NEW MEXICO
Public Education Department

HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

Cluster Statement: A: Interpret the structure of expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.
Note, the A-SSE domain is especially important in the high school content standards overall as a widely applicable prerequisite.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. * • HSA.SSE.B.3.A: Factor a quadratic expression to reveal the zeros of the function it defines. • HSA.SSE.B.3.B: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. • HSA.SSE.B.3.C: Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* *Note: Algebra 1 focuses on quadratic and exponential.*	SMP3: Students can construct viable arguments to explain whether two expressions are equivalent or not. SMP 7: Students can look for and make use of structure in expressions by using equivalent forms of expressions identify important components of functions, such as where zeros may occur or end behavior.	• Write a quadratic expression with rational coefficients in an equivalent form by factoring and by completing the square. • Identify and use the zeros to solve or explain familiar problems. • Use properties of exponents to write equivalent forms of exponential functions with one or more variables, integer coefficients, and nonnegative rational exponents involving operations of addition, subtraction and multiplication, including distributing an exponent across terms within parentheses. • Find the maximum or minimum values of a quadratic function. • Choose an appropriate equivalent form of an expression in order to reveal a property of interest when solving problems. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
<u>Previous Learning Connections</u> • Connect to knowing and apply the properties of integer exponents to generate equivalent, simplified numerical expressions using the properties of exponents. **(8.A.1)**	**<u>Current Learning Connections</u>** • Connect to recognizing and flexibly writing expressions (or rewriting) to use that expression and solve the problem at hand. **(HSA.SSE.A.2)**	**<u>Future Learning Connections</u>** • Connect to factoring polynomial functions of varying degrees. **(HSA.APR.3)** • Connect to completing the square to solve quadratic equations with imaginary solutions. **(HSN.CN.7)**

		• Connect to rewriting exponential equations as logarithmic equations. **(HSF.LE.4)**

Clarification Statement

HSA.SSE.B.3: The Standards emphasize purposeful **transformation of expressions** into **equivalent forms** that are suitable for the purpose at hand. The Standards avoid talking about **simplification**, because it is often not clear what the simplest form of an expression is, and even in cases where that is clear, is in not obvious that the simplest form is desirable for a given purpose.

There are three commonly used forms for a quadratic expression:

- **Standard form**, e.g., $x^2 – 2x – 3$
- **Factored form**, e.g., $(x + 1)(x – 3)$
- **Vertex form** (a **square** plus or minus a **constant**), e.g. $(x – 1)^2 – 4$

Rather than memorize the names of these forms, students need to gain experience with them and their different uses. The traditional emphasis on simplification as an automatic procedure might lead students to automatically convert the second two forms to the first, rather than convert an expression to a form that is useful in each context.

The introduction of **rational exponent**s and systematic practice with the **properties of exponents** in high school widen the field of operations for manipulating expressions.

Common Misconceptions

- When factoring a quadratic where $a > 0$, students may look at c only when determining which factors to use, rather than looking for the factors of the product a and c.
- When completing the square, students may forget to subtract the number that was added inside the parentheses.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying writing expressions in equivalent forms to solve problems because they will be asked to combine several skills that they had previously learned independently of each other, like rewriting exponents, into one larger problem so reviewing these individual skills will help them be more confident in the larger problem.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.EEA.1: This standard provides a foundation for work with writing expressions in equivalent forms to solve problems because they need to be able to manipulate expressions in a basic sense of linear equations if they are going to be successful at manipulating more complex expressions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features like the font or the format in which the problem is presented (e.g. worksheet, whiteboard, grouping cards because the variety will relieve some of the monotony of accessing problems in the same way all the time and stimulate different aspects of their brains as they encounter the material in the different formats like up on the wall verses always being on a paper in front of them.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because there is not one best right way to write and equation to solve a problem and the nuance need to be highlighted so that students can improve their work over time.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) because students are often asked to interpret a scenario with information given in one form to answer a question and if they don't understand the context of what is happening, it makes it nearly impossible to complete the task.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because the concept is to be able to look at the different forms, not the incidental calculations needed to do so.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because the path forward is not always clear when trying to find the best way to solve a problem, with the most obvious path not always being the most efficient path to the solution.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on writing expressions in equivalent forms to solve problems by critiquing student approaches/solutions to make connections through a short mini-lesson because there are a variety of ways to solve problems and looking at the ways that other students are solving the problems can help the students to make connections between their preferred methods and another that could help them become more efficient at solving similar problems in the future.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit writing expressions in equivalent forms to solve problems by offering opportunities to understand and explore different strategies because different strategies of looking at the equivalent forms are more efficient for certain tasks and exploring when it is most appropriate to use a particular form will help them become more flexible in their problem solving skills. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying writing expressions in equivalent forms to solve problems because some students will pick up on the technical mechanics of a particular technique quickly, so having them go more deeply into why it works will help them gain a better understanding of the overall intricacies of the method. For example, factoring using a variety of methods, like factoring by grouping and how it relates to factoring a traditional trinomial into two binomials.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Supporting Productive Struggle in Learning Mathematics: The standard for mathematical practice, makes sense of mathematics and persevere in solving them is the foundation for supporting productive struggle in the mathematics classroom. "Too frequently, historically marginalized students are overrepresented in classes that focus on memorizing and practicing procedures and rarely provide opportunities for students to think and figure things out for themselves. When students in these classes struggle, the teacher often tells them what to do without building their capacity for persistence." Teachers need to provide tasks that challenge students and maintain that challenge while encouraging them to persist. This encouragement or "warm-demander" requires a strong relationship with students and an understanding of the culture of the students. For example, when studying writing expressions in equivalent forms to solve problems supporting productive struggle is critical because students will come to this cluster with a variety of knowledge about how to manipulate equations and there are a lot of correct ways to do so, therefore, they need to be encouraged to work through the process to find the ways that are more effective on their own instead of being asked to memorized rote procedures for a given situation.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1474142

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to d. solve polynomial equations in one variable that are written in factored form;	No Calculator

$x^2+x-12=0$

If a is a solution of the equation above and $a>0$, what is the value of a ?

Rationale

The correct answer is 3. The solution to the given equation can be found by factoring the quadratic expression. The factors can be determined by finding two numbers with a sum of 1 and a product of -12. The two numbers that meet these constraints are 4 and -3. Therefore, the given equation can be rewritten as $(x+4)(x-3)=0$. It follows that the solutions to the equation are $x=-4$ or $x=3$. Since it is given that $a>0$, a must equal 3.

Question Difficulty: ■■□

Profit of a company (HSA.SSE.B.3): http://tasks.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/434

Relevance to families and communities:

During a unit focused on writing expressions in equivalent forms to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, asking students to share about structures that they encounter in their everyday that they manipulate, such as changing the formatting of a picture or video so that it is sharable on different device platforms.

Cross-Curricular Connections:

Science: Finding the zeros and maximum for a model that created a projectile motion equation may also require students to rewrite a quadratic in an equivalent form. Consider providing a connection for students to experiment with projectile motion by tossing objects themselves, possibly using technology, and then exchanging equations with another classmate or group to identify key components.

Social Studies: In high school the New Mexico Social Studies Standards state students should "understand basic economic principles." Consider providing a connection for students to rewrite the model $P(1+r)^t$ for compound interest to identify the quarterly, monthly or weekly interest rate.