

HS: NUMBER AND QUANTITY- QUANTITIES

Cluster Statement: A: Reason quantitatively and use units to solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text

HSN.Q.A.1: Use units to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Foundation for work with expressions, equations and functions

Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by utilizing appropriate units and/or quantities in the context of the problems.

SMP 2: Students can reason abstractly and quantitatively by applying appropriate units and/or quantities in the context of the problems.

Students who demonstrate understanding can:

- Choose the units in a formula.
- Correctly scale a graph with unit increments and identify a quantity from a graph with a scale in unit increments of a specified measurement.
- Use units to guide the solution of a familiar multi-step problem with scaffolding.
- Make measurement conversions between compound units.

Webb's Depth of Knowledge: 1-2

Standard Text

HSN.Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.

Foundation for work with expressions, equations and functions

Standard for Mathematical Practices

SMP 1: Students can make sense of problems and persevere in solving them by defining different quantities in a problem and its solution.

SMP 4: Students can model mathematically using appropriate quantities in the context of the problems.

Bloom's Taxonomy: Understand, Apply

Students who demonstrate understanding can: • Identify important inform

- Identify important information, plan, and develop strategies to solve a problem in a context.
- Define appropriate quantities to construct a model

Webb's Depth of Knowledge: 1-2

Bloom's Taxonomy: Understand, Apply



Standard Text

HSN.Q.A.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Foundation for work with expressions, equations and functions

Standard for Mathematical Practices

SMP 5: Use appropriate tools strategically by experiencing different measurement tools digitally and concretely to observe measurement error.

SMP 6: Students can attend to precision by using the measurement of the same object multiple times to determine an acceptable level to report.

Students who demonstrate understanding can:

- Determine whether a measurement is appropriate in each context. (e.g., measuring the length of a desk in inches versus yards).
- Determine the appropriate level of precision of measurement in each context.
- Write solutions using appropriate units and rounding techniques based on the context of the problem.

Webb's Depth of Knowledge: 1-2

Current Learning Connections

 Connect to application problems using linear, quadratic, and exponential models. (HSF.IF.4- 6)

Bloom's Taxonomy:

Understand, Apply, Analyze

Future Learning Connections

- Connect will continue to use and expand upon the use of units to make sense of problems and use the context of a problem to create and label graphs using appropriate scales. (HSF.IF.4-7)
 Focus on using key features to
 - Focus on using key features to guide selection of appropriate type of model function)

Clarification Statement

units. (8.F.4-5)

and y-axes with

Previous Learning

Connect to rounding.

Connect to finding unit

Connect to labeling x-

appropriate scales and

(4.NBT.3, 5.NBT.4)

Connections

rates.

HSN.Q.A.1: Reasoning quantitatively includes knowing when and how to convert units in computations, such as when adding and subtracting quantities that measure the same attribute but are expressed in different units and other computations with measurements in different units or converting units for derived quantities such as density and speed. Reasoning quantitatively can also include analyzing the units in a calculation to reveal the units of the answer. This can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed (MP.2).

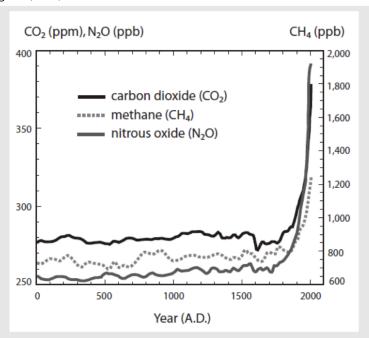
Students should specify units when defining variables and attend to units when writing **expressions** and **equations** (MP.6).

In **applications**, **formulas** are often used, and errors can occur in the use of the formulas if units are not attended to carefully. The formula d=vt notwithstanding, a car driving at 25 mph for 3 minutes does not cover 25 x 3 miles. Conversely, if the student does attend carefully to units, the result can be a deeper understanding of a formula or a situation.

A good **quantitative understanding** of [a real-life situation] helps a student make sound choices for the **scale** and **origin** of a **graph** or a **display**. In a map of arable land area, for example, there is no sense in having a scale that extends to **negative values**, in a graph showing the concentration of atmospheric



carbon dioxide over the past 2000 years, the choice of origin in the **vertical scale** is an important editorial decision. These considerations apply to graphs, **data tables**, **scatter plots**, and other visual displays of **numerical data**. It should go without saying that graphs and displays must be properly **labeled**, or else they are meaningless (MP.6)



Concentration of carbon dioxide and other gases in the atmosphere over the past 2,000 years. Source: Forster et al., 2007, Changes in Atmospheric Constituents and in Radiative Forcing. In Solomon et al. (Eds.), *Climate Change 2007: The Physical Science Basis*, Figure 1, p. 135, http://www.ipcc.ch/pdf/assessment-report/ar4/wg1/ar4-wg1-chapter2.pdf.

