NEW MEXICO
Public Education Department

<table>
<tr><td colspan="3">HS: FUNCTIONS — LINEAR, QUADRATIC, & EXPONENTIAL MODELS*</td></tr>
<tr><td colspan="3">Cluster Statement: A: Construct and compare linear, quadratic, and exponential models and solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.</td></tr>
<tr><td rowspan="3">Standard Text

HSF.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
• HSF.LE.A.1.A: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
• HSF.LE.A.1.B: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
• HSF.LE.A.1.C: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td><td rowspan="3">Standard for Mathematical Practices

SMP 3: Students can construct viable arguments and critique the reasoning of others when comparing linear and exponential functions, proving that linear functions grow by equal differences over equal intervals and exponential functions grow by equal factors over equal intervals.

SMP 8: Students look for and express regularity in repeated reasoning by showing that the rate of change over any given interval of a linear function is the same.</td><td>Students who demonstrate understanding can:
• Compare linear and exponential functions in various ways.
• Show that linear functions have a common difference and that exponential functions have a common ratio.
• Determine when a relationship is growing by a constant difference.
• Determine when a relationship grows by a common ratio.</td></tr>
<tr><td>Webb's Depth of Knowledge: 1-3</td></tr>
<tr><td>Bloom's Taxonomy:
Understand, Apply, Analyze, Evaluate</td></tr>
<tr><td>Standard Text

HSF.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td><td>Standard for Mathematical Practices

SMP 3: Students can construct viable arguments by convincing classmates when a set of data represents linear or exponential relationships.

SMP 5: Students can use tools strategically by determining whether a given description of a numerical relationship is linear or exponential using a table, graph or verbal description.</td><td>Students who demonstrate understanding can:
• Write linear and exponential functions (including arithmetic and geometric sequences) based on a graph.
• Write linear and exponential functions (including arithmetic and geometric sequences) based on a description of a relationship.
• Write linear and exponential functions (including arithmetic and geometric sequences) based</td></tr>
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		on two ordered pairs (including from a table). • Decide whether a relationship is linear, or exponential given a table, graph or verbal description.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze
Standard Text HSF.LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	**Standard for Mathematical Practices** SMP 5: Students can use tools strategically by using technology to determine relation rates of change and make conclusions. SMP 8: Students look for and express regularity in repeated reasoning by comparing exponential, quadratic and linear functions (either with graphs or tables) to realize that a quantity increasing exponentially will exceed a quantity growing linearly or quadratically.	**Students who demonstrate understanding can:** • Explore rates of change of different functions using graphs or tables. • Generalize that an exponential growth function will exceed a linear or quadratic function eventually. • identify situations where this phenomenon is occurring.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply, Analyze
<u>Previous Learning Connections</u> • Connect to determining the growth of a linear expression by taking the ratio of rise over run for any two distinct points on the same line. **(8.EE.6)** • Connect to relating the information gathered by the ratio of rise over run to the linear equation in terms of input and output. **(8.F.4)**	**<u>Current Learning Connections</u>** • Connect to examining contextual information and distinguishing if the solution can be modeled with linear or exponential functions. **(HSA.CED.3)** • Connect to writing arithmetic and geometric sequences both recursively and with an explicit formula to model situations. **(HSF.BF.1)** • Connect to relating the knowledge of linear functions to exponential and polynomial functions and comparing their behaviors. **(HSF.If.9)**	**<u>Future Learning Connections</u>** • Connect to extending their knowledge of linear, quadratic and exponential situations to different types of functions and making comparisons. **(HSF.IF.7-9)**

Clarification Statement

- HSE.LE.A.1: Students should be able to differentiate between **exponential** and **linear functions** by determining whether given relationships have a **common difference** or a **common ratio**. Students have to know the differences between linear functions and exponential functions. In simplest terms, a linear function one that takes the **form y = mx + b** and an exponential function is one in which **y = ax**.

- HSE.LE.A.2: When given a variety of descriptions (whether words, **graphs**, or **tables**), student should write linear and exponential functions. Students should determine and explain (orally and in writing) whether **relationships**—in descriptions, tables, **equations**, or graphs—are functions. In a table, students should recall that when the **difference** in **interval** is **constant**, we can presume that our equation is most likely linear. In this case it is simply a matter of $f(x) = x + 1$. Students should understand that when graphs are involved, students should **plot** points. That way, students can assemble a list of **input and output values** from the graph. As for descriptions, words to watch out for are "exponential," "linear," "**multiple**," "**constant**," and "**factor.**"
- HSE.LE.A.3: Students should understand that a function **growing** exponentially will eventually overtake or grow faster that either a linear or quadratic function. Students should be able to compare linear and exponential relationships by performing calculations and by interpreting graphs that show two **growth patterns**. Students should be able to **prove** that eventually, as long as the functions are headed in the same direction, a quantity increasing exponentially will "beat" linear, quadratic, and polynomial functions.

