for Grade 8 Statistics and Probability

8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

- Learning Target: I can interpret the line of best fit from a scatterplot of data to make a prediction.
- Webb's Depth of Knowledge: 2

Albert works in a store that sells T-shirts. He made this graph to show the relationship between the number of customers that come into the store each day and the number of T-shirts the store sells that day.



Based on the graph, about how many T-shirts would be sold on a day when 100 customers come into the store?

A. 150
B. 175
C. 200
D. 22

Relevance to families and communities:

During a unit focused on studying patterns of association in bivariate data, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, having students survey their

Cross-Curricular Connections:

Science: Students can conduct experiments in connection with NGSS science standards, collect bivariate data, represent that data in a two-way table, and hypothesize correlations between the two variables.

families, bring back the data and share with the class. The class can then create a two-way frequency table and a scatter plot that represents their data and the classroom's data. They can use the data to see if there is a correlation between their data and the classroom data. (Height and shoe size).	Social Studies: Study trends in overtime such as populations, the stock market or gross domestic product.

Section 3: Resources, References, and Glossary

Resources

Evidence-Based Resources	English Learner Resources	MLSS Resources	Mathematics Standard Resources
What Works Clearinghouse Best Evidence Encyclopedia Evidence for Every Students Succeeds Act Evidence in Education Lab	World-Class Instructional Design and Assessment (WIDA) Standards USCALE Language Routines for Mathematics English Language Development Standards Spanish Language Development Standards	NM Multi-Layered System of Supports (MLSS) Universal Design for Learning Guidelines Achieve the Core: Instructional Routines for Mathematics Project Zero Thinking Routines	Focus by Grade Level and Widely Applicable Prerequisites High school Coherence Map College-and Career Ready Math Shifts Fostering Math Practices: Routines for the Mathematical Practices

Planning Guidance for Multi-Layered Systems of Support: Core Instruction[9]

Core Instructional Planning must reflect and leverage scientific insights into how humans learn in order to ensure all students are ready for success, thus the following guidance for optimizing teaching and learning is grounded in the **Universal Design Learning (UDL) Framework**

Key design questions, planning actions, and potential strategies are provided below, with respect to guidance for minimizing barriers to learning and optimizing (1) universal ACCESS to learning experiences, (2) opportunities for students to BUILD their understanding of the **Learning Goal**, and (3) INTERNALIZATION of the Learning Goal.

Optimizing Universal ACCESS to Learning Experiences	
ENGAGEMENT ? How will you provide multiple options for recruiting interest?	**Recruiting Student Interest**: ? What do you anticipate in the range of student interest for this lesson? ▯ Plan for options for recruiting student interest: ❒ provide choice (e.g. sequence or timing of task completion) ❒ set personal academic goals ❒ provide contextualized examples connected to their lives ❒ support culturally relevant connections (i.e home culture) ❒ create socially relevant tasks ❒ provide novel & relevant problems to make sense of complex ideas in creative ways

[9] Adapted from: CAST (2018). *Universal Design for Learning Guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org

	❒ provide time for self-reflection about content & activities ❒ create accepting and supportive classroom climate ❒ utilize **instructional routines** to involve all students
REPRESENTATION ? How will you reduce barriers to perceiving the information presented in this lesson?	**Perception**: ? What do you anticipate about the range in how students will perceive information presented in this lesson? ▯ Plan for different modalities and formats to reduce barriers to learning: ❒ display information in a flexible format to vary perceptual features ❒ offer alternatives for auditory information ❒ offer alternatives for visual information
ACTION & EXPRESSION ? How will the learning for students provide a variety of methods for navigation to support access?	**Physical Action**: ? What do you anticipate about the range in how students will physically navigate and respond to the learning experience? ▯ Plan a variety of methods for response and navigation of learning experiences by offering alternatives to: ❒ requirements for rate, timing, speed, and range of motor action with instructional materials, manipulatives, and technologies ❒ physically indicating selections ❒ interacting with materials by hand, voice, keyboard, etc.

