NEW MEXICO
Public Education Department

HS: ALGEBRA - ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: A: Perform arithmetic operations on polynomials.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.APR.A.1" Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *Note: Algebra 1 focuses on linear and quadratic*	SMP 4: Students can model with mathematics by using algebra tiles to model polynomial operations. SMP 7: Students can look for and make use of structure by seeing how the structure of arithmetic is also used when working with polynomials.	• Relate integer arithmetic to polynomial arithmetic. • Demonstrate polynomials are closed under addition, subtraction, and multiplication (but not division). • Add, subtract, and multiply multi-variable polynomials of any degree. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember, Understand
<u>Previous Learning Connections</u> • Connect to combining like terms and simplifying expressions using the distributive property **(6.EE.3)**	**<u>Current Learning Connections</u>** • Connect to using the properties of operations to write expressions in different but equivalent forms. **(HSA.SSE.A.2)**	**<u>Future Learning Connections</u>** • Connect to dividing polynomials. **(HSA.APR.2-3)** • Connect to performing operations with rational expressions **(HSA.APR.7)**

Clarification Statement

HSA.APR.A.1: The development of **polynomials** and **rational expressions** in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions for the same numbers 8 + 3 and 11, or 3/4 and 0.75, as referring to different entities: 8 + 3 might be seen as describing a calculation and 11 is its answer; 3/4 is a fraction and 0.75 is a decimal. They come to understand that these different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but **equivalent forms**, and that all of these numbers can be represented as points on the number line. In middle grades, they come to see numbers as forming a unified system, the number system, still represented by points on the number line. The whole numbers expand to the integers—with extensions of addition, subtraction, multiplication, and division, and their properties. Fractions expand to the rational numbers—and the four operations and their properties are extended. A similar evolution takes place in algebra. At first algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing.

Common Misconceptions

- Students might think polynomials are only monomial, binomial, or trinomial.

- Students may not confuse the impact of adding and subtracting polynomials on the degree of the variable.
- Students may not fully distribute the multiplication of polynomials and only multiply like terms.
- When adding and multiplying like terms students may initially confuse $x + x$ as x^2 instead of $2x$.
- Students may not think $x^2 \cdot x = x^3$ is not an example of closure for polynomial multiplication since the result has a different exponent than the factors.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems and rewriting rational expressions. because students may have unfinished learning when identifying and combining like terms, understanding the relationship between a zero and a factor, and division of numerical expressions. Students need to understand the connection between numerical and variable expressions. Also, students may have unfinished learning on identifying the parts, such as, coefficient, variable, constant of a variable expression and would benefit from targeted pre-teaching.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.A.4 provides a foundation for work with performing arithmetic operations of polynomials because students learned to identify two expressions as equivalent written in the form y + y + y and 3y by substituting a fixed value for y. Students should use this same reasoning to add and subtract polynomials. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because this cluster is the building blocks for future mathematical content. Students have difficulty performing operations with polynomials using the algorithms. It is beneficial to link operations with polynomials back to operations with numerical expressions to help build conceptual understanding. Students will benefit from novel and relevant problems using multiple entry points and various strategies, colored pencils, area models, algebra tiles, technology, etc. It is essential that students gain a conceptual understanding of polynomials to include all operations adding, subtracting, multiplying, and dividing as well as a conceptual understanding of zeros of a polynomial and how the zeros help provide a sketch of the graph.

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Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because students need to view errors and wrong answers as a tool for learning how to validate solutions as viable outcomes. When students can make sense of the problem and understand solutions as viable/non-viable they are able to use reasoning to justify their solution or use strategies to improve their solution.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because making connections to prior learning is essential for conceptual understanding of this domain. Connections such as linking perimeter to a variable with an exponent of one, which is a linear unit, area to a variable with an exponent of two, which is a square unit, long division of numerical expressions to long division of polynomials, and zeros of a polynomial function to the x-intercept (0,y) helps students make those relevant connections which solidifies their understanding of performing operations on polynomials and identifying zeros.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because making connections to area models through algebra tiles help students understand adding and subtracting like terms, multiplying and dividing polynomials. Providing students with the flexibility to use actual algebra tiles, virtual algebra tiles, or sketching area models using colored pencils allows all students to access the content and build conceptual understanding, so the algorithms make sense when applied.

