NEW MEXICO
Public Education Department

HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: A: Understand the concept of a function and use function notation.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *Note: Algebra 1 focuses on learning as general principle; focus on linear and exponential and on arithmetic and geometric sequences*	SMP 2: Students can reason abstractly and quantitatively by interpreting the relation of domain and range of functions related to a problem given algebraically or graphically within a context. SMP 4: Students can model with mathematics by creating examples of what is and what is not a function using different representations, including graphs, tables, symbols and contexts.	• Distinguish between functions and nonfunctions from a graph. • Distinguish between functions and nonfunctions from a table. • Distinguish between functions and nonfunctions from an equation • State the domain and range given a graph. • Understand when an equation is a function y is replaced with f(x) **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember and Understand
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *Note: Algebra 1 focuses on learning as general principle; focus on linear and exponential and on arithmetic and geometric sequences*	SMP 2: Students can reason abstractly and quantitatively by using and interpret function notation to identify relationships between domain and range. SMP 6: Students can attend to precision by accurately and appropriately using vocabulary and symbols to write functions in function notation and describe different characteristics of graphs.	• Evaluate a function given in function notation for an input. • Find the input for a given output when given in function notation. • Identify the domain and range for any given function presented in function notation or given as a verbal description in terms of a context. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand and Apply

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* *Note: Algebra 1 focuses on learning as general principle; focus on linear and exponential and on arithmetic and geometric sequences*	SMP 4: Students can model with mathematics by interpreting parts of sequences in relation to a context. SMP 7: Students can look for and make use of structure by determining that a sequence with which they are working is a function by analyzing the data they are given, whether it is in list form or a graph.	• Observe a sequence as a function whose domain consists of integers. • Observe a sequence that is defined recursively whose domain consists of only integers. **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember and Understand
<u>Previous Learning Connections</u> • Connect to analyzing proportional relationships and solving real-world math problems using numerical and algebraic expressions and equations. **(7.RPA.2-3)** • Connect to describing the functional relationship between two quantities qualitatively by analyzing a graph. **(8.F.5)** • Connect to constructing a function to model a linear relationship. **(8.F.4)**	**<u>Current Learning Connections</u>** • Connect to writing recursive and explicit formulas for arithmetic and geometric sequences. **(HSF.BF.2)** • Connect to writing functions for linear, quadratic, and exponential relationships. **(HSF.BF.1- 2)** •	**<u>Future Learning Connections</u>** • Connect to use function notation with all types of functions. **(HSF.IF.2)** • Connect to deriving the formula for a geometric series. **(HSA.SSE.4)**

Clarification Statement

- HSF.IF.A.1: Students need to understand that a **function** from one **set** (called the **domain**) to another set (called the **rang**e) **assigns** to each **element** of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $\boldsymbol{f(x)}$ denotes the **output** of f **corresponding** to the **input** x. The **graph** of f is the graph of the **equation** $y = f(x)$. 8.F.A are foundational standards; however, this is students' first opportunity to work with function notation as it is explicitly left out of the Grade 8 standards.
- HSF.IF.A.2: Students should be able to use function notation in a flexible way such as knowing how to **plug in a value** and get the corresponding output. They should also be able to understand and use x and F(x) interchangeably with x and y when explaining the context of a problem. Students should know that all they must do is **isolate an equatio**n for y and then replace it with f(x) (read as "**f of x**").
- HSF.IF.A.3: Students should recognize that **sequences** are functions, sometimes defined **recursively**, whose domain is a **subset** of the **integers**. Students should see **patterns** emerge when comparing the **x and y values** to each other. Students should know that these patterns are not coincidences and, students should know that these patterns can be thought of as sequences, or a **list** of numbers. Sequences can be either **arithmetic** (where the same number is added or subtracted) or **geometric** (where the same number is multiplied or divided).

Common Misconceptions

- Students do not recognize f(x) = is the same as y =. They also will often confuse f(x) with the product of f and x and not recognize that it is a form of notation.

- Students often show a lack of understanding for what 'n' represents. Students often struggle understanding the notation of recurrence sequences. Using different values of n for a given term.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the concept of a function and function notation because the foundation for this cluster is developed in 8th grade. Students are introduced to functions as relationships having a unique output for input. Building from this idea is a crucial connection for students developing a deeper understanding of functions and function notation.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.1: This standard provides a foundation for work with the concept of a function and function notation because it is the foundational concept of the function. Understanding the definition of a function is crucial to making sense of the more complicated functions that are seen in Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because it is often difficult to relate sequences to an Algebra 1 student's daily life. By using tasks that involve things like video game scores to work with sequences as functions students can engage in mathematics related to something that many of them work with or use/play daily.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with the concepts of functions and function notation benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that models how to incorporate evaluation, including identifying patterns of errors and wrong answers, into positive strategies for future success because such feedback is invaluable to developing a depth of understanding with this cluster of conceptual standards.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because the foundation for this standard is laid in 8th grade. Because it is conceptual in nature and includes symbolic representations that will be used throughout the students mathematical career, making the connections to prior understandings is vital to guiding students onward through the more complicated functions introduced in Algebra 1 and later in Algebra 2.