Opportunities for Students to BUILD their Understanding	
ENGAGEMENT ? How will the learning for students provide options for sustaining effort and persistence?	**Sustaining Effort & Persistence**: ? What do you anticipate about the range in student effort? ▯ Plan multiple methods for attending to student attention and affect by: ❒ prompting learners to explicitly formulate or restate learning goals ❒ displaying the learning goals in multiple ways ❒ using prompts or scaffolds for visualizing desired outcomes ❒ engaging assessment discussions of what constitutes excellence ❒ generating relevant examples with students that connect to their cultural background and interests ❒ providing alternatives in the math representations and scaffolds ❒ creating cooperative groups with clear goals, roles, responsibilities ❒ providing prompts to guide when and how to ask for help ❒ supporting opportunities for peer interactions and supports (e.g. peer tutors) ❒ constructing communities of learners engaged in common interests ❒ creating expectations for group work (e.g., rubrics, norms, etc.) ❒ providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge ❒ providing feedback that: ❒ emphasizes effort, improvement, and achieving a standard rather than on relative performance ❒ is frequent, timely, and specific ❒ is informative rather than comparative or competitive

	❒ models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success
REPRESENTATION ⍰ How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners?	**Language & Symbols**: ⍰ What do you anticipate about the range of student background experience and vocabulary? ▯ Plan multiple methods for attending to linguistic and nonlinguistic representations of mathematics to ensure universal clarity by: ❒ pre-teaching vocabulary and symbols in ways that promote connection to the learners' experience and prior knowledge ❒ graphic symbols with alternative text descriptions ❒ highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to structure ❒ embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) ❒ embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) ❒ highlighting structural relations or make them more explicit ❒ making connections to previously learned structures ❒ making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas, etc.) ❒ allowing the use of text-to-speech and automatic voicing with digital mathematical notation (math ml) ❒ allowing flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) ❒ clarification of notation through lists of key terms ❒ making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf ❒ linking key vocabulary words to definitions and pronunciations in both dominant and heritage languages ❒ defining domain-specific vocabulary (e.g., "map key" in social studies) using both domain-specific and common terms ❒ electronic translation tools or links to multilingual web glossaries ❒ embedding visual, non-linguistic supports for vocabulary clarification (pictures, videos, etc) ❒ presenting key concepts in one form of symbolic representation (e.g., math equation) with an alternative form (e.g., an illustration, diagram, table, photograph, animation, physical or virtual manipulative) ❒ making explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams
ACTION & EXPRESSION ⍰ How will the learning provide multiple	**Expression & Communication**: ⍰ What do you anticipate about the range in how students will express their thinking in the learning environment? ▯ Plan multiple methods for attending to the various ways in which students can express knowledge, ideas, and concepts by providing:

modalities for students to easily express knowledge, ideas, and concepts in the learning environment?	❒ options to compose in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video ❒ use of social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations) ❒ flexibility in using a variety of problem solving strategies ❒ spell or grammar checkers, word prediction software ❒ text-to-speech software, human dictation, recording ❒ calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper ❒ sentence starters or sentence strips ❒ concept mapping tools ❒ Computer-Aided-Design (CAD) or mathematical notation software ❒ virtual or concrete mathematics manipulatives (e.g., base-10 blocks, algebra blocks) ❒ multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches) ❒ multiple examples of novel solutions to authentic problems ❒ different approaches to motivate, guide, feedback or inform students of progress towards fluency ❒ scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) ❒ differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners)

Optimizing INTERNALIZATION of the Learning Goal	
ENGAGEMENT ? How will the design of the learning strategically support students to effectively cope and engage with the environment?	**Self-Regulation**: ? What do you anticipate about barriers to student engagement? ▯ Plan to address barriers to engagement by promoting healthy responses and interactions, and ownership of learning goals: ❒ metacognitive approaches to frustration when doing mathematics ❒ increase length of on-task orientation through distractions ❒ frequent self-reflection and self-reinforcements ❒ address subject specific phobias and judgments of “natural” aptitude (e.g., “how can I improve on the areas I am struggling in?” rather than “I am not good at math”) ❒ offer devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the math practices for the purpose of monitoring and improving ❒ use activities that include a means by which learners get feedback and have access to alternative scaffolds (e.g., charts, templates, feedback displays) that support understanding progress in a manner that is understandable and timely
REPRESENTATION ? How will the learning support transforming accessible information into usable knowledge	**Comprehension**: ? What do you anticipate about barriers to student comprehension? ▯ Plan to address barriers to comprehension by intentionally building connections to prior understandings and experiences, relating meaningful information to learning goals,