Internalize

Executive Functions: *How will the learning for students support the development of executive functions to allow them to take advantage of their environment?*

- For example, learners engaging with performing arithmetic operations on polynomials, understanding the relationship between zeros and factors of polynomials, using polynomial identities to solve problems, and rewriting rational expressions benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as using templates that guide self-reflection on quality and completeness because students must have time for self-reflection in order to understand how to self-assess work and determine if their solution makes sense and is a viable solution. It is imperative students check work for quality and completeness because students will make errors when performing operations with polynomials. Also, by allowing students time to communicate and discuss solutions with peers provides students strategies to improve their solution.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on performing arithmetic operations on polynomials, by examining tasks from a different perspective through a short mini-lesson because students need to understand the parts of the expression are related to the outcome <i.e. Sum, difference, product, quotient>. Given the outcome and one of its parts, students can find the other part. Example: $(4x + 6) + ? = 8x - 10$

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit performing arithmetic operations on polynomials by offering opportunities to understand and explore different strategies because some students may need support strategies, such as using colored pencils to color code like terms, using algebra tiles to perform operations on polynomials, or the use of calculators to assist in the adding or subtracting of integers. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying performing arithmetic operations on polynomials because students need to expand their algebraic thinking to gain a deeper understanding of polynomials by generating their own equivalent expressions. Students' understanding of integer sums, differences, products and quotients will be reinforced when students are asked to use reasoning to generate their own equivalent expressions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

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Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; adding, subtracting, and multiplying polynomials the types of mathematical tasks are critical because students need to build procedural fluency by practice that is embedded in tasks that build their conceptual understanding. Students need to understand conceptually like terms, how and why the result of adding and multiplying polynomials is different, how multiplying polynomials is connected to an area model, and how adding polynomials connects to a linear model like perimeter. Algebra tiles, area models, tasks involving perimeter and area will help students build conceptual understanding while improving their procedural fluency.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19948

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Equivalent expressions	2. Fluently add, subtract, and multiply polynomials.	Calculator

19948

$$(-3x^2+5x-2)-2(x^2-2x-1)$$

If the expression above is rewritten in the form ax^2+bx+c, where a, b, and c are constants, what is the value of b ?

Rationale
The correct answer is 9. To rewrite the difference $(-3x^2+5x-2)-2(x^2-2x-1)$ in the form ax^2+bx+c, the expression can be simplified by using the distributive property and combining like terms as follows:

$(-3x^2+5x-2)-(2x^2-4x-2)$

$(-3x^2-2x^2)+(5x-(-4x))+(-2-(-2))$

$-5x^2+9x+0$

Non-Negative Polynomials: https://drive.google.com/drive/u/0/folders/1QLXJMWDXpb4MRU4Oj7QDVPkypf67usG9

Powers of 11: https://drive.google.com/drive/u/0/folders/1QLXJMWDXpb4MRU4Oj7QDVPkypf67usG9

Relevance to families and communities:

During a unit focused on understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; adding, subtracting, and multiplying polynomials, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, by discussing how their

Cross-Curricular Connections:

Industrial Arts: Often construction makes use of multiplying polynomials in deciding how to design various aspects of a house or office to fit a predetermined area. Consider providing a connection for students to design something like a sliding door given a specific frame to height ratio and surrounding framework and then plugging in to find the total area for different input values.

family culture celebrates an event. Use that event to show some elements of each families' celebration may be similar and different but each is valid and link it to learning. Some students may need to use area models, some may need colored pencils, some students may prefer to add horizontally, and some will prefer to add vertically. Although we learn in different ways, our learning is valid.	Art: Often students like to "play" with manipulatives. Consider having students make a work of art using algebra tiles and then create a polynomial expression to represent their artwork.