NEW MEXICO
Public Education Department

HS: FUNCTIONS- BUILDING FUNCTIONS

Cluster Statement: B: Build new functions from existing functions.

Standard Text **HSF.BF.B.3: Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.** *Note: Algebra 1 focuses on linear, exponential, quadratic, and absolute value*	**Standard for Mathematical Practices** SMP 5: Students can use tools by using graphing calculators or technology to experiment with parent functions and the results when different transformations are applied. SMP 8: Students look for and express regularity in repeated reasoning by exploring different expressions for transformations of f(x) and generalizing the effects.	**Students who demonstrate understanding can:** • Identify vertical transformations from a function or a graph. • Identify horizontal transformations from a function or a graph. • Identify a shrink or a stretch from a function or a graph. • Write the results from such transformations. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text HSF.BF.B.4: Find inverse functions. • HSF.BF.B.4.A: Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.* *Note: Algebra 1 focuses on linear only*	**Standard for Mathematical Practices** SMP.6 Students can attend to precision by understanding that some functions do not have an inverse unless there is some sort of restriction on the domain. SMP 7: Students can look for and make use of structure by recognizing that the ordered pair (x, y) is reversed for a function's inverse.	**Students who demonstrate understanding can:** • Write the inverse of a simple function. • Relate using an inverse as an operation that undoes another operation. • Determine restrictions on the domain to allow for an inverse to exist. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Previous Learning Connections • Connect to recognizing and understanding that all linear functions can be written in the form y = mx + b. **(8.F.3)** • Connect to graphing linear relationships. **(8.F.5)**	**Current Learning Connections** • Connect to graphing linear, quadratic, and exponential relationships. **(HSF.IF.4)**	**Future Learning Connections** • Connect to extending transformation patterns to all functions. **(HSF.BF.3)** • Connect to graph transformations and compositions of transformations on a coordinate plane. **(HSF.BF.1)**

Clarification Statement

- HSF.BF.B.3: Students should describe the effect of **stretches, shrinkages**, **vertical and horizontal transformations** of **linear, quadratic** and **exponential functions**. They should be able to find the value of the **transformation** when given a **graph** and be able to explain effects of transformations using technology. Students should know that adding a **constant** k to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If y = f(x) is changed to y = f(x) + k, the curve will **shift** vertically (up for $k > 0$, down if $k < 0$). Adding k to x such that y = f(x + k) will shift the curve horizontally (left for $k > 0$, right for $k < 0$). Multiplying f(x) by a constant k stretches ($k > 1$) or **squishes** ($0 < k < 1$) the graph vertically. If $k < 0$, the graph is also **flipped** over the **x-axis**. Multiplying x by k stretches ($k > 0$) or squishes ($k < 0$) the graph horizontally.
- HSF.BF.B.4: Students should be able to find the **inverse** of **simple linear functions** and recognize that other functions may not have an inverse unless there are **restrictions** placed on the **domain**. If f(x) = y is a function, the inverse function can be found by switching the place of x and y (f(y) = x), and then solving for y so that f-1(x) = y. For instance, if the function f(x) is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of x and y ($x = 2y^3$) and then solving for y.

Common Misconceptions

- Students often have difficulty determining the direction of the horizontal shifts.
- Students often confuse the notation for the inverse and negative numbers.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying building new functions from existing functions because students will need to make connections to the previous standard. If they still have misconceptions it is better to address before the new standard is introduced to reduce the amount of future confusion.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.3: This standard provides a foundation for work with building new functions from existing functions because students identify the type of relationship the two quantities have (linear, non-linear, exponential) and they can create new functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user.*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials because students can compare the functions using various displays.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence.*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as such as providing feedback that is substantive and informative rather than comparative or competitive because students will begin to see the relationship between parent functions and how changes to the parent function move the graph. When students look for and express regularity in repeated reasoning by exploring different expressions for transformations of f(x) and generalizing the effects (MP.8) they begin to understand how changing a function rule moves the graph of the function.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as using technology (graphing calculators, desmos.com) because students can explore function types, how changes to the function rule moves the graph of the function and begin to see the relationship between the function rule and the graph.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as allowing access to graphing calculators, graph paper, colored pencils, desmos.com provides students with the opportunity to choose appropriate tools (MP.4) They can focus on the mathematics and show their representation of their solution. In the real-world technology is used most of the time when graphing is needed.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with building new functions from existing functions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students must have the opportunity to choose the appropriate tool to determine if their thinking, planning and process is helping them understand the problem and how to solve it.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on Build new functions from existing functions by critiquing student approaches/solutions to make connections through a short mini-lesson because by having students critiquing their work or others they are able to make connections which they can use to help them build new functions.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after the unit building new functions from existing functions by addressing conceptual understanding because this will inform the teacher what the student understands and why it is important to understand why building new functions from existing functions is useful.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying building new functions from existing functions because some students can do the assignments but sometimes do not fully understand the concept. This will allow them to focus on the concept and not just on finishing the problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying building new functions from existing functions, the use of mathematical representations within the classroom is critical because students will need different representations when creating new functions from existing functions. Students will need to make connections to their previous "mathematical and cultural "knowledge.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 4788993

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Nonlinear functions	2. For a quadratic or exponential function, e. make connections between tabular, algebraic, and graphical representations of the function, by iii. determining how a graph is affected by a change to its equation, including a vertical shift or scaling of the graph.	Calculator

4788993

In the xy-plane, which of the following changes to the graph of the equation $y = x^2 + 3$ will result in the graph of the equation $y = (x^2 + 3) - 6$?

A. A shift 6 units to the left

B. A shift 6 units to the right

C. A shift 6 units upward

D. A shift 6 units downward

Rationale

Choice D is correct. The graph of the equation $y = (x^2 + 3) - 6$ is similar to the graph of $y = x^2 + 3$ in the xy-plane. The constant -6 in the equation $y = (x^2 + 3) - 6$ indicates a vertical shift of 6 units downward from the graph of $y = x^2 + 3$. Therefore, a shift of 6 units downward of the graph of $y = x^2 + 3$ will result in the graph of $y = (x^2 + 3) - 6$.

Choice A is incorrect. A shift of 6 units to the left is equivalent to the graph of $y = (x + 6)^2 + 3$. Choice B is incorrect. A shift of 6 units to the right is equivalent to the graph of $y = (x - 6)^2 + 3$. Choice C is incorrect. A shift 6 units upward is equivalent to the graph of $y = (x^2 + 3) + 6$.

https://docs.google.com/a/hobbsschools.net/viewer?a=v&pid=sites&srcid=c2pjaXNkLm9yZ3xjb21tb24tY29yZS1tYXRoZW1hdGljcy1hc3Nlc3NtZW50LWZpZWxkLXRlc3Rpbmctc2l0ZXxneDo1ODhmMGI5NGNlNGU3MDYy

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on building new functions from existing functions., consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example compare functions that represent your community that you can find on line. This can create a strong connection between your school tasks and your community.	Science: The equation for velocity, M(v) = $6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of M(v) cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.