

HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: B: Solve equations and inequalities in one variable.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

<p>Standard Text</p> <p>HSA.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p> <p><i>Note: Algebra 1 focuses on linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP 2: Students can reason abstractly and quantitatively by solving equations with and without models.</p> <p>SMP 8: Students look for and express regularity in repeated reasoning by connecting the steps to solve an equation or inequality with variable coefficients to an equation or inequality with integer coefficients.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Solve linear equations, including ones that require using the distributive property, combining like terms, variables on both sides and rational coefficients. Solve literal equations to isolate a specific variable (e.g., rewriting point slope form to solve for m). Solve linear inequalities, including ones with negative coefficients.
		<p>Webb’s Depth of Knowledge: 1-2</p>
		<p>Bloom’s Taxonomy: Understand, Apply</p>
<p>Standard Text</p> <p>HSA.REI.B.4: Solve quadratic equations in one variable.</p> <ul style="list-style-type: none"> HSA.REI.B.4.A: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. HSA.REI.B.4.B: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the 	<p>Standard for Mathematical Practices</p> <p>SMP 5: Students can use tools by using pictures, algebra tiles and/or symbols to explain the concept underlying completing the square.</p> <p>SMP 7: Students can look for and make use of structure by choosing the most efficient method to solve a quadratic equation.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Derive the quadratic formula from the general form of a quadratic equation. Solve quadratic equations in one variable with real solutions by inspection, taking square roots, completing the square, using the quadratic formula and factoring. Identify the number and types of solutions of a quadratic equation using the discriminant.
		<p>Webb’s Depth of Knowledge: 1-3</p>
		<p>Bloom’s Taxonomy: Understand, Apply, Analyze, Evaluate</p>

<p>equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.</p> <p><i>Note: Algebra 1 focuses on linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions</i></p>		
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to solving equations and inequalities in one variable. (7.EE.4, 8.EE.7) Connect to solving equations involving squares and square roots. (8.EE.2) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to solving quadratic equations and relating solutions to the graph of the function. (HSF.IF.7) Connect to use completing the square and factoring to rewrite quadratic functions in vertex and intercept form to identify key features of the graph. (HSS.SSE.3) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to solving additional types of nonlinear equations. (HSA.REI.2) Connect to relating knowledge of solving quadratic equations to complex numbers, solving rational equations, trigonometric equations, and trigonometric form. (HSN.CN.7, HSA.REI.2, HSF.TF.5, 7) Connect to understanding the need for a variety of methods (factoring, completing the square, and using quadratic formula) when solving other types of equations, such as parabolas, hyperbolas, and ellipses. (HSG.GPE.A)
<p>Clarification Statement</p> <ul style="list-style-type: none"> HSE.REI.B.3: With an understanding of solving equations as a reasoning process, students can organize the various methods for solving different types of equations into a coherent picture. For example, solving linear equations involves only steps that are reversible (adding a constant to both sides, multiplying both sides by a non-zero constant, transforming an expression on one side into an equivalent expression). Therefore, solving linear equations does not produce extraneous solutions. HSE.REI.B.4a: The key step in completing the square involves at its heart factoring. And the quadratic formula is nothing more than an encapsulation of the method of completing the square, expressing the actions repeated in solving a collection of quadratic equations with numerical coefficients with a single formula. (MP.8) HSE.REI.B.4b: It is traditional for students to spend a lot of time on various techniques of solving quadratic equations, which are often presented as if they are completely unrelated (factoring, completing the square, the quadratic formula). Students with an understanding of the underlying reasoning behind all these methods are opportunistic in their application, choosing the method that best suits the situation at hand. 		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> Since the steps for solving addition and subtraction equations and inequalities are similar, students often forget to change the direction of the inequality sign when multiplying or dividing by a negative coefficient. Students will often gravitate toward one solution method or another and try to use it in every possible situation given rather than paying attention to the structure of the equation and choosing the method that is most appropriate to use based on its structure. 		

