HS: ALGEBRA- REASONING WITH EQUATIONS AND INEQUALITIES

Cluster Statement: D: Represent and solve equations and inequalities graphically.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *Note: Algebra 1 focuses on linear and exponential; learn as general principle*	SMP 2: Students can reason abstractly and quantitively by relating graphical representations to contextualized situations. SMP 4: Students can model with mathematics by using graphical approaches to represent solutions to a two-variable equation.	• Explain and verify that every point (x, y) on the graph of a linear or exponential equation represents all values for x and y that make the equation true. • Identify points that are solutions to an equation given a graph of a linear or exponential equation. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember, Understand
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* *Note: Algebra 1 focuses on linear and exponential; learn as general principle*	SMP 3: Students can construct viable arguments by explaining how the x-coordinate of a solution to the system $y = f(x)$ and $y = g(x)$ solves $f(x) = g(x)$. SMP 5: Students can use tools by finding solution(s) of system of equations from graph or tables. SMP 7: Students look for and make use of structure by explaining in their own words how and when a solution is given as a point (x, y) versus a value (x = a).	• Recognize what the solution $y = f(x)$ and $y = g(x)$ means on a graph. • Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. • Find approximate solutions for the system $y = f(x)$ and $y = g(x)$ using graphs and tables. • Find successive approximations and use them to solve the system $y = f(x)$ and $y = g(x)$. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Analyze, Evaluate

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. *Note: Algebra 1 focuses on linear and exponential; learn as general principle*	SMP 2: Students can reason abstractly and quantitatively by creating graphs and solving systems of inequalities with their models. SMP 5: Students can use tools by using graph paper and/or technology to graph linear inequalities and systems of linear inequalities.	• Determine whether the boundary line of a linear inequality is inclusive (solid) or is exclusive (broken) of the solution. • Determine which half-plane is the solution to a linear inequality • Graph the solution set to a system of linear inequalities in two variables as the intersection. of the corresponding half-planes. • Identify points that are a solution or non-solution to a linear inequality or system of linear inequalities. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Previous Learning Connections • Connect to using variables to write expressions, equations, and inequalities. **(6.EE.2)** • Connect to graphing one variable inequalities on a number line. **(7.EE.4)** • Connect to graphing linear equations. **(8.EE.5)** • Connect to graphing systems of linear equations. **(8.EE.8)**	**Current Learning Connections** • Connect to interpreting statements, key features, and solutions of linear, quadratic, and exponential functions in terms of context. **(HSA.CED.1,3)** • Connect to graphing linear, quadratic, and exponential functions. **(HSA.CED.2)** • Connect to creating linear, quadratic, and exponential functions. **(HSA.CED.1-2)** • Connect to using graphs of linear, quadratic, and exponential functions to solve real-world contexts. **(HSA.CED.2)**	**Future Learning Connections** • Connect to apply these principles to different types of functions. (HSA.REI.11)

Clarification Statement

HSA.REI.D.11: Just as the **algebraic** work with **equations** can be reduced to a series of algebraic moves unsupported by reasoning, so can the **graphical visualization of solutions**. The simple idea that an equation f(x) = g(x) can be **solved (approximately)** by graphing y = f(x) and y = g(x) and finding the **intersection points** involves a number of pieces of conceptual understanding. [This method] seeks to convert an equation in one variable, f(x) = g(x), to a s**ystem of equations** in two **variables**, y = f(x) and y = g(x), by introducing a second variable y and **setting it equal** to each side of the equation. If x is a solution to the original equation, then f(x) and g(x) are equal, and thus **(x, y)** is a solution to the new system.

Common Misconceptions

- Students often interpret the solutions to an equation or graphical representation of an equation as only integer values.

