NEW MEXICO
Public Education Department

HS: FUNCTIONS- BUILDING FUNCTIONS

Cluster Statement: A: Build a function that models a relationship between two quantities.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.BF.A.1: Write a function that describes a relationship between two quantities.* • HSF.BF.A.1.A Determine an explicit expression, a recursive process, or steps for calculation from a context. • HSF.BF.A.1.B Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 4: Students can model with mathematics by discovering patterns in each contextual problem and creating verbal, symbolic or explicit symbolic rules to describe them. SMP 7: Students can look for and make use of structure by using the operations of math to combine functions.	• Write an explicit expression to model linear, exponential and quadratic relationships. • Write a recursive expression to model linear, exponential and quadratic relationships. • Add, subtract multiply and divide functions related to a given context. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. * *Note: Algebra 1 focuses on linear, exponential, and quadratic.*	SMP 4: Students can model with mathematics by writing using recursive steps to model a context by writing the formula. SMP 8: Students can look for and express regularity in repeated reasoning by using the recursive steps to recognize relationships between the	• Write arithmetic sequences recursively and explicitly. • Write geometric sequences recursively and explicitly. • Translate between recursive and explicit formulas. • Model situations using the formulas. • Relate arithmetic sequences to linear function and geometric sequences to exponential functions.

	pattern and the symbolic representation of the pattern.	**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
<u>Previous Learning Connections</u> • Connect to interpreting the equation y = mx + b as defining a linear function. (8.F.3) • Connect to comparing properties of two functions, each represented in a different way. (8.F.2)	**<u>Current Learning Connections</u>** • Connect to identifying patterns in the function's rate of change, specifying intervals of increase and decrease, and graphing to model functions. **(HSF.IF.4,6)** • Connect to discussing the relative strength sand weaknesses of each representation and which are most efficient to be able to assist them in making symbolic functions. **(HSF.IF.9)** • Connect to recognizing situations that grow by a constant rate or percent. **(HSF.LE.1)**	**<u>Future Learning Connections</u>** • Connect to continuing to write arithmetic and geometric sequences. **(HSF.LE.2)** • Connect to using geometric series to find the sum. **(HSA.SSE.4)** • Connect to performing operations with all parent functions and composing functions. **(HSF.BF.1)**

Clarification Statement

- HSF.BF.A.1: Students should write **functions** for given **relationships** between **quantities**. Students can use functions to **model real-life situations** and make **predictions**. Students should be able to use functions describe relationships between two quantities, usually x and f(x), where f(x) is some **output** value that depends on the **input** value x. Within a context, students should be able to express a given relationship as a function.
- HSF.BF.A.2: Students should write **formulas** for **arithmetic** and **geometric sequences** with both **explicit** and **recursive formulas**. They should be able to relate these to a context they represent and be able to **transition** from one form to the other. Students should know that they can write explicit functions recursively, too. For instance, with every year that passes, your age increases by 1. It can be interpreted as constantly adding 1 to the age you were before. In other words, write your age as f(x) = f(x – 1) + 1 starting with f(1) = 1. Students should know how to recognize that arithmetic functions that take the explicit form A(n) = A(1) + (n – 1)d have the recursive form A(n) = A(n – 1) + d and geometric functions with the form G(n) = G(1) × r^{n-1} have the recursive form G(n) = G_{n-1} × r.

Common Misconceptions

- Students may want to try to use a linear function, specifically the slope-intercept form for every situation.
- Students may tend to focus on the symbolic form of a function and may need additional support in working with other forms.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying building a function that models a relationship between two quantities because to discuss possible strategies and viable solutions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.3 This standard provides a foundation for work with building a function that models a relationship between two quantities because students identify the type of relationship the two quantities have (linear, non-linear, exponential). If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features given an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials because students will be able to recognize various situations and can create representations that will allow them to understand and be able to share their understanding with others. When a student recognizes a situation, they are then able to create a table, equation, and graph to explain and show their thinking using various forms (paper, technology).