- HSN.Q.A.2: In modeling situations (MP.4), defining the **key quantity of interest** might be part of the task.
 For example, in a situation that involves crop productivity, a student might choose to examine the number of tons of fertilizer per acre as the variable of interest. In a situation that involves content development for a web site, a choice might arise as to whether the number of posts per day or the number of words per day is the key productivity **variable**.
- HSN.Q.A.3: Quantitative reasoning includes choosing an **appropriate level of accuracy** when reporting quantities. For example, if the doctor measures your height as 73 inches and your weight as 210 pounds, then your Body Mass Index (BMI) is (weight in pounds)/(height in inches ²) x 703 = (210)/(73²) x 703 ≈ 27.7031 ≈ 28. There is no point in reporting a value more **precise** than 28 here, because any value between 25 and 30 is considered overweight. * (See reference under Figure #).

Common Misconceptions

- Students may have difficulty with multi-step problems.
- Students frequently confuse precision with accuracy.



Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?

For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying using units to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations because students need time to determine relevant information and the unit's importance of the units given in the context to help guide their approach. Students also need to use reasoning skills to determine the level of accuracy appropriate to the limitations of their problem. Students need to make sense of the problem, use reasoning to create a plan and use precision to develop a solution that makes sense in the context of the problem. Students should be given multiple opportunities to apply these skills.

Pre-teach (intensive): What critical understandings will prepare students to access the mathematics for this cluster?

• 6.RP.A.1 This standard provides a foundation for work with using units as a way to understand problems and to guide the solution of multi-step problems because understanding the concept of a ratio and use ratio language to describe a ratio relationship between two quantities is the building blocks for proportional reasoning and graphs. Students can gain confidence in their problem-solving ability by attempting a problem based on prior learning. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

For example, learners engaging with reasoning quantitatively and use units to solve problems benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features < give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials > because this domain is embedded throughout algebra. Students use units to understand problems, guide their solutions, justify solutions as viable/non-viable, and understand the accuracy as well as the limitations of their solutions. Emphasis should be placed on the core of this domain when displays such as graphs, tables, anchor charts, videos, etc. For example, showing a video modeling volume is cubic units. Showing a video modeling the formula distance equals rate time time. Displaying an anchor chart that connects linear, square, and cubed units to a physical model will make connections for students.



Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

• For example, learners engaging with benefit when learning to reason quantitatively and use units to solve problems experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students need to understand the importance of their approach not just their solution. Providing students with feedback that allows improvement will build their reasoning and problem-solving abilities. Students should understand what part of their approach viable and which part of their approach needs improvement in order to persevere in solving a contextual problem.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

• For example, learners engaging with reason quantitatively and use units to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making relationships between elements explicit (e.g., highlighting the transition words in an argument, links between ideas in a concept map, etc.) because students need a structure when reasoning and persevering in solving contextual problems. Using strategies such as highlighting important information, crossing out irrelevant information, and using a graphic organizer to organize relevant information help students make sense of the information and bring understanding on how the information is related.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

For example, learners engaging with reason quantitatively and use units to solve
problems benefit when learning experiences attend to the multiple ways students
can express knowledge, ideas, and concepts such as solving problems using a variety
of strategies because students will gain an understanding of the best approach for a
given contextual problem and also see some contextual problems offer many
different approaches. Students need to be provided opportunities to see the
limitations of an approach for a given context.

Internalize

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

For example, learners engaging with reason quantitatively and use units to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as offering devices, aids, or charts to assist students in learning to collect, chart and display data about the behaviors such as the mathematical practices for the purpose of monitoring and improving because students need their learning recorded in an organized manner to access the learning in the future. Students need access to devices such as, anchor charts, concept maps, and charts displaying the mathematical practices and problem-solving approaches to



use as a reference. These resources allow students the opportunity to learn from their mistakes and not repeat the same mistakes. Problem solving should be a fluid concept in all classrooms. Students should increase their "bag of tricks" to approaching problems through their ability to improve their work by using multiple approaches.

Re-teach

Re-teach (targeted): What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?

For example, students may benefit from re-engaging with content during a unit on using units as a way to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations by critiquing student approaches/solutions to make connections through a short mini-lesson because providing students with feedback not only on their solution but on their approach will engage students in discussions that will lead to clarifying the best approach for a given context.

Re-teach (intensive): What assessment data will help identify content needing to be revisited for intensive interventions?

