

HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: C: Solve systems of equations.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

<p>Standard Text HSA.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p><i>Note: Algebra 1 focuses on Linear-linear and linear-quadratic.</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP3: Students can construct viable arguments to verify why replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p>SMP 8: Students look for and express regularity in repeated reasoning by working with different sets of equations to come to understand why the elimination method works.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Transform a given system of two equations in two variables into an equivalent system that has the same solutions as the original system. Prove that both systems have the same solution. <p>Webb’s Depth of Knowledge: 2-3</p> <p>Bloom’s Taxonomy: Apply, Analyze, Evaluate</p>
<p>Standard Text HSA.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p><i>Note: Algebra 1 focuses on Linear-linear and linear-quadratic.</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP 3: Students can construct viable arguments by explaining why the graphing, elimination, or substitution method is the best method to solve a system of equations.</p> <p>SMP 7: Students can look for and make use of structure by recognizing systems of equations that have no solution or infinite solutions.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Solve a system of linear equations using substitution. Solve a system of linear equations using elimination. Solve a system of linear equations by graphing by hand. Solve a system of linear equations using graphing technology (or Desmos) to estimate more complicated solutions (non-terminating rational solutions). Differentiate among situations where one solution, no solutions or infinite solutions occur. <p>Webb’s Depth of Knowledge: 1-2</p> <p>Bloom’s Taxonomy: Understand, Apply, Analyze</p>

<p>Standard Text</p> <p>HSA.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</p> <p><i>Note: Algebra 1 focuses on Linear-linear and linear-quadratic.</i></p>	<p>Standard for Mathematical Practices</p> <p>SMP 5: Students can use tools by graphing by hand or using technology and using algebraic methods or with CAS.</p> <p>SMP 7: Students can look for and make use of structure by recognizing systems consisting of one linear and one quadratic equation that have no solution, one solution or two solutions.</p>	<p>Students who demonstrate understanding can:</p> <ul style="list-style-type: none"> Solve a simple system of a linear equation and a quadratic equation algebraically. Solve a simple system of a linear equation and a quadratic equation by graphing by hand. Differentiate among situations where one solution, no solutions or two solutions occur. <p>Webb’s Depth of Knowledge: 1-2</p> <p>Bloom’s Taxonomy: Understand, Apply, Analyze</p>
<p>Previous Learning Connections</p> <ul style="list-style-type: none"> Connect to solving systems of linear equations with a focus on graphing and substitution. (8.EE.8) 	<p>Current Learning Connections</p> <ul style="list-style-type: none"> Connect to creating a system of linear equations or inequalities in a real- world context. (HSA.CED.3) 	<p>Future Learning Connections</p> <ul style="list-style-type: none"> Connect to using matrices to solve systems of linear equations. (HSA.REI.8-9)
<p>Clarification Statement</p> <ul style="list-style-type: none"> HSA.REI.C.5: Student work with solving systems of equations starts the same way as work with solving equations in one variable; with an understanding behind the various techniques. An important step is realizing that a solution to a system of equations must be a solution [to] all the equations in the system simultaneously. Then the process of adding one equation to another is understood as "if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum of the left sides of the two equations is equal to the sum of the right sides." Since this reasoning applies equally to subtraction, the process of adding one equation to another is reversible, and therefore leads to an equivalent system of equations. HSA.REI.C.6: [Systems of two linear equations with two variables] also have the advantage that a good graphical visualization is available; a pair (x,y) satisfies two equations in two variables if it is on both their graphs, and therefore an intersection point of the graphs. HSA.REI.C.7: Another important method of solving systems is the method of substitution. Again, this can be understood in terms of simultaneity; if (x, y) satisfies two equations simultaneously, then the expression for y in terms of x obtained from the first equation should form a true statement when substituted into the second equation. Since a linear equation can always be solved for one of the variables in it, this is a good method when just one of the equations in a system is linear. 		
<p>Common Misconceptions</p> <ul style="list-style-type: none"> Students may not realize that a solution to a system of equations must be a solution of all the equations in the system simultaneously and find a pair (x, y) that only fits one of the equations. Students may incorrectly apply basic integer operations when substituting one equation into another one. Students may believe that a line and a circle have nothing in common. 		

