NEW MEXICO
Public Education Department

HS: NUMBER AND QUANTITY- THE REAL NUMBER SYSTEM

Cluster Statement: B: Use properties of rational and irrational numbers.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSN.RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	SMP 3: Students can construct viable arguments by explaining the result of computations with rational and irrational numbers by providing examples and counterexamples as justification and explain why their conjectures work. SMP 7: Students can look for and make sure of structure by examining the underlying structure of number arithmetic and applying it to real numbers.	• Identify the difference between a rational and an irrational number. • Perform operations on rational and irrational numbers. • Explain that the sum and product of two rational numbers is rational. • Explain that the sum and product of a rational number and a nonzero irrational number are irrational. **Webb's Depth of Knowledge:** 2 **Bloom's Taxonomy:** Apply, Analyze

Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• Connect to identifying and comparing rational and irrational numbers. **(8.NS.2)** • Connect to computing rational and irrational values when working with volume, surface area, and circles. **(7.G.4,6)**	• Connect to using the same strategies as classifying one number as rational or irrational to classify sums and products.	• Connect to rationalizing denominators using an understanding of products of irrational numbers. **(HSN.CN.5)** • Connect to working with irrational numbers when solving equations. **(HSN.CN.7)** • Connect to simplifying radicals using an understanding of irrational numbers. **(HSA.REI.2)** • Connect to calculating and interpreting measurements using irrational numbers. **(HSF.TF.1-3)**

Clarification Statement

HSN.RN.B.3: An important difference between **rational** and **irrational numbers** is that rational numbers form a **number system**. If you **add**, **subtract**, **multiply**, or **divide** two rational numbers, you get another rational number (provided the **divisor** is not 0 in the last case). The same is not true of irrational numbers.

Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with **finite decimal approximations** (and therefore with rational numbers), thinking about the **properties** of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments and attending to precision. (MP.3, MP.6).

Common Misconceptions

- Students may think that the quotient of two rational numbers isn't always rational because some quotients do not appear to terminate or repeat.
- Students may wrongly believe that a single explanation is an explanation or proof of a property.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying properties of rational and irrational numbers because some students have either unfinished learning or misconceptions such as confusing repeating with non-terminating regarding rational and irrational numbers. Assessing students prior learning of rational and irrational numbers and addressing student misconceptions is imperative to avoid further misconceptions on classifying sums and products of rational and irrational numbers.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.NS.A.1 This standard provides a foundation for work with the classification of sums and products of rational and irrational numbers because this standard introduces the concept that a number can't be both rational and irrational simultaneously. In prior grades, students were presented with only the rational number system. Students must understand the difference between a rational and irrational number before they can classify expressions. Allowing time for those discussions and addressing misconceptions regarding the real number system will diminish further misconceptions from developing. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences include ways to recruit interest such as providing time for self-reflection about the content and activities because students need to communicate their thinking verbally and in writing to solidify the outcomes of sums and products of rational and irrational numbers. Students need time to reflect on their learning to solidify their understanding of this domain.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to students attention and affect

to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because students need to work in a community of learners with a common goal to understand how to identify sums and products of rational and irrational numbers. For example, students should be given specific questions, such as, Is the outcome of the sum of two irrational numbers always irrational? What about the products of rational and irrational numbers? Does your conjecture hold true for all cases? Can you find a counterexample to disprove your claim? to target the learning goals and allow time within a community to make a conjecture and see if it holds true in all cases.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as pre-teaching vocabulary and symbols, especially in ways that promote connection to the learners' experience and prior knowledge because students must understand how to classify numbers as rational or irrational before they can classify the sums and products of rational and irrational numbers. Pre-teaching rational and irrational numbers by providing students with several rational and irrational numbers and having students collaboratively come up with the definition allow students to use reasoning skills and communicate their thinking. Then by allowing students the opportunity to classify sums and products of rational and irrational numbers in this same manner, solidifies the students conceptual understanding of rational and irrational expressions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because for students to gain conceptual knowledge of classifying expressions as rational or irrational, calculators are useful as an efficiency tool. Students can use the calculator to visually see when a decimal terminates, has repeating value (leads to discuss why 1.6666… shows as 1.666666667 in the calculator), or is non-terminating, non-repeating. The use of a calculator when determining if an expression is rational or irrational can be used as a tool to prove a student's conjecture or disprove their conjecture by finding a counterexample. Students can then apply their conceptual understanding of the characteristics of a rational or irrational sum or product and use it to classify expressions using reasoning skills without the use of technology.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with use properties of rational and irrational numbers benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new

learning; and, applying learning to new contexts such as offering opportunities over time to revisit key ideas and linkages between ideas because students need to build on the concept of rational and irrational outcomes when dealing with precision in future mathematics. Students need to understand when a mathematical relationship calls for an exact answer versus an approximate answer. In solving quadratics using the quadratic formula, students are presented with irrational solutions and need to understand the exact and approximate solution when identifying the zeros of the function. The irrational number square root of 17 is close to 4 since the square root of 16 is 4. so, an approximate solution of square root of 17 is about 4.1.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on Classifying Expressions as Rational or Irrational by revisiting student thinking through a short mini-lesson because students cannot classify expressions as rational or irrational before they have developed conceptual understanding of rational and irrational numbers. Revisiting student thinking before the presentation of this concept will show student's unfinished learning, student misconceptions, and students' level of reasoning. Understanding student thinking is essential to present this concept in a way that students can extend their thinking.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit Classifying Sums and Products as Rational or Irrational by helping students move from specific answers to generalizations for certain types of problems because students need time to reason and apply their thinking using generalizations to develop conceptual understanding. For example, students should be given time and tools (calculators) to investigate whether the sum of two irrational numbers are always, sometimes, or never rational. Students can explore sums of different irrational numbers to determine if they are always irrational or can a counterexample be found. Students then should be allowed time to communicate their thinking verbally and in writing to write a general statement regarding the posed question.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying the classification of sums and products of rational and irrational Numbers because students use reasoning skills to make conjectures and provide counterexamples to disprove conjectures and develop deep understanding of the concept. For example, Does the product of a rational and irrational number always produce an irrational product? If not, can you provide a case where it does NOT hold true.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational the types of mathematical tasks are critical because students must develop conceptual understanding of this concept making conjectures regarding the sum and products of rational and irrational numbers , conduct investigations by exploring many cases, providing counter examples if possible to refute the conjecture, and justifying their claims through verbal and written communication. Students who are given rules, do not remember them unless they make a personal connection to the rule. Discovery is the connection students need to truly understand and remember the outcomes of sums and products of rational and irrational numbers.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/
Operations with Rational and Irrational Numbers http://tasks.illustrativemathematics.org/content-standards/HSN/RN/B/3/tasks/690

Relevance to families and communities:

During a unit focused on why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the student interests, how they are able to explore their interests and make connections that provide meaning to their interests can help students learning occurs from the ability to make connections and making sense of how it connects to their world.

Cross-Curricular Connections:

Science: Two irrational numbers that are of great importance in physics are e and π. Consider providing a connection for students to explore irrational numbers in this context, and the fact that whenever we compute a number answer we must use rational numbers to do it, most generally a finite-precision decimal representation.

Social Studies: In high school the New Mexico Social Studies Standards state students should explain and analyze "tension and cooperation between religion and new scientific discoveries". Consider providing a connection for students to learn about Hippias's who was rumored to have been murdered for divulging the existence of irrational numbers.