For example, some students may benefit from intensive extra time during and after a
unit using units as a way to understand problems and to guide the solution of multistep problems; Using descriptive modeling and choosing a level of accuracy
appropriate to the limitations by offering opportunities to understand and explore
different strategies because students with unfinished learning need ample
opportunities to explore different strategies to determine the validity of each
strategy given a specific context. Students need opportunities to solve contextual
problems that involve using units to understand and solve problems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

For example, some learners may benefit from an extension such as open-ended tasks linking multiple disciplines when studying using units as a way to understand problems and to guide the solution of multi-step problems; Using descriptive modeling and choosing a level of accuracy appropriate to the limitations because through open ended tasks linking multiple disciplines students begin to understand the relationship between mathematics and other disciplines. Students engage in using problem solving approaches to address problems in a context other than mathematics. Students will extend their thinking to contextual situations to reinforce their understanding of using units to understand and persevere through all problems.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?



Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying using units as a way to understand problems and to guide the solution of multi-step problems, choosing and interpreting units consistently in formulas, choosing and interpreting the scale and the origin in graphs and data displays, defining appropriate quantities for the purpose of descriptive modeling, and choosing a level of accuracy appropriate to limitations on measurement when reporting quantities the use of mathematical representations within the classroom is critical because students approaches as well as their solutions need to be validated. For example, multi-entry tasks allow students to choose the tools and approaches best suited for the situation. Allowing for discourse regarding the tools and approach selected provides students' knowledge that there are limitations to tools and approaches. When selecting an approach or tools to attempt a mathematical task, students use their reasoning skills to determine if their approach is valid for the situation and whether there are limits to the approach. Also, students' awareness that many approaches or tools may be accurate for the situation, but some are more precise than others.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5207812

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Analysis in Science		Heart of Algebra	Linear functions	4. Make connections	Calculator
						between verbal, tabular,	
						algebraic, and graphical	
						representations of a	
						linear function, by a.	
						deriving one	
						representation from the	
						other;	

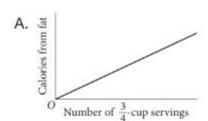
Questions 18-20 refer to the following information.

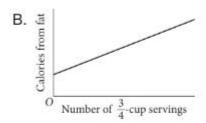
Jennifer bought a box of Crunchy Grain cereal. The nutrition facts on the box state that a serving size of the cereal is $\frac{3}{4}$ cup and provides 210 calories, 50 of which are calories from fat. In addition, each serving of the cereal provides 180 milligrams of potassium, which is 5% of the daily allowance for adults.

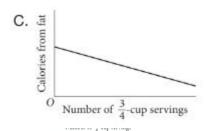
Which of the following could be the graph of the number of colories from fat in Crupaby Crain coreal as a function

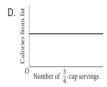
Which of the following could be the graph of the number of calories from fat in Crunchy Grain cereal as a function of the number of $\frac{3}{4}$ -cup servings of the cereal?











Rationale

Choice A is correct. There are 0 calories in 0 servings of Crunchy Grain cereal so the line must begin at the point (0,0). Point (0,0) is the origin, labeled O. Additionally, each serving increases the calories by 250. Therefore, the number of calories increases as the number of servings increases, so the line must have a positive slope. Of the choices, only choice A shows a graph with a line that begins at the origin and has a positive slope. Choices B, C, and D are incorrect. These graphs don't show a line that passes through the origin. Additionally, choices C and D may result from misidentifying the slope of the graph.

Weed Killer (HSN.Q.A.1, HSN.Q.A.2, HSN.Q.A.3): http://tasks.illustrativemathematics.org/content-standards/HSN/Q/A/2/tasks/81

Relevance to families and communities:

During a unit focused on using units as a way to understand problems and to guide the solution of multi-step problems, choosing and interpreting units consistently in formulas, choosing and interpreting the scale and the origin in graphs and data displays, defining appropriate quantities for

Cross-Curricular Connections:

Science: In high school the NGSS state students should "carefully format data displays and graphs, attending to origin, scale, units, and other essential items." Consider providing a connection for students to choose and interpret the scale and the origin in graphs and data displays that they are working with in science.



the purpose of descriptive modeling, and choosing a level of accuracy appropriate to limitations on measurement when reporting quantities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, discussing how different cultures eat food will show that although certain cultures choose different tools, such as, forks, chopsticks, tortillas, they are all possible approaches, but some may be more precise than others. Also, with practice other approaches can be useful. The connection can be made that although trying something new, as in a new approach to a mathematical task, may be uncomfortable, but with practice it becomes more useful.

Social Studies: In high school the New Mexico Social Studies Standards state students should "explain how to use technological tools to research data, verify facts and information, and communicate findings." Consider providing a connection for students to look at the accuracy/precision of measurement data.