Common Misconceptions

- Students may not realize when a table or set of points increases by an interval other than 1 and not take the effect of this into account when finding the common difference or ration.
- Students may find it difficult to attend to direction and rates of change, making it hard to then compare the graphs.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that <rehearses prior learning when studying Construct And Compare Linear, Quadratic, And Exponential Models And Solve Problems because this allows students to go over what they previously learned and think about the process and skills needed to construct models of functions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.2 This standard provides a foundation for work with Construct And Compare Linear, Quadratic, And Exponential Models And Solve Problems because students compared properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). This sets up the concept they will need for this cluster. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: *How will the learning for students provide a variety of methods for navigation to support access?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard because when students are given different options of constructing function models and they can identify the model they created then students are able to see the mathematics behind the problems.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems. benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing prompts that guide learners in when and how to ask peers and/or teachers for help because when students are given the opportunity to question and explore more deeply their work is that they can be better understood and more thoughtful opinions and decisions are made.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because the students will be able to solve the problem and not be confused about what the problem is requiring them to do since they know the vocabulary and the symbols needed. Students will be more prepared and therefore more confident as they work thru the problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solve problems. benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because students like instant feedback and teachers been able to have different forms of giving feedback will only keep the students engaged, because they like to hear different form of been told that they are doing a good job instead of "yes, you are correct or good job."

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with constructing and comparing linear, quadratic, and exponential models and solving problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") because when students are constructing models and they can explain them to others then the students know if they are on the right path of solving the problem. If there are misconceptions, they can find them as they explain their model and they can make changes as needed.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on constructing and comparing linear, quadratic, and exponential models and solving problems by critiquing student approaches/solutions to make connections through a short mini lesson because students will be given an opportunity to hear vocabulary and revisit concepts and skills needed to construct various models.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on constructing and comparing linear, quadratic, and exponential models and solving problems offering opportunities to understand and explore different strategies because when students are able to have various opportunities to understand and explore different strategies then they are able to think of the different models that they have learned about and use that connection to solve the problem by choosing the strategy that they understand the best.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying constructing and comparing linear, quadratic, and exponential models and solving problems because students can design their own learning path and select the resources, guides and information they will need to discover new information ad think critically about it.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying constructing and comparing linear, quadratic, and exponential models and solve problems the types of mathematical tasks are critical because the tasks need to be engaging and allow students to use multiple solution strategies which will give the students opportunities to make comparisons.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5440666 ×

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	■■■	Heart of Algebra	Linear functions	2. Create a linear function to model a relationship between two quantities.	Calculator

5440666

The dwarf planet Makemake completes one orbit around the Sun every 310 years. Which of the following functions r models the number of orbits of Makemake in t years?

A. $r(t) = 310 + t$

B. $r(t) = 310t$

C. $r(t) = \frac{t}{310}$

D. $r(t) = \frac{310}{t}$

Rationale

Choice C is correct. It's given that Makemake completes one orbit around the Sun every 310 years. This can be represented by the ratio $\frac{1 \text{ orbit}}{310 \text{ years}}$. The number of orbits r in t years can be determined by setting up an equivalent ratio and solving the proportion for r: $\frac{1 \text{ orbit}}{310 \text{ years}} = \frac{r \text{ orbits}}{t \text{ years}}$. Cross-multiplying yields $310r = 1t$, or $r = \frac{t}{310}$. Therefore, the function $r(t) = \frac{t}{310}$ models the number of orbits of Makemake in t years.

Choice A is incorrect and results from adding the time needed for one orbit and the number of years t. Choice B is incorrect and results from multiplying the time needed for one orbit by the number of years t. Choice D is incorrect and results from dividing the time needed for one orbit by the number of years t.

https://www.map.mathshell.org/tasks.php?unit=HN08&collection=9&redir=1

Relevance to families and communities:

During a unit focused on constructing and comparing linear, quadratic, and exponential models and solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example while looking at the current events that are occurring in your community or in the world create a model and determine the type of model it represents. This will create a strong connection on math tasks and current events that affect your life.

Cross-Curricular Connections:

Science: Exponential functions can model population growth. However, they will ultimately be limited by resource availability. Consider providing a connection where students track and/or predict when and why this will happen for a given population.