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Public Education Department

HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: C: Analyze functions using different representations.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. * • HSF.IF.C.7.A: Graph linear and quadratic functions and show intercepts, maxima, and minima. • HSF.IF.C.7.B: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. • HSF.IF.C.7.E: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise-defined.*	SMP 4: Students can model with mathematics by graphing linear and quadratic functions using by-hand and technology methods. SMP 7: Students can look for and make use of structure by recognizing linear and quadratic families from their symbolic and graphical forms.	• Graph functions expressed symbolically showing key features of the graph by hand in simple cases and with technology for more complicated cases. • Graph linear functions showing intercepts. • Graph quadratic functions showing intercepts, maxima and minima. • Graph piecewise defined functions (step functions and absolute value functions) showing intercepts, maxima, and minima. • Compare and contrast linear, quadratic and exponential functions. • Explain issues of domain, range and usefulness when examining piecewise-defined functions.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. • HSF.IF.C.8.A: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay. *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise-defined.*	SMP 4: Students can model with mathematics by interpreting zeros, intervals where the function is increasing or decreasing, extrema and symmetry within a context. SMP 7: Students can look for and make use of structure by using rearranging functions to highlight key features.	• Rewrite a function to find and highlight key features. • Factor a quadratic expression to find zeros, extrema and symmetry • Interpret the meaning of zeros, extrema and symmetry within the context of a problem. • Complete the square for a quadratic function to reveal its key features. • Interpret the key features of a quadratic expression in terms of the context it represents. • Use properties of exponents to relate parts of an exponential function to its context (e.g., describe the initial value, growth/decay rate or factor and the growth period). • Identify how key features of an exponential function relate to characteristics in a real-world context. • Classify real-world problems as an exponential growth or decay. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply, Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined.*	SMP 5: Students can use tools by working flexibly with multiple representations. SMP 7: Students can look for and make use of structure by comparing the similarities and differences of linear, quadratic, and exponential functions.	• Make comparisons between functions in different forms using their knowledge of key features. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply,Analyze

NEW MEXICO
Public Education Department

Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to graphing linear functions. **(8.F.5)** • Connect to comparing properties of linear functions represented in different ways. **(8.F.2)** • Connect to identifying and using key features of linear functions. **(8.F.4)** • Connect to writing linear equations. **(8.F.4)**	• Connect to writing linear, quadratic, and exponential functions to describe relationships between quantities. **(HSA.CED.1-3)** • Connect to analyzing transformations of parent functions for linear, quadratic, and exponential functions. **(HSF.BF.3)**	• Connect to graphing all parent functions by hand and using technology and identifying their key features. **(HSF.IF.7)** • Connect to factoring to complete the square with quadratic functions with complex zeros. **(HSN.CN.7)**

Clarification Statement

- HSF.IF.C.7: Students should be able to describe the significant features of different **functions graphically** and **algebraically**. Students should be able to use the **significant features** to **sketch** the graph of the function. Students should graph **linear** and **quadratic functions** and show **intercepts**, **maxima**, and **minima.** Students should know the **slope-intercept form** of linear functions, y = mx + b, and how to extract enough information from the equation to be able to draw it. When graphing **roots**, remember that for $\sqrt[n]{x}$, if n is **even,** the domain includes all **positive integers**. Otherwise, **negative** values are included as well. When graphing roots of the for $y = a\sqrt{x} + b$, remember the **y-intercept** is b. Students should remember that roots are **fractional exponents.** Students should know to look at the **highest degree of the polynomial** and its **coefficient,** ax^n. If n is even, the function will extend either **up** or **down** on both ends (as x goes to **positive** or **negative infinity**). If n is **odd**, they'll go in **opposite directions**. If a is positive, the even powered functions will go up and the odd powered functions will start down and go up. If a is negative, the even powered functions will go down, and the odd powered functions will start up and go down.
- HSF.IF.C.8: Students should be able to rewrite quadratic and **exponential functions** in different ways to find key features of the expression and interpret those key features in terms of the context they represent. Students should be able to find the **x-intercepts** of a quadratic function using both **factoring** and **completing the square**.
- HSF.IF.C.9: Students should be able to compare two given functions (linear, exponential, quadratic) whether that be as a function or **equation**, in a **table,** in a **graph,** or by **verbal description**. Students should start by knowing the difference between linear, quadratic and exponential functions, and be able to identify them by equation and by graph. Students should be able to compare two functions even when they're both represented differently. To do this successfully, they must be able to **translate** between an equation, a graph, words, and a table of values, and understand how certain aspects of one **representation** impact the rest.

Common Misconceptions

- Students may have difficulty identifying the key features needed to sketch the graphs or identifying those features algebraically.
- Students may have difficulty with contextualizing and decontextualizing expressions.
- Students will often confuse functions given in a table as a representation of a finite set of numbers rather than a subset of the entire function. They also may have difficulty with the abstractness of determining what is happening with a function over intervals of the domain that they cannot see.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when analyzing of functions using different representations because it is in this cluster that students begging to broaden the scope of the functions they are working with. It is in this section that students are specifically introduced to the ideas of quadratic and exponential functions as well as piecewise defined an absolute value function. In allowing them time to struggle and grapple with the mathematics we are allowing them to make sense of the functions and internalize the understanding of the functions key features when given them in various ways.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.A.3: This standard provides a foundation for work analyzing functions using different representations because this standard lays the foundation for order of operations and understanding the idea of equivalent expressions. The ideas presented in this standard allow students to start slow with expressions that are linear in nature leading up to the use of the distributive property as well as associative and commutative properties that are the precursors for factoring and rearranging higher order functions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information such as because this provides multiple entry points for students. It also supports the cluster in that the standards within it require multiple representations of functions. If students start with the one they understand best and then work their way through the others we build ownership and then deeper engagement and understanding of the multiple ways in which we represent functions (symbolically, graphically, or in a table).

