NEW MEXICO
Public Education Department

HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: A: Perform arithmetic operations on polynomials.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *Note: Algebra 1 focused on linear and quadratic, Algebra 2 focuses on polynomials beyond quadratic.*	SMP 2: Students can reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations.	• Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. • describe the similarities between the set of integers and the system of polynomials. • add, subtract, and multiply polynomials. • determine whether a set or system is closed under a given operation **Webb's Depth of Knowledge:** 1 **Bloom's Taxonomy:** Remember, Understand

<u>**Previous Learning Connections**</u>	<u>**Current Learning Connections**</u>	<u>**Future Learning Connections**</u>
• Connect to applying the properties of <u>integer</u> exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **(8.EE.A.1)**	• Connect to using the properties of operations to write expressions in different but equivalent forms. **(HSA.SSE.A.2)**	• Connect to performing operations with rational expressions **(HSA.APR.7)** • Connect to deriving the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. *For example, calculate mortgage payments.* **(HS.A.SSE.B.4)**

Clarification Statement

HSA.APR.A.1: The development of **polynomials** and **rational expressions** in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions for the same numbers 8 + 3 and 11, or 3/4 and 0.75, as referring to different entities: 8 + 3 might be seen as describing a calculation and 11 is its answer; 3/4 is a fraction and 0.75 is a decimal. They come to understand that these different expressions are different names for the same numbers, that properties of operations allow numbers to be written in different but **equivalent forms**, and that all of these numbers can be represented as points on the number line. In middle grades, they come to see numbers as forming a unified system, the number system, still represented by points on the number line. The whole numbers expand to the integers—with extensions of addition, subtraction, multiplication, and division, and their properties. Fractions expand to the rational numbers—and the four operations and their properties are extended. A similar evolution takes place in algebra. At first algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing. As they see polynomial expressions as quantities rather than operations to be performed, they can perform operations such as adding, subtracting and multiplying two polynomials and identify that these operations will yield another polynomial, thus making the system of polynomials closed.

Common Misconceptions

- Students might think polynomials are only monomial, binomial, or trinomial.
- Students may not confuse the impact of adding and subtracting polynomials on the degree of the variable.
- Students may not fully distribute the multiplication of polynomials and only multiply like terms.
- When adding and multiplying like terms students may initially confuse $x + x$ as x^2 instead of $2x$.
- Students may not think $x^2 \cdot x = x^3$ is not an example of closure for polynomial multiplication since the result has a different exponent than the factors.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying arithmetic operations on polynomials because the structure of the four basic operations hold true for arithmetic operations on polynomials.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.EE.A.1: This standard provides a foundation for work with arithmetic operations on polynomials because the student must know and apply the properties of integer exponents. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because the student can bridge conceptual ideas within a familiar context to procedural mechanics of the cluster.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing prompts that guide learners in when and how to ask peers and/or teachers for help because as students perform arithmetic operations on polynomials they can have scripted questions to focus the intent of support provided.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because the structure of the four basic operations holds true for polynomials as it does for integers.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills because integer arithmetic and polynomial arithmetic are related and the student becomes fluent in the latter as (s)he is gradually released.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with performing arithmetic operations on polynomials benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because students use connections with arithmetic of polynomials and functions to explore equivalent expressions and create functions that meet specific conditions.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on arithmetic operations on polynomials by providing specific feedback to students on their work through a short mini-lesson because looking at integer rules for arithmetic operations apply directly to arithmetic operations with polynomials.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on arithmetic operations on polynomials by confronting student misconceptions because integer rules concerning positives and negatives are common errors that lead to misconceptions when performing arithmetic operations on polynomials.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying arithmetic operations on polynomials because addition and subtraction are inverse operations as are multiplication and division.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying performing arithmetic operations on polynomials, the types of mathematical tasks are critical because they build on prior knowledge of arithmetic operations. Time spent on conceptual understanding of the four basic operations (addition, subtraction, multiplication, division) using integers can bridge to the conceptual understanding of those operations of polynomials. From here time can be spent on procedural fluency of the mechanics of the operations with polynomials. As students will be expected to demonstrate proficiency on End of Course exams and the SAT in English, an opportunity presents itself to bridge home language to the language of these exams.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 5094624

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■□	Passport to Advanced Mathematics	Equivalent expressions	2. Fluently add, subtract, and multiply polynomials.	No Calculator

Which of the following is equivalent to the sum of $3x^4+2x^3$ and $4x^4+7x^3$?

A. $16x^{14}$

B. $7x^8+9x^6$

C. $12x^4+14x^3$

D. $7x^4+9x^3$

Rationale

Choice D is correct. Adding the two expressions yields $3x^4+2x^3+4x^4+7x^3$. Because the pair of terms $3x^4$ and $4x^4$ and the pair of terms $2x^3$ and $7x^3$ each contain the same variable raised to the same power, they are like terms and can be combined as $7x^4$ and $9x^3$, respectively. The sum of the given expressions therefore simplifies to $7x^4+9x^3$.
Choice A is incorrect and may result from adding the coefficients and the exponents in the given expressions. Choice B is incorrect and may result from adding the exponents as well as the coefficients of the like terms in the given expressions. Choice C is incorrect and may result from multiplying, rather than adding, the coefficients of the like terms in the given expressions.

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on performing arithmetic operations on polynomials, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, in becoming a critical thinker and problem solver. Students take a skill familiar to them (arithmetic operations with integers) and apply it to something new, arithmetic operations on polynomials. This unit practices learning something new from existing knowledge.	History: The history of exponents dates back many centuries and Euclid are credited with the first known usage of exponents. He used the term 'power' to represent what we know today, how many times a number is multiplying by itself. The ancient Greek mathematicians used, and many other mathematicians added onto the use of exponents as they learned more about their use. Archimedes generalized the same idea of powers and later mathematicians in the Islamic golden age utilized powers of two and three in their work in algebra. In our project, you will see many other mathematicians and their contributions to the development of exponents from the 14th century up to the use of exponents today. https://www.sutori.com/story/history-of-exponents--wNbwYExXdzFNYPh1zFUYhbDc