HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: B: Interpret functions that arise in applications in terms of the context.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

<table>
<tr>
<td rowspan="3">Standard Text

HSF.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *

Note: Algebra 1 focuses on linear, exponential, and quadratic</td>
<td rowspan="3">Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by thinking through the meaning of the key features in graphs as relative to a given context.

SMP 4: Students can model with mathematics by creating an approximate graph that could model a given context.</td>
<td>Students who demonstrate understanding can:
• Identify intercepts of a function.
• identify intervals where the function is increasing.
• Identify intervals where the function is decreasing.
• Identify intervals where the function is positive.
• Identify intervals where the function is negative.
• Identify relative maximums of a function.
• Identify relative minimums of a function.
• Identify symmetries in the functions.
• Identify end behavior of the functions.
• Sketch graphs given a list of key features or a verbal model.
• Sketch functions that model key feature behavior.
• Label intercepts and intervals of a graph.
• Interpret where the function is increasing, decreasing, positive, or negative.
• Interpret relative maximums and minimums.
• Interpret various symmetries, end behaviors, and periodicity.</td>
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<td>Webb's Depth of Knowledge: 1-2</td>
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<td>Bloom's Taxonomy:
Understand, Apply, Analyze</td>
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Standard Text **HSF.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.** *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** *Note: Algebra 1 focuses on linear, exponential, and quadratic*	**Standard for Mathematical Practices** SMP 3: Students can construct viable arguments by explaining why the domain of a given context is discrete or continuous. SMP 4: Students will model with mathematics by connecting a function to the context it represents using quantities.	**Students who demonstrate understanding can:** • Make connections between a graph of a function and its domain. • Make connections between the graph of a function and the context it describes. • Identify when the domain of a given context is discrete or continuous and explain why. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text **HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *** *Note: Algebra 1 focuses on linear, exponential, and quadratic*	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by interpreting the average rate of change within the context of a problem. SMP 5: Students can use tools by using tables and graphs to determine the average rate of change over a specified interval.	**Students who demonstrate understanding can:** • Calculate the average rate of change of a function over a specified interval presented symbolically. • Calculate the average rate of change of a function over a specified interval presented in a table. • Interpret the average rate of change of a function over a specified interval presented symbolically for a given context. • Interpret the average rate of change of a function over a specified interval presented in a table for a given context. • Estimate the rate of change of a function from a graph. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
<u>Previous Learning Connections</u> • Connect to interpreting the equation y =mx + b as a linear function and using the equation to solve problems in context. **(8.F.3)** • Connect to interpreting key features of linear equations in	**<u>Current Learning Connections</u>** • Connect to discovering features of families of functions. **(HSF.IF.7)** • Connect to distinguishing between situations modeled by linear and exponential functions. **(HSF.LE.1)**	**<u>Future Learning Connections</u>** • Connect to finding key features of the entire family of functions. **(HSF.IF.4)**

relation to a contextual situation. **(8.F.4)**		

Clarification Statement

- HSF.IF.B.4: Students interpret the **key features** of the different **functions** listed in the standard. When given a **table** or **graph** of a function that models a **real-life situation**, explain the meaning of the characteristics of the table or graph in the **context** of the problem. Key features of a **linear function** are **slope** and **intercepts**, of a **quadratic function** are **intervals of increase/decrease**, **positive/negative**, **maximum/minimum**, **symmetry**, and **intercepts**, of an **exponential function** include **y-intercept** and **increasing/decreasing intervals** and of an **absolute value** include **y-intercept**, **minimum or maximum**, **increasing or decreasing intervals**, and **symmetry**.
- HSF.IF.B.5: Students should focus their attention on possible **input and output values**, framing them as the **domain and range of a function**. When given a description of a function that represents a situation, the students should determine reasonable domain and range. Students relate the domain of a function to its graph and, where applicable, to the **quantitative relationship** it describes. Students need to explain the reasonableness of a domain for a given context. Students should understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs. Also looking at if a function is **continuous** (time, amount of liquid filling a container) or discrete (number of people or things) and connecting back to number classifications
- HSF.IF.B.6: Students will calculate and interpret the **average rate of change** of a linear, quadratic, **piecewise linea**r (to include absolute value), and exponential function (presented **symbolically** or as a table) over a specified **interval.** Students will **estimate** the rate of change from a graph. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect **data** from **experiments** or **simulations** (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.