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as using prompts or scaffolds for visualizing desired outcomes because while students may find one method of representing a function or another more approachable they have to make sense of them all. As we guide them through their work, we can help them build confidence and perseverance with analyzing functions.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

NEW MEXICO Public Education Department

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because when students understand the expectations of representations they will be asked to create they are more likely to engage in the work and complete it with more understanding.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video because the primary goal of this cluster is to take a function within its context and make sense of it. Students can use multiple algebraic methods of describing the key features of a function - but they can use the ideas listed here to help tell the story of the function.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because students need to engage in this cluster in many ways. They first see it with linear functions in 8th grade. In Algebra 1 they will see it with linear, quadratic, and exponential functions. They will work with this cluster further in Algebra 2. Students will need to practice, practice, practice. Making sense of functions and their key features is a life skill and the more problems they see and work with the more widely applicable they will see that standard is.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on analyzing functions using different representations by critiquing student approaches/solutions to make connections through a short mini-lesson because so much of this cluster can be learned through student choice of solution method. When we allow students to share their thinking and make connections between their work and that of others, they are encouraged to try a solution method that they hadn't tried before and might be more efficient. They can also see their errors and make revisions. Jo Boaler (youcubed.com) tells us that brain research suggests that we learn more from when we make mistakes than we do when we get things right all the time. Therefore, constructive criticism and feedback that is more meaningful than just a percentage, and vital for our students' success in learning mathematics.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit analyzing functions using different representations by offering opportunities to understand and explore different strategies because as stated above students will approach the problems with the method that makes the most sense to them at first even if it isn't the most efficient strategy. Looking at ideas from other students allows kids to engage in math practice 5 and perhaps make more meaning of different more efficient strategies. We know that we can pick the best strategy from the outset of the problem - but it's because we have a lot of practice and often, we can't necessarily explain why we chose a specific method. It is helpful for our students to think about why one strategy might be better than another and learning when to use specific strategies based on the problem type, they are given when analyzing functions given in different ways.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when analyzing functions given in different ways because this cluster is widely applicable to other disciplines such as science and statistics. If students can explore something of interest to them related to this cluster, they may think of ways to analyze functions that makes more sense to them and their peers. Allowing them to explore the widely applicable nature of functions given in multiple representations will also allow them to become more informed citizens of our society at large.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when analyzing functions using different representations the use of mathematical representations within the classroom is critical because it is the focus of this cluster. Students must be able to connect a table to the algebraic written function, and its graph (in any order). All three representations are vital to making sense in mathematics applications. Students often come to us with strengths using one or more of those representations and we can build on those strengths and extend them to the other representations. In connecting what they already know to what they need to add to their "toolbox", students build strength in mathematical representations.

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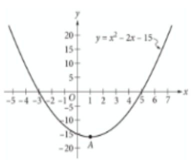
Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 18494

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Nonlinear functions	2. For a quadratic or exponential function, d. determine the most suitable form of the expression representing the output of the function to display key features of the context, including i. selecting the form of a quadratic that displays the initial value, the zeros, or the extreme value;	Calculator

18494



Which of the following is an equivalent form of the equation of the graph shown in the xy-plane above, from which the coordinates of vertex A can be identified as constants in the equation?

A. $y=(x+3)(x-5)$

B. $y=(x-3)(x+5)$

C. $y=x(x-2)-15$

D. $y=(x-1)^2-16$

Rationale

Choice D is correct. Any quadratic function q can be written in the form $q(x)=a(x-h)^2+k$, where a, h, and k are constants and (h,k) is the vertex of the parabola when q is graphed in the coordinate plane. This form can be reached by completing the square in the expression that defines q. The equation of the graph is $y=x^2-2x-15$. Since the coefficient of x is -2, this equation can be written in terms of $(x-1)^2=x^2-2x+1$ as follows: $y=x^2-2x-15=(x^2-2x+1)-16=(x-1)^2-16$. From this form of the equation, the coordinates of the vertex can be read as $(1,-16)$.

Choices A and C are incorrect because the coordinates of the vertex A do not appear as constants in these equations. Choice B is incorrect because it is not equivalent to the given equation.

Interpret the graph: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/tasks/636
The Customers: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624
The Parking Lot:
http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/832/original/public_task_588.pdf?1462392946

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on analyzing functions using different representations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, allowing students to look at home budgets, utility bills (the cost as a function of usage etc.) or even bringing in examples of functions from various careers represented at home can help students make connections between the abstract idea of functions and how/where they exist in real life. You can then extend these functions by having students make tables, graphs, and write functions related to what they find at home.	Science: In high school the NGSS build on K–8 experiences and progresses to using, synthesizing, and developing models to predict and show relationships among variables between systems and their components in the natural and designed worlds. Consider providing a connection for students to use a model based on evidence to illustrate the relationships between systems or between components of a system.