- Students may believe an estimate of a value between two integer points is sufficient, but the standard states that students should find successive approximations to approximate the solution.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying representing and solving equations and inequalities graphically because they will be taking the graphing of single points to graphing lines and equations as a set.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work to represent and solve equations and inequalities graphically because substituting numerical values into an equation to determine if the equation is true, the student will comprehend that the answer is a solution. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because students understand how equations and inequalities can be used to model real-life problems. Students relate and interpret the solution(s) of the equations or inequalities in the context of the problems.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because students persist on finding the solutions of equations and inequalities using the graphs constructed. Students check and interpret the solutions in the context of the problem to make sense of their mathematical thinking.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate because students understand the multiple representations of the solutions and make connections to the graphs that represents the equations and inequalities. Students connect the multiple representations to make sense of the meaning of the solution in the context of the problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because students use multiple ways, including graphing calculators and graph paper, to construct the graphic representations of the equations and inequalities. Students compare and verify the solutions from different representations to defend the solutions.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply their interpretation and knowledge of the solutions to new problems in the context of the situation. Students also extend their knowledge of solving equations and inequalities graphically to solving equations and inequalities algebraically.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on representing and solving equations and inequalities graphically by clarifying mathematical ideas and/or concepts through a short mini-lesson because helping students to understand what the different parts of the graph are telling them will help them to make better understanding of the graphs themselves.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit representing and solving equations and inequalities graphically by confronting student misconceptions because graphs can be misleading if read incorrectly and lead to quite a number of misconceptions, especially when it comes to how accurate the answers you are getting from them are. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying representing and solving equations and inequalities graphically because some students will pick up on the nuances of graphing quite quickly by comparison and could investigate further along points of inquiry such as how changing windows, scaling, or other aspects of the graph effects the readability and usefulness of it as a tool.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying representing and solving equations and inequalities graphically the use of mathematical representations within the classroom is critical because students are given a situation in two variables and they must find the value of one variable given the value of the other, create an equation to represent the situation, use technology to create a graph, and interpret each representation. Understanding how lines and tables represent solution sets of linear relationships will help students make sense of graphs of and solutions to linear inequalities, and later, to make sense of solutions to systems of linear equations in their Algebra 1 class.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

NEW MEXICO
Public Education Department

Question ID 422195

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to f. solve systems of linear and nonlinear equations in two variables, including relating the solutions to the graphs of the equations in the system.	No Calculator

422195

$y = x^2$

$2y + 6 = 2(x + 3)$

If (x, y) is a solution of the system of equations above and x > 0, what is the value of xy ?

A. 1

B. 2

C. 3

D. 9

Rationale

Choice A is correct. Substituting x^2 for y in the second equation gives $2(x^2) + 6 = 2(x + 3)$. This equation can be solved as follows:

$2x^2 + 6 = 2x + 6$	Apply the distributive property.
$2x^2 + 6 - 2x - 6 = 0$	Subtract 2x and 6 from both sides of the equation.
$2x^2 - 2x = 0$	Combine like terms.
$2x(x - 1) = 0$	Factor both terms on the left side of the equation by 2x.

Thus, $x = 0$ and $x = 1$ are the solutions to the system. Since $x > 0$, only $x = 1$ needs to be considered. The value of y when $x = 1$ is $y = x^2 = 1^2 = 1$. Therefore, the value of xy is $(1)(1) = 1$.

Choices B, C, and D are incorrect and likely result from a computational or conceptual error when solving this system of equations.

Ideal Gas Law: http://tasks.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/1925

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on representing and solving equations and inequalities graphically, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, bringing in the different languages spoken in the home and connecting it to the tools available to translate different languages, <i.e. Google translate, closed captions on televisions,	Computer Science: Computer programs use functions to define the points used to graph the animation on a computer. Consider providing a connection where students can write from scratch or compile premade selections to create code that will result in their own animations. Social Studies: In high school the New Mexico Social Studies Standards state students should “use quantitative data to analyze economic information”. Consider

etc.> make connections that show that in the culture of mathematics, tools are used to translate mathematics and help us make sense of what we are seeing.	providing a connection for students to work with a context that compares two situations that each include a standard base fee and additional charges per unit of some quantity.