NEW MEXICO
Public Education Department

HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

Cluster Statement: A: Interpret the structure of expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.
Note, the A-SSE domain is especially important in the high school content standards overall as a widely applicable prerequisite.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.SSE.A.1: Interpret expressions that represent a quantity in terms of its context.* • **HSA.SSE.A.1.A: Interpret parts of an expression, such as terms, factors, and coefficients.** • **HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity.** ***For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*** *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 4: Students can model with mathematics by identifying the meaning of the terms, factors, and coefficients of linear, exponential and quadratic expressions in context. SMP 7: Students can look for and make use of structure in expressions by seeing how the structure of an algebraic expression reveals properties of the function it defines.	• Identify parts of an expression, such as terms, factors, coefficients, exponents, etc. • Interpret simple compound expressions by viewing one or more of their parts as a single entity. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Remember, Understand, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. ***For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*** *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 3: Students can construct viable arguments by explaining whether expressions are equivalent using mathematical justifications. SMP 8: Students look for and express regularity in repeated reasoning by connecting exponents to repeated multiplication.	• Recognize equivalent forms of expressions. • Use the structure of an expression to identify ways to rewrite it. • Make generalizations about the possible equivalent forms expressions can have (e.g., a quadratic expression can always be represented as the product of two factors containing its roots). • Rewrite expressions to identify important components, such as where zeros may occur or end behavior.

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		Webb's Depth of Knowledge: 1-2 **Bloom's Taxonomy:** Remember, Understand, Apply
Previous Learning Connections • Connect to identifying and interpreting slope and y-intercept for linear representations. (8.F.3-4) • Connect to rewriting standard linear equation to slope-intercept form for systems of equations. (8.EE.8)	**Current Learning Connections** • Connect to rewriting quadratic functions to find specific key features. (HSA.SSE.B.3) • Connect to rewriting formulas to highlight quantities of interest. (HSA.CED.4)	**Future Learning Connections** • Connect to work with expressions of all function types. (HSA.SSE.A.1-2 - *polynomial and rational*)

Clarification Statement

- HSA.SEE.A.1: The middle grades standards in Expressions and Equations build a ramp from **arithmetic expressions** in elementary school to more sophisticated work with **algebraic expressions** in high school. As the complexity of expressions increases, students continue to see them as being built out of **basic operations**; they see expressions as **sums** of **terms** and **products** of **factors**. In "Animal Populations" students compare P + Q and 2P by seeing 2P as P + P. They distinguish between (Q–P)/2 and Q – P/2 by seeing the first as the **quotient** where the **numerator** is a **difference** and the second as a difference where the second term is a quotient. [This example] illustrates how students are able to see complicated expressions as built out of simpler ones.

Animal Populations

Suppose P and Q give the sizes of two different animal populations, where $Q > P$. In 1–4, say which of the given pair of expressions is larger. Briefly explain your reasoning in terms of the two populations.

1. $P + Q$ and $2P$
2. $\frac{P}{P+Q}$ and $\frac{P+Q}{2}$
3. $(Q - P)/2$ and $Q - P/2$
4. $P + 50t$ and $Q + 50t$

Task from Illustrative Mathematics. For solutions and discussion, see http://www.illustrativemathematics.org/illustrations/436.

- HSA.SEE.A.2: Seeing structure in expressions entails a dynamic view of an algebraic expression, in which potential rearrangements and manipulations are ever present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which not.