Common Misconceptions

- Students may confuse independent and dependent variables.
- Students may believe that the domain for all functions is all real numbers.
- Students may struggle with the concepts of rate of change and slope.
- Students may focus on the y values of the graph instead of the x values of the interval, when identifying key features of a graph.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that previews new contexts for tasks within the unit (e.g., cell phone plans) when studying the interpretations of functions that arise in applications in terms of a context because understanding the key aspects of a context is the key to unlocking a problem for students.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.B.5: This standard provides a foundation for work with interpreting functions that arise in applications in terms of a context because the given standard is the foundational piece of interpreting linear functions. Once students can efficiently and accurately interpret linear functions, they can apply that knowledge to the more complex quadratic and exponential functions of Algebra 1. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis because the important part of this cluster is first making sense of the application problem and then interpreting the functions related to the problem. These accommodations can take away visual barriers for students with visual acuity issues.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because students often struggle with making sense of application problems. When they are adequately supported and encouraged to persevere it builds their confidence in applying mathematics (which is what this cluster is all about - application problems involving functions).

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making all key information available in English also available in first languages (e.g., Spanish) for English Learners and in ASL for learners who are deaf because often seeing/reading/hearing the application problem in a student's first language allows them to make more sense of the given problem and then break it down in order make the necessary interpretations. In this case we are taking away ONE of the two translations they need to make.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of novel solutions to authentic problems because when students are able to make sense of a mathematics problem (especially with functions and applications) they are more likely to retain the skills that they learn. In allowing them to use multiple procedures or multiple modalities of explaining their thinking or displaying their thinking in approaching a problem we are giving them ownership of learning.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with interpreting functions that arise in applications in terms of a context benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because students struggle with interpreting functions in context. When we build in supports, design and ask appropriate guiding questions, and support them through a productive struggle (wrestling mathematics) with the mathematics their success will build up their confidence and understanding of how mathematics in the real-world works.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpreting functions that arise is applications in terms of the context by providing specific feedback to students on their work through a short mini-lesson because in interpreting functions within a context providing feedback and allowing students to revise their work can be a powerful tool in deepening their understanding.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interpreting functions that arise in applications in terms of the context by confronting student misconceptions because as in the section on re-teach targeted in this cluster students need feedback for learning. They need to see what they don't understand, celebrate their successes and revise their work to deepen their understanding of functions and interpreting functions in context.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying the interpretation of functions that arise in applications in terms of the contest because there are so many opportunities to make connections between the abstract and isolated nature of functions in mathematics and applications in science, history, psychology, sociology and other topics that may be of greater interest to students. If they can research functions in other disciplines they will have more "buy in" to the importance of interpreting functions.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Task: When planning with your HQIM consider how to modify tasks to represent the prior experiences, culture, language and interests of your students to "portray mathematics as useful and important in students' lives and promote students' lived experiences as important in mathematics class." Tasks can also be designed to "promote social justice [to] engage students in using mathematics to understand and eradicate social inequities (Gutstein 2006)." For example, when interpreting functions that arise in applications in the terms of a context the types of mathematical tasks are critical because student engagement in this area leads to greater understanding of the key features of functions and how they relate to the context. When students are beginning to make sense of the parts of a function (or its various representations), they need it to be related to an idea they already understand. In doing this we aren't trying to teach the concept in the application and the mathematics because the students already understand the context and can focus on the mathematics of the task.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 1054722

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■□	Heart of Algebra	Linear functions	3. For a linear function that represents a context a. interpret the meaning of an input/output pair, constant, variable, factor, term, or graph based on the context, including situations where seeing structure provides an advantage;	No Calculator

$A = 1{,}321 + 0.3433m$

The equation above can be used to estimate the body surface area A, in square centimeters, of a child with mass m, in grams, where $3{,}000 \le m \le 30{,}000$. Which of the following statements is consistent with the equation?

A. For each increase of 1 gram in mass, A increases by approximately 0.3433 square centimeter.

B. For each increase of 0.3433 gram in mass, A increases by approximately 1 square centimeter.

C. For each increase of 1 gram in mass, A increases by approximately 1,321 square centimeters.

D. For each increase of 1,321 grams in mass, A increases by approximately 1 square centimeter.

Rationale Choice A is correct. The equation can be represented by a linear graph, where the slope represents the ratio of the change in the dependent variable for every 1-unit change in the independent variable. In this context, the slope is the change in A, the body surface area in square centimeters, for every increase by 1 in m, the mass in grams. In the equation, the slope is represented by the coefficient m of the independent variable, which is 0.3433. Thus, for each increase of 1 gram in mass, A increases by approximately 0.3433 square centimeter. Choice B is incorrect and may result from incorrectly interpreting the slope in this context. Choices C and D are incorrect. These choices compare a mass, in grams, to an area, in square centimeters. However, the value 1,321 refers only to the initial estimated body surface area A, in square centimeters. Interpret the graph: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/tasks/636 The Customers: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624 The Parking Lot: http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/832/original/public_task_588.pdf?1462392946

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on Interpreting functions that arise in applications in terms of a context, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life.	Science: Average rate of change can be modeled in contexts involving temperature, speed or height. Consider providing a connection where students collect bivariate data and that make a contextualized explanation of an average rate of change for a model they have created.