that is accessible for future learning and decision-making?	providing a process for meaning making of new learning, and applying learning to new contexts: ❒ incorporate explicit opportunities for review and practice ❒ note-taking templates, graphic organizers, concept maps ❒ scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) ❒ explicit, supported opportunities to generalize learning to new situations (e.g., different types of problems that can be solved with linear equations) ❒ opportunities over time to revisit key ideas and connections ❒ make explicit cross-curricular connections ❒ highlight key elements in tasks, graphics, diagrams, formulas ❒ outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ❒ multiple examples & non-examples ❒ cues and prompts to draw attention to critical features ❒ highlight previously learned skills that can be used to solve unfamiliar problems ❒ options for organizing and possible approaches (tables and representations for processing mathematical operations) ❒ interactive representations that guide exploration and new understandings ❒ introduce graduated scaffolds that support information processing strategies ❒ tasks with multiple entry points and optional pathways ❒ "Chunk" information into smaller elements ❒ remove unnecessary distractions unless essential to learning goal ❒ anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) ❒ pre-teach critical prerequisite concepts via demonstration or representations ❒ embed new ideas in familiar ideas and contexts (e.g., use of analogy, metaphor, drama, music, film, etc.) ❒ advanced organizers (e.g., KWL methods, concept maps) ❒ bridge concepts with relevant analogies and metaphors
ACCESS ACTION & EXPRESSION ? How will the learning for students support the development of executive functions to allow them to take advantage of their environment?	**Executive Functions**: ? What do you anticipate about barriers to students demonstrating what they know? ▯ Plan to address barriers to demonstrating understanding by providing opportunities for students to set goals, formulate plans, use tools and processes to support organization and memory, and analyze their growth in learning and how to build from it: ❒ prompts and scaffolds to estimate effort, resources, difficulty ❒ models and examples of process and product of goal-setting ❒ guides and checklists for scaffolding goal-setting ❒ post goals, objectives, and schedules in an obvious place ❒ embed prompts to "show and explain your work" ❒ checklists and project plan templates for understanding the problem, prioritization, sequences, and schedules of steps ❒ embed coaches/mentors to demonstrate think-alouds of process ❒ guides to break long-term goals into short-term objectives ❒ graphic organizers/templates for organizing information & data ❒ embed prompts for categorizing and systematizing ❒ checklists and guides for note-taking ❒ asking questions to guide self-monitoring and reflection ❒ showing representations of progress (e.g., before and after photos, graphs/charts showing progress, process portfolios)

	❒ prompt learners to identify type of feedback or advice they seek ❒ templates to guide self-reflection on quality & completeness ❒ differentiated models of self-assessment strategies (e.g., role-playing, video reviews, peer feedback) ❒ assessment checklists, scoring rubrics, and multiple examples of annotated student work/performance examples

Planning Guidance for Culturally and Linguistically Responsive Instruction[10]

In order to ensure our students from marginalized cultures and languages view themselves as confident and competent learners and doers of mathematics within and outside of the classroom, educators must intentionally plan ways to counteract the negative or missing images and representations that exist in our curricular resources. The guiding questions below support the design of lessons that validate, affirm, build, and bridge home and school culture for learners of mathematics:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language and the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

In addition, Aguirre and her colleagues[11] define **mathematical identities** as the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. Many students see themselves as "not good at math" and approach math with fear and lack of confidence. Their identity, developed through earlier years of schooling, has the potential to affect their school and career choices.

Five Equity-Based Mathematics Teaching Practices[12]

Go deep with mathematics. Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

Leverage multiple mathematical competencies. Use students' different mathematical strengths as a resource for learning.

Affirm mathematics learners' identities. Promote student participation and value different ways of contributing.

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[10] This resource relied heavily on the work of: Hollie, S. (2011). Culturally and linguistically responsive teaching and learning. Teacher Created Materials. (see also, https://www.culturallyresponsive.org/vabb)

[11] Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices. Reston, VA: National Council of Teachers of Mathematics (p. 14).

[12] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc. (p.6). (adapted from Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013) (p. 43).

Challenge spaces of marginality. Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

Draw on multiple resources of knowledge (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

The following lesson design strategies support Culturally and Linguistically Responsive Instruction, specific examples for each cluster of standards can be found in part 2 of the document. These were adapted from the Promoting Equity section of the Taking Action series published by NCTM.[13]

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement.

Mathematical Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher.

Modifying Mathematical Tasks: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)."

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics.

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages.

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we

[13] Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians.

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others."

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time."

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students.

References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.

Boston, M., Dillon, F., & Miller, S. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. (M. S. Smith, Ed.). Reston, VA: National Council of Teacher of Mathematics, Inc.

Cardone, T. (n.d.). Nix the Tricks. Retrieved from https://nixthetricks.com/

CAST (2018). *Universal Design for Learning Guidelines version 2.2.* Retrieved from http://udlguidelines.cast.org

Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/

English Learners Success Forum. (2020). ELSF | Resource: Analyzing Content and Language Demands. Retrieved from https://www.elsuccessforum.org/resources/math-analyzing-content-and-language-demands

Gifted and talented | NZ Maths. (n.d.). Retrieved from https://nzmaths.co.nz/gifted-and-talented

Gojak, L. M. (Ed.) (2015-2018). *The Common Core Mathematics Companion: The Standards Decoded: What They Say, What They Mean, How to Teach Them (Corwin Mathematics Series)* (1st ed.). Thousands Oak, CA: Corwin.

