NEW MEXICO
Public Education Department

HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: A: Understand solving equations as a process of reasoning and explain the reasoning.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Note: Algebra 1 focuses on mastering linear; learn as general principle*	SMP 3: Students can construct viable arguments and critique the reasoning of others by comparing and justifying the set of steps used to solve equations and obtain solutions. SMP 5: Students can use tools by using algebra tiles, tape diagrams, multi-colored chips and more to develop the formal application of algebra as an abstract tool for solving mathematical problems.	• Explain why an equation is equivalent when performing operations to isolate a variable. • Construct arguments for equality using visual representations. • Justify reasoning for elimination of coefficients and/or constants and other steps using multiple types of operations, including multiplication of fractions. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Evaluate
Previous Learning Connections • Connect to applying the associative, commutative, distributive, and identity properties. **(3.OA.5)** • Connect to learning math properties and their names.**(7.NS.1-2)** • Connect to using variables to write expressions and equations. **(6.EE.2)** • Connecting to solving linear equations. **(7.EE.4, 8.EE.7)**	**Current Learning Connections** • Connect to creating and solving equations and inequalities in one variable. **(HSA.CED.1, HSA.REI.3)**	**Future Learning Connections** • Connect to justifying steps in solving rational and radical equations. **(HSA.REI.2)** • Connect to justifying steps in writing proofs for geometry. **(HSG.CO.9-11, HSG.SRT.4-5)**

Clarification Statement

HSA.REI.A.1: A written sequence of steps to **solve** an **equation** is code for a narrative line of **reasoning** using words like "**if**," "**then**," "**for all**," " and "**there exists**." In the process of learning to solve equations, students learn certain standard **"if-then" moves**, for example "if x = y then x + 2 = y + 2." The danger in learning **algebra** is that students emerge with nothing but the moves, which may make it difficult to detect incorrect or made-up moves later. Thus, the first requirement in the standards in this domain is that students understand that solving equations is a process of reasoning. This does not necessarily mean that they always write out the full text; part of the advantage of **algebraic notation** is its compactness. Once students know what the code stands for, they can start writing in code.

Fragments of reasoning

$$x^2 = 4$$
$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = 2, -2$$

This sequence of equations is short-hand for a line of reasoning:

> If x is a number whose square is 4, then $x^2 - 4 = 0$. Since $x^2 - 4 = (x-2)(x+2)$ for all numbers x, it follows that $(x-2)(x+2) = 0$. So either $x - 2 = 0$, in which case $x = 2$, or $x + 2 = 0$, in which case $x = -2$.

More might be said: a justification of the last step, for example, or a check that 2 and -2 actually do satisfy the equation, which has not been proved by this line of reasoning.

Common Misconceptions

- Students do not recognize equality is a relationship between two quantities or, more generally two mathematical expressions, asserting that the quantities have the same value, or that the expressions represent the same mathematical object.
- Students may perform inappropriate operations on polynomials. Students may subtract from coefficients and constants when subtracting on both sides of an equation or multiply only coefficients when multiplying both sides of an equation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying solving equations and explaining each step because students may need to justify the inverse operation used in each step with viable arguments. Students may practice expressing their mathematical thinking verbally and symbolically.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work with reasoning and solving one-variable equations because students need to understand each step of solving one-variable equations and explain the reason for each step. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding and explaining the reasoning of solving equations benefit when learning experiences include ways to recruit interest such as providing choices in their strategies of solving equations and in their reasoning because students make connections of their prior knowledge of solving equations in different problems. By showing a different order of applying the inverse operations to the equations, students gain new skills and knowledge of solving complex equations and deeper understanding of solving equations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with explaining reasoning of each step of solving equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because students engage in meaningful discourse to construct viable arguments with the support of the cooperative learning group. Students justify and make connections with reasoning of other strategies used by other learners in the cooperative learning groups.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with explaining the reasoning of each step of solving equations and constructing viable argument benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students build their reasoning and viable argument using their prior knowledge of solving one-step or two-step equations. Students connect their understanding of inverse operation to justify each step of solving equations.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with explaining the reasoning of solving equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students justify solving the equations in multiple ways and communicate their mathematical thinking verbally and symbolically. By presenting their mathematical thinking in multiple ways, students make connections of conceptual knowledge and gain fluency in procedural knowledge.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with explaining the reasoning of solving equations with viable arguments benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply the knowledge of solving one-variable equations to solving literal equations. Students identify the patterns of solving equations and make generalization of solving and rearranging equations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on explaining the reason of each step of solving equations by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to understand why the specific inverse operation is used and develop the viable argument using properties of equality.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit explaining the steps of solving equations by offering opportunities to understand and explore different strategies because students need to understand why some steps are interchangeable when solving the equations. Students need to explain the order of applying the inverse operations and how that relates to the order of operation of the equations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying solving complex equations and explaining the steps because students may deepen their understanding of inverse operation, such as logarithm as the inverse operation of exponent. Students explore strategies of solving equations with complex operations and justify their reason in cooperative learning groups.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying understanding solving equations as a process of reasoning and explaining the reasoning facilitating meaningful mathematical discourse is critical because students practice expressing their mathematical thinking using the content language. Students compare and evaluate different entry points of solving equations. Students defend their strategies by constructing viable arguments and build confidence in math.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1053197

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math		Passport to Advanced Mathematics	Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to b. solve simple rational and radical equations in one variable;	No Calculator

1053197

If $\frac{8}{x} = 160$, what is the value of x ?

A. 1,280

B. 80

C. 20

D. 0.05

Rationale

Choice D is correct. Multiplying both sides of the given equation by x yields $160x = 8$. Dividing both sides of the equation $160x = 8$ by 160 results in $x = \frac{8}{160}$. Reducing $\frac{8}{160}$ to its simplest form gives $x = \frac{1}{20}$, or its decimal

Zero Product Property 1

http://tasks.illustrativemathematics.org/content-standards/HSA/REI/A/1/tasks/2141

Relevance to families and communities:

During a unit focused on solving equations as a process of reasoning consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different ways of relating the steps of solving equations in real-life application of using equations. Students might work backward to solve for the unknown quantity is the same as students use inverse operations to solve the equation.

Cross-Curricular Connections:

Language Arts: Justifying reasoning is a form of persuasive writing, as students are trying to get others to agree that their solving process is appropriate and accurate. Consider providing a connection for students to write out the full text (as referenced above) in more of an essay format.

Science: When students write up a lab report they often must detail how they tested their hypothesis and clarify why they performed their experiment in a specific way. Consider providing a connection where students must make some "prediction" or hypothesis about an equation prior to solving and then write up their solving method in a format like a lab report.