NEW MEXICO
Public Education Department

HS: NUMBER AND QUANTITY- THE REAL NUMBER SYSTEM

Cluster Statement: A: Extend the properties of exponents to rational exponents.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Explain how integer exponent properties apply to rational exponent properties. • Show how a rational exponent (whose numerator is not one) can be expanded as a whole number multiplied by a fraction. • Justify that raising the base to power and then taking the root is equivalent to taking the root and then raising the base to a power. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply, Analyze
HSN.RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)^3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	SMP 3: Students can construct viable arguments by explaining the meaning of rational exponents using the properties of exponents. SMP 7: Students can look for and make sure of structure by extending the properties of integer exponents to multiply and divide expressions with rational exponents that have common denominators.	
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Apply properties of exponents to simplify algebraic expressions with fractional exponents. • Apply power of zero, negative exponent, product, quotient, power to a power, product to a power, and quotient rules of exponents to simplify or write equivalent expressions. • Convert radical expression to expressions with rational exponents and vice versa. • Identify the exponent property used when rewriting expressions and recognize when laws of exponents cannot be used to rewrite an expression. **Webb's Depth of Knowledge:** 1-2
HSN.RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.	SMP 1: Students can make sense of problems and persevere in solving them by making meaning of rational exponents in terms of exponent and root and applying that meaning to various problems SMP 3: Students can construct viable arguments by explaining the meaning of rational exponents in terms of radicals and roots. SMP 7: Students can look for and use patterns and structures in exponential expressions by applying properties of exponents to rational exponents.	

		Bloom's Taxonomy: Understand, Apply, Analyze
Previous Learning Connections • Connect to using square root and cube root symbols. **(8.EE.2)** • Connect to understanding and applying the properties of integer exponents. **(8.EE.1**)	**Current Learning Connections** • Connect to applying the properties of exponents to rewrite exponential functions. **(HSA.SSE.3)**	**Future Learning Connections** • Connect to solving equations using rational exponents and radical operations. **(HSA.REI.2)** • Connect to continuing to use exponent properties as they arise in various situations. **(HSF.IF.8, HSF.LE)**

Clarification Statement

- RN.A.1: **Exponent notation** is a remarkable success story in the expansion of mathematical ideas. It is not obvious at first that a number such as sqrt(2) can be represented as a **power of 2**. But reflecting that $(\text{sqrt}(2))^2=2$ and thinking about the **properties of exponents**, it is natural to define $2^{(1/2)}=\text{sqrt}(2)$ since if we follow the rule $(a^b)^c=a^{(bc)}$ then $(2^{(1/2)})^2=2^{((1/2)*2)}=2^1=2$.
- RN.A.2: Because **rational exponents** have been introduced in such a way as to preserve the **laws of exponents**, students can now use those laws in a wider variety of situations. For example, they can rewrite the **formula** for the **volume** of a **sphere** of **radius** r, $V=(4/3)(\text{pi})(r^3)$ to express the radius **in terms of** the volume, $r=((3/4)(V/\text{pi}))^{(1/3)}$.