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying solving systems of equations because understanding common errors will help clarify understanding and avoid making the same mistakes.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8. EE.C.8 Analyze and solve pairs of simultaneous linear equations. This standard provides a foundation for work with solving equations simultaneously graphically, algebraically, or with a matrix because understanding that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with solving systems of equations benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students struggle with the solving systems of equations and then resort to wondering why they have to learn it. When we make the problems relevant, they can see the applicable nature of this cluster and the necessity of learning how to solve symptoms of equations in many ways.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because students need to not only know what mistakes they made but they need to learn from them. When we give substantive and informative feedback it is feedback FOR learning rather than just an arbitrary number (or letter) for a grade.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because whether you teach solution

methods in isolation or in some other way you can build on what students already know and understand. You can take the methods and structures they are more familiar with in solving linear equations and inequalities in one variable and apply them to systems of linear equations and other types of systems of equations. In this way students can build on what they already know about solving and realize that they are using similar ideas, with similar results on a more complicated problem type.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because the standards within this cluster indicate that students should use all three methods for solving systems of equations (graphically, with substitution and elimination). However, they tend to gravitate to the solution method that comes easiest to them or the one that they understand the most. Because of this we need to accept their first attempt and encourage them to try another method with the same problem. In this way they again can see that different approaches will produce the same results. In turn they will build their confidence with using different solution methods and recognize that just because they approach a problem in a different way from their peers does not mean that they've done the problem incorrectly especially if they come up with the same solution.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with solving systems of equations benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing interactive representations that guide exploration and new understandings because students will be able to differentiate that there is more than one way to solve a problem in the real world and know how to pick the best solution method.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on solving system of equations by revisiting student thinking through a short mini lesson because sometimes students need a refresher in prior knowledge to help them continue in the task.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit system of equations by confronting student misconceptions because learning from other students' mistakes can help develop their own understanding and help them to not continue to make the same mistakes. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying solving systems of equations because they can relate the concept to a real world problem and see how this will benefit in real life. Making connections with them and applying solving a system to a real-life situation will allow them to make connections to other concepts as well.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying solving system of equations the pattern of questions within the classroom is critical because promoting student learning in It should connect students' lived experiences and interests (their only resources for learning something new) to disciplinary problems in the world, systems can be used when trying to determine if you'll make more money at one job or another, taking multiple variables into account, such as salary, benefits and commissions. For example, how would you describe in writing the graphic and algebraic solutions to systems of linear equations using key, technical vocabulary in expanded and some complex sentence? This allows students to really see if they understand the concept. How can you create your own real-world problem?

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: <https://satsuitequestionbank.collegeboard.org/>

Question ID 5346050 ×

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■ ■ ■	Heart of Algebra	Systems of two linear equations in two variables	6. Fluently solve a system of linear equations in two variables.	No Calculator

5346050

$$\begin{aligned} -3x + 5y &= 1 \\ 2x - 3y &= 2 \end{aligned}$$

If (x,y) is the solution to the given system of equations, what is the value of x ?

Rationale
The correct answer is 13. The solution to a system of two linear equations is the point (x,y) that satisfies both equations. Rewriting the equations so the coefficients of y are additive inverses, then adding the equations can eliminate y . Multiplying both sides of the first equation by 3 and multiplying both sides of the second equation by 5 yield an equivalent system of equations: $-9x + 15y = 3$ and $10x - 15y = 10$. Adding these equations to eliminate y yields $x = 13$.

Products and Reciprocals: <http://tasks.illustrativemathematics.org/content-standards/HSA/REI/C/tasks/911>

Relevance to families and communities:

During a unit focused on solving system of equations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, connecting systems to their future will allow students the opportunity to understand there are many variables that influence their future goals and some variables are dependent on other variables. For example, whether they attend or where they attend college is dependent on money, grades, etc. By connecting systems of equations to their future goals, students learn how variables are connected and influence each other.

Cross-Curricular Connections:

Science: Projectile motion is modeled by quadratic functions and height is modeled by linear functions. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging their system equations with another classmate or group to solve.

Social Studies: In high school the New Mexico Social Studies Standards state students should "use quantitative data to analyze economic information". Consider providing a connection for students to work with system of equations involving economic data.