Common Misconceptions

- Students may confuse the parts of an expression, such as counting variables and not terms and therefore misidentifying the number of terms an expression has.
- Students may not have a conceptual basis for patterns, such as an area model for difference of squares, and therefore struggle to recognize and apply them to new situations.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying interpreting the structure of expressions because when student feel comfortable with the vocabulary being used, they are more likely to use it and using the correct terminology when discussing the structure of an equation allows everyone (bother students and teachers) to communicate their ideas and understanding more clearly.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 5.OA.A.2: This standard provides a foundation for work with interpreting the structure of expressions because students write out the numerical expression without the calculation. Students become comfortable with using the vocabulary words: difference, greater than, multiple, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.
- 6.EE.A.4: This standard provides a foundation for work with interpreting the structure of expressions because being able to tell if two expressions are equivalent is the building blocks for being able to construct and deconstruct expressions to use their structure. Being able to tell if what you have done to an expression essentially changes it or not leads to the understanding of how to use these changes to manipulate the expressions and equations to better understand their structure. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because using the structure of expressions is mostly about making sense of the problems that they have been given an using different parts of the problem to be able to be creative and solve the problem.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because seeing a variety of different representations allows the students to more easily see the connections between the different parts of the expressions and their possible meanings.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because the more that students can see the building block of expressions and equations and how they can be combined to give a larger amount of information they will be more prepared to make their own sense of the structure by combining the building blocks in their own way.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing differentiated feedback (e.g., feedback that is accessible because it can be customized to individual learners) because students might see the various parts of the structure differently or approach them differently so they will need feedback that helps them to access the other ways of seeing the structure and this will depend greatly on the individual student.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as offering opportunities over time to revisit key ideas and linkages between ideas because learning how to use the structure of an expression in one setting doesn't mean that the students will understand it in all settings so offering the opportunity to revisit the idea in a different setting will allow students to better understand the concept as a whole with regards to the various branches of mathematics.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because the structure of an expression can be looked at in many different ways and you don't students to get locked into one way of thinking about equations, like understanding that slope-intercept, point-slope, and standard form are all useful ways of looking at linear equations and can tell you different things about the equation.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because the structure of an expression can be looked at in many different ways and you don't students to get locked into one way of thinking about equations, like understanding that slope-intercept, point-slope, and standard form are all useful ways of looking at linear equations and can tell you different things about the equation.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying interpreting the structure of expressions because looking at the structure of the different types of equations and disciplines will help reinforce concepts such as the inverse relationship between logarithmic and exponential functions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying interpreting the structure of expressions the types of mathematical tasks are critical because the conceptual part of interpreting the structure of expressions is foundational for being able to build and understand equations later on and is not something that is going to be culturally relevant to most students' home lives.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19789

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■□	Heart of Algebra	Linear functions	3. For a linear function that represents a context a. interpret the meaning of an input/output pair, constant, variable, factor, term, or graph based on the context, including situations where seeing structure provides an advantage;	Calculator

The average number of students per classroom at Central High School from 2000 to 2010 can be modeled by the equation $y = 0.56x + 27.2$, where x represents the number of years since 2000, and y represents the average number of students per classroom. Which of the following best describes the meaning of the number 0.56 in the equation?

A. The total number of students at the school in 2000

B. The average number of students per classroom in 2000

C. The estimated increase in the average number of students per classroom each year

D. The estimated difference between the average number of students per classroom in 2010 and in 2000

Rationale

Choice C is correct. In the equation $y = 0.56x + 27.2$, the value of x increases by 1 for each year that passes. Each time x increases by 1, y increases by 0.56 since 0.56 is the slope of the graph of this equation. Since y represents the average number of students per classroom in the year represented by x, it follows that, according to the model, the estimated increase each year in the average number of students per classroom at Central High School is 0.56.
Choice A is incorrect because the total number of students in the school in 2000 is the product of the average

Equivalent Expressions: http://tasks.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/87

Relevance to families and communities:

During a unit focused on interpreting the structure of expressions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, ask them about structures that they encounter in their everyday lives that help them parse information, such as knowing which gaming platform a video game is operating on or how to substitute ingredients in a recipe based on someone's food allergies.

Cross-Curricular Connections:

Science: Many science formulas take on linear, exponential and quadratic forms. For example, *F* = *ma*. Consider providing a connection for students to explore these formulas and identify their structure and how knowing that structure helps them make sense of the context.

Social Studies: In high school the New Mexico Social Studies Standards state students should "understand basic economic principles." Consider providing a connection for students to use expressions to model cost and revenue.