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because students will think about what was their thinking process solving the problem instead of asking "is this correct?" students will talk and discuss with other students and demonstrate how they came about to the conclusion that they are displaying. If students use (MP 3) Construct viable arguments and critique the reasoning of others. The students themselves will determine if their conclusion is the correct one for the given problem.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because students can go back and use the support that will aid them in clarifying and understanding what the problem wants them to do. If students do not understand what they are solving for and how they need to represent it is usually the first cause of the students getting confused and frustrated.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because when students are given appropriate tools the students are then able to demonstrate what they are thinking without having to worry about if they are drawing the graph correctly. They can focus on the mathematics and show their representation of their solution. In the real-world technology is used most of the time when graphing is needed.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with building a function that models a relationship between two quantities benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students need tools that will allow them to be able to determine if their thinking, planning and process is helping them understand the problem and how to solve it.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on building a function that models a relationship between two quantities by revisiting student thinking through a short mini lesson because some students have trouble writing their thinking and they just need more time to explain what they are thinking.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on building a function that models a relationship between two quantities by helping students move from specific answers to generalizations for certain types of problems because students need to understand that the content used in this unit is not only useful for one relationship between two quantities. It is a concept that they will continue to use every time that they have a relationship between two quantities.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying building a function that models a relationship between two quantities because students will be able to explore relationships between two quantities in different perspectives. With the problem being open-ended it allows the students to explain and describe their thinking without feeling pressured to one specific answer.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying building a function that models a relationship between two quantities the pattern of questions within the classroom is critical because by posing purposeful questions you will be able to scaffold the activity to provide multiple entry points meeting students where they are at.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1474177

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra		Heart of Algebra	Linear functions	5. Write the rule for a linear function given two input/output pairs or one input/output pair and the rate of change.	Calculator

Population of Greenleaf, Idaho

Year	Population
2000	862
2010	846

The table above shows the population of Greenleaf, Idaho, for the years 2000 and 2010. If the relationship between population and year is linear, which of the following functions P models the population of Greenleaf t years after 2000?

A. $P(t) = 862 - 1.6t$

B. $P(t) = 862 - 16t$

C. $P(t) = 862 + 16(t - 2{,}000)$

D. $P(t) = 862 - 1.6(t - 2{,}000)$

Rationale Choice A is correct. It is given that the relationship between population and year is linear; therefore, the function that models the population t years after 2000 is of the form $P(t) = mt + b$, where m is the slope and b is the population when $t = 0$. In the year 2000, $t = 0$. Therefore, $b = 862$. The slope is given by $m = \frac{P(10) - P(0)}{10 - 0} = \frac{846 - 862}{10 - 0} = \frac{-16}{10} = -1.6$. Therefore, $P(t) = -1.6t + 862$, which is equivalent to the equation in choice A. Choice B is incorrect and may be the result of incorrectly calculating the slope as just the change in the value of P. Choice C is incorrect and may be the result of the same error as in choice B, in addition to incorrectly using t to represent the year, instead of the number of years after 2000. Choice D is incorrect and may be the result of incorrectly using t to represent the year instead of the number of years after 2000. https://docs.google.com/a/hobbsschools.net/viewer?a=v&pid=sites&srcid=c2pjaXNkLm9yZ3xjb21tb24tY29yZS1tYXRoZW1hdGljcy1hc3Nlc3NtZW50LWZpZWxkLXRlc3Rpbmctc2l0ZXxneDoyNjA3Mzc0N2YwZDBkZDMz

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on building a function that models a relationship between two quantities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, finding something that you do in your family and create a function model to show someone can create a strong connection between your school tasks and your life tasks.	Science: In high school the NGSS students should apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population. Consider providing a connection for students to examine scientific data and predict the effect of a change in one variable on another. https://www.nextgenscience.org/topic-arrangement/hsinheritance-and-variation-traits