Gonzales, Lorenzo (2004). *Building personal knowledge of rational numbers through mental images, concepts, language, facts, and procedures*. Unpublished Doctoral Dissertation, New Mexico State University, College of Education, Las Cruces, New Mexico.

Hollie, S. (2011). *Culturally and linguistically responsive teaching and learning*. Teacher Created Materials.

Howard County Public School System: Mathematics. (n.d.). Mathematics – HCPSS. Retrieved from https://www.hcpss.org/academics/mathematics/

Illustrative Mathematics. (2014, February 14). Standards for Mathematical Practice: Commentary and Elaborations for K–5. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf

Illustrative Mathematics. (2014b, May 6). Standards for Mathematical Practice: Commentary and Elaborations for 6–8. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/05/2014-05-06-Elaborations-6-8.pdf

Kansas State Department of Education. (2018). 2018 Kansas Mathematics Flip Books. Retrieved from https://community.ksde.org/Default.aspx?tabid=5646

Kaplinsky, R. (n.d.). SIOP Content and Language Objectives for Math. Retrieved from http://robertkaplinsky.com/wp-content/uploads/2017/04/SIOP-Content-and-Language-Objectives-for-Math.pdf

Louisiana Department of Education . (n.d.). K-12 Math Planning Resources. Retrieved from https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Los Angeles, CAUSA. (n.d.). VABB™. Retrieved from https://www.culturallyresponsive.org/vabb

New Visions for Public Schools. (2020). Re-engagement Lessons. Retrieved from https://curriculum.newvisions.org/math/nv-math-team/curriculum-components/re-engagement-lessons/

North Carolina Department of Instruction. (2016–2019). North Carolina Unpacked Content for Mathematics. Retrieved from https://www.dpi.nc.gov/teach-nc/curriculum-instruction/standard-course-study/mathematics

Seeley, C. L. (2016). Math is Supposed to Make Sense. In *Making sense of math: How to help every student become a mathematical thinker and problem solver*. Alexandria, VA, USA: ASCD.

South Dakota Department of Education. (n.d.). South Dakota Content Standards - Math. Retrieved from https://doe.sd.gov/contentstandards/math.aspx#ratios+

Student Achievement Partners. (n.d.). College- and Career-Ready Shifts in Mathematics. Retrieved from https://achievethecore.org/page/900/college-and-career-ready-shifts-in-mathematics
Student Achievement Partners. (n.d.-a). Coherence Map. Retrieved from https://achievethecore.org/coherence-map/

Trundley, R. (2018). Changing lives and providing equity through pre-teaching and assigning competence. *Association of Teachers of Mathematics (ATM)*, *July*, 31–34. Retrieved from https://www.atm.org.uk/write/MediaUploads/Journals/MT262/MT26213.pdf

Utah Education Network. (2016). Mathematics Core. Retrieved from https://www.uen.org/core/math/

Glossary[14]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances
from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten.
For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

[14] Glossary and tables taken from: Common Core State Standards Initiative. (2020). Mathematics Glossary | Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/Math/Content/mathematics-glossary/
[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line.
Also known as a dot plot.[17]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[17] Adapted from Wisconsin Department of Public Instruction, op. cit.

[18] To be more precise, this defines the arithmetic mean.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin,
selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a/b* or – *a/b* for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also*: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[19]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar
model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

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[19] Adapted from Wisconsin Department of Public Instruction, op. cit.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also*: median, first quartile, interquartile range.

Table 1: Common addition and subtraction.[1]

	RESULT UNKNOWN	**CHANGE UNKNOWN**	**START UNKNOWN**
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN[2]**
PUT TOGETHER / TAKE APART[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2
COMPARE	**DIFFERENCE UNKNOWN**	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
	(“How many more?” version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2: Common multiplication and division situations.[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	3 x ? = 18, and 18 ÷ 3 = ?	? x 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a x b = ?	a x ? = p and p ÷ a = ?	? x b = p, and p ÷ b = ?

[1]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3: The properties of operations.

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a

Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that a x $1/a$ = $1/a$ x a = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Table 4: The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: a < b, a = b, a > b.
If a > b and b > c then a > c.
If a > b, b < a.
If a > b, then -a <-b.
If a > b, then a ± c > b ± c.
If a > b and c > 0, then a x c > b x c.
If a > b and c < 0, then a x c < b x c.
If a > b and c > 0, then a ÷ c > b ÷ c.
If a > b and c < 0, then a ÷ c < b ÷ c.