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying solving equations and inequalities in one variable because knowing this will help prevent errors when solving this type of problem. Students will know what to look for and be aware of when approaching the problems.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.: This standard provides a foundation for work with solving the equations and inequalities because students will learn that solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners solving equations and inequalities in one variable benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features using different background colors, or changing the size of the text used to present the equations or inequalities because the contrast in font size or background color will help students more accurately depict where their focus should be. It also makes it easier for students to read taking at least one potential barrier out of their way.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because this cluster is a continuation of standards from earlier grades. Students are deepening and broadening their skills with solving equations in Algebra 1. The more representations, examples, and problem types they see, the better they will become at solving equations and inequalities. Additionally, students need to be given multiple entry points and

different strategies based on their learning style when approaching problems within this cluster. This will help them to internalize their learning as well as take ownership of it.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because pre-teaching or reteaching what students have learned in prior grades will give you an idea of what they remember about solving equations and inequalities and what might need to be retaught. Some students are experts while others have no clue where to begin. In pre-teaching with simpler equations students have a lower entry point into the lesson and you can build on the skills they have from prior courses.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because the more ideas and solution methods students are exposed to the more likely they are to find a few that they can use successfully. Students need multiple ways to address problems and when introduced at this early stage it shows students that there is more than one way to approach the problem and that different doesn't always mean wrong.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with solving equations and inequalities in one variable benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need repetition in this foundational skill in order to internalize it. Solving equations and inequalities is a process that can take on many different forms. The more that students can practice the more strategies they will learn and make sense of. With additional practice students are also able to better decide which strategy is more suited to which problem type.

Re-teach

Re-teach (targeted): What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?

- For example, students may benefit from re-engaging with content during a unit on solving equations and inequalities in one variable by providing specific feedback to students on their work through a short mini-lesson because < completing a task that compares equations and inequalities side by side and using the previous learned steps in solving both problems allows them to practice the skills that they have learned previously and reinforce them.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit solving equations and inequalities in one variable by offering opportunities to understand and explore different strategies> because students need opportunities to explore different methods and find which one works best for them. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open ended tasks linking multiple disciplines when studying solving equations and inequalities in one variable because making connections help students appreciate learning the concept more and gives them opportunities to see where it may be going.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying Solving equations and inequalities in one variable the pattern of questions within the classroom is critical because promoting student learning in It should connect students' lived experiences and interests (their only resources for learning something new) to disciplinary problems in the world. For example, how are verbal and algebraic models and formulas used to represent real life situations? This allows students to come up with their own ideas and make it personable.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <https://satsuitequestionbank.collegeboard.org/>

Question ID 4788931

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■ □ □	Heart of Algebra	Linear equations in two variables	3. For a linear equation in two variables that represents a context b. given a value of one quantity in the relationship, find a value of the other, if it exists.	Calculator

The equation $y = 0.1x$ models the relationship between the number of different pieces of music a certain pianist practices, y , during an x -minute practice session. How many pieces did the pianist practice if the session lasted 30 minutes?

- A. 1
- B. 3
- C. 10
- D. 30

Rationale

Choice B is correct. It's given that the equation $y = 0.1x$ models the relationship between the number of different pieces of music a certain pianist practices, y , and the number of minutes in a practice session, x . Since it's given that the session lasted 30 minutes, the number of pieces the pianist practiced can be found by substituting 30 for x in the given equation, which yields $y = 0.1(30)$, or $y = 3$. Choices A and C are incorrect and may result from misinterpreting the values in the equation. Choice D is incorrect. This is the given value of x , not the value of y .

Reasoning with linear inequalities: <http://tasks.illustrativemathematics.org/content-standards/HSA/REI/B/3/tasks/807>

Relevance to families and communities:

During a unit focused on solving equations and inequalities in one variable, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students the autonomy to choose and create problems relevant to their home culture provides students a connection to the world of mathematics.

Cross-Curricular Connections:

Science: Projectile motion is modeled by quadratic functions. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging equations with another classmate or group to solve.

Language Arts: Explaining a process is a form of expository writing, as students are trying to give facts and information. Consider providing a connection for students to write out the derivation of the quadratic formula from standard form to help them see and explain how the two forms are related.