Expression and Communication: H*ow will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as using concept mapping tools because students can link their understanding of a function from 8th grade examples to function notation. Since this cluster of standards is conceptual in nature any "linking" that we can do to broaden and deepen their understanding is the key to this cluster.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with the concepts of a function and function notation benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because connecting ideas to what students already know and moving their thinking forward (from what they say in 8th grade to Algebra 1 and on through Algebra 2) with functions and function notation will support a deeper and greater understanding of those ideas.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on the concept of a function and function notation by clarifying mathematical ideas and/or concepts through a short mini lesson because the cluster is conceptual in nature. Making sense of the concepts is key to analyzing them and interpreting in the next two clusters.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit the concept of a function and function notation by addressing conceptual understanding because this cluster is conceptual in nature. Anything that we do to deepen students understanding of the concept of function and function notation will be a key to extend their understanding of functions and function notation in additional standards within this domain.

Extension

	What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM? • For example, some learners may benefit from an extension such as addressing conceptual understanding when studying the concept of a function and function notation because students gain a deeper understanding of functions once, they see applications in real life disciplines other than mathematics. In making cross curricular links students will not only deepen their understanding of the widely applicable nature of functions but also prepare themselves for the next levels of analyzing and interpreting functions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying the concept of a function and function notation the types of mathematical tasks are critical because this cluster is conceptual in nature. The types of vocabulary introduced/continued within this cluster are vital to success in future mathematics especially those within the domain of interpreting functions. Students who are unfamiliar with the idea of a function or the concept of function notation will struggle with these foundational ideas if explicit instruction is neglected.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5441675

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■□□	Heart of Algebra	Linear functions	3. For a linear function that represents a context a. interpret the meaning of an input/output pair, constant, variable, factor, term, or graph based on the context, including situations where seeing structure provides an advantage;	Calculator

5441675

Questions 11 and 12 refer to the following information.

t C(t)
1 8.5
2 11
3 13.5
4 16

The length C(t), in inches, of a channel catfish in an Iowa river *t* years after the first year of life can be approximated by the linear function *C*. Some values of C(t) are given in the table above.

$F(t) = 3t + 4$

The length F(t), in inches, of a flathead catfish in the same Iowa river *t* years after the first year of life can be approximated by the linear function *F*, defined by the equation above.

According to the model, which of the following is closest to the expected age, to the nearest whole year, of a flathead catfish that is 31 inches long?

A. 10 years old

B. 13 years old

C. 98 years old

D. 106 years old

Rationale

Choice A is correct. It is given that the length $F(t)$, in inches, of a flathead catfish in the river t years after the first year of life can be approximated using the function $F(t) = 3t + 4$. The question asks for the expected age when a catfish is 31 inches long, which can be represented by substituting 31 for $F(t)$, which gives $31 = 3t + 4$. Subtracting 4 from both sides of this equation gives $27 = 3t$, and then dividing both sides by 3 gives $9 = t$. This means that approximately 9 years after the first year of life, or at the age $1 + 9 = 10$ years old, a flathead catfish is expected to be 31 inches long.

Choice B is incorrect and may result from substituting 31 for $F(t)$ in the linear function F, but solving for t by adding 4 to both sides of the equation, rather than subtracting 4, before dividing both sides by 3 and adding 1 to the result. Choice C is incorrect and may result from substituting 31 for t, rather than for $F(t)$, in the linear function F, then solving for $F(t)$ and adding 1 to the result. Choice D is incorrect and may result from substituting 31 for t, rather than for $F(t)$, in the linear function F, then rewriting the right-hand side of the function as $3(31 + 4)$ and adding 1 to the result.

Interpret the graph: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/tasks/636

The Customers: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624

The Parking Lot: http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/832/original/public_task_588.pdf?1462392946

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on the concept of a function and function notation, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.	Science: Radioactive decay is a function that is a sequence. Consider providing a connection where students know the half-life and starting amount of a substance and use that to define a function and determine the amount left after a certain amount of time.