Common Misconceptions

- Struggle to connect rational exponents to its radical form. Students tend to multiply the number by the exponent.
- When using the Power of a Power Property some students may forget to multiply the entire quantity by the exponent and only multiply the variable.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying the rewriting of expressions involving radicals and rational exponents by applying the properties of exponents because students may have unfinished learning simplify expressions with exponents using the exponent properties and would benefit from the access of that prior learning. Re-visiting expanded form and connecting to exponent properties as well as anchor charts would be beneficial to provide students with access to this content.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.EE.A.1 This standard provides a foundation for work with extending exponent properties to rewriting expressions with radicals and rational exponents because this was the first-time students were introduced to applying the exponent properties in simplifying and generating equivalent expressions. Students will benefit from time to access and apply this prior knowledge. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational numbers benefit when learning experiences include ways to recruit interest such as providing choices in their learning <give an example such as the sequence or timing of task completion because students must see the connection of the properties of exponents to rational exponents. Students have difficulty with the exponent properties because they have the flexibility to simplify expressions involving integer exponents multiple ways. Students must be provided choices when learning the exponent properties, communicate and discuss multiple ways to simplify the same expression, and make the connection between the exponent properties to expressions with rational exponents.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational numbers benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because for this domain it is imperative students work in cooperative groups to strengthen their understanding of simplifying an expression using the exponent properties provides students with multiple entry points that end in the same result. Students must know and understand the role of their group and their own individual role of the group. Students must be provided many opportunities to simplify expressions using the exponent properties and rewriting expressions with rational exponents for them to persevere and solidify their knowledge and understanding of this concept. When students gain the confidence of simplifying expressions with the exponent properties, they can extend that knowledge to rewrite expressions with rational exponents into radical form.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational number benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because accessing the prior knowledge of simplifying exponential expressions using the exponent properties is essential for students to rewrite expressions with rational exponents in radical form. Understanding the entry point of students' prior knowledge of this concept will allow for the level of reteaching before students rewrite expressions with rational exponents.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational number benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use differing approaches, strategies, skills, etc.) because students have a difficult time when mathematical problems provide students with multiple entry points. Students must be provided with multiple ways to simplify an expression using the exponent properties to understand their connection to rewriting expressions with rational exponents. Providing students examples of expanded form and having the students make the connection to the properties <i.e. product rule -keep the base and add the exponent, quotient rule- keep the base and subtract the exponent, etc..> is important for students to gain conceptual understanding of the concept. This also allows all students access to the content, the opportunity to reason and make connections.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with extending the properties of exponents to rational exponents and using properties of rational and irrational number benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of "natural" aptitude (e.g., "how can I improve on the areas I am struggling in?" rather than "I am not good at math") because students struggle with the flexibility of this domain and might need to have more focused learning with this standard. For instance, many students have great difficulty with the negative exponent property of exponents. Approaching this standard by ensuring all exponents are positive as the first step in simplifying helps students manage their anxiety. Focusing on problems with just one or two properties will help some learners be able to persevere in this domain instead of becoming overwhelmed and giving up.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on rewriting radical and rational exponent expressions by clarifying mathematical ideas and/or concepts through a short mini-lesson because students must build a conceptual understanding of the meaning of a rational exponent by decomposing the exponent into parts. Students must understand the parts of the rational exponent ⅔; ⅔ can be rewritten as 2 times ⅓; the numerator of 2 is the power and the denominator 3 of (⅓) is the cube root.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit of rewriting radical expressions and expressions with rational exponent by offering opportunities to understand and explore different strategies because students must make the connection that a rational exponent can be broken down to

the root and the power. Students need time to explore performing different operations first to understand the mathematical relationships between inverse operations. <i.e. (5 ^3)^(⅓) is 5>

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the application of and development of abstract thinking skills when studying how to rewrite radical expressions and expressions with rational exponents because students extend this thinking by using the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents and rewriting expressions involving radicals and rational exponents using the properties of exponents eliciting and using student thinking is critical because students need to experience mathematics that allow students to use different approaches to find the same end result. When rewriting expressions with rational exponents in order to simplify expressions, by using student thinking, students will be provided the opportunity to see the order in which you perform the root and the power does not make a difference with the end result. Also, by using student thinking about simplifying exponential expressions using the exponent properties, students are provided the opportunity to reason, communicate their reasoning and justify their solution. Students will have the opportunity to build on the knowledge that mathematics is a powerful tool and all approaches should be validated.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

NEW MEXICO
Public Education Department

Question ID 1053933

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Equivalent expressions	1. Make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including b. rewriting expressions with rational exponents and radicals;	No Calculator

If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\frac{a}{b}$?

Rationale

The correct answer is $\frac{7}{6}$. The value of $\frac{a}{b}$ can be found by first rewriting the left-hand side of the given equation as $\frac{x^{\frac{5}{2}}}{x^{\frac{4}{3}}}$. Using the properties of exponents, this expression can be rewritten as $x^{\left(\frac{5}{2}-\frac{4}{3}\right)}$. This expression can be rewritten by subtracting the fractions in the exponent, which yields $x^{\frac{7}{6}}$. Thus, $\frac{a}{b}$ is $\frac{7}{6}$. Either 7/6, 1.16, or 1.17 can be entered as the correct answer.

Evaluating Exponential Expressions: http://tasks.illustrativemathematics.org/content-standards/HSN/RN/A/1/tasks/1866

Relevance to families and communities:

During a unit focused on how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents and rewriting expressions involving radicals and rational exponents using the properties of exponents, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about what systems are in place at home to organize and simplify the household can provide students' with a powerful connection on why we simplify expressions.

Cross-Curricular Connections:

Science: Rational exponents can be applied to scientific notation. Consider providing a connection for students to explore rational exponents in this context, such as to determine the maximum distance of each planet from the sun.

Music: The frequencies in the musical range of a various instruments can be modeled using rational exponents. Consider providing a connection for students to find the highest and lowest frequencies.