NEW MEXICO Public Education Department

HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: B: Understand the relationship between zeros and factors of polynomials.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.APR.B.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	SMP 3: Students can construct viable arguments and critique the reasoning of others by explaining in their own words what it means to factor an expression, what a zero of an equation represents and how it relates to its graph. Students will be able to explain how the quotient and remainder of a polynomial division problem are related.	• Define the Remainder Theorem. • Use the Remainder Theorem to show the relationship between a factor and a zero. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	SMP 3: Students can construct viable arguments and critique the reasoning of others by explaining in their own words what it means to factor an expression, what a zero of an equation represents and how it relates to its graph. Students will be able to explain how the quotient and remainder of a polynomial division problem are related.	• Use polynomial identities to prove numerical relationships. • Determine the degree of a polynomial and the number of possible zeros of that polynomial. • Simplify polynomials into factored forms. • Identify the zeros of the polynomial using the factors. • Plot the zeros of the polynomial on a graph. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Apply, Analyze
Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• Connect to factoring and completing the square and using the Remainder Theorem (standards A-APR.B.2, F-IF.C.8.a)	• Connect to calculating the zero in the Remainder Theorem or by factoring to graph the zeros of a polynomial function (standards A-APR.B.2, A-APR.B.3)	• Connect to graphing key features of polynomial functions to identifying zeros and sketching a graph (standards F-IF.C.7)

Clarification Statement
The zeros of a polynomial are turned into linear factors and can be used to factor polynomials of any power. The degree of a polynomial will indicate the maximum number of zeros of the polynomial.

Common Misconceptions

- Division problems never have a remainder; it is okay to write R-value.
- Students often forget to distribute the –1 which is equivalent to subtraction, to terms inside the parenthesis.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division. When listing the coefficients, there may be missing degrees and students will forget to write a zero.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying understanding the relationship between zeros and factors of polynomials because when using the Remainder Theorem students must use the opposite sign of the factor in the dividend. Students must also know how to factor polynomials when the leading coefficient is equal to 1 and not equal to 1 which can lead to common misconceptions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- A.SSE.B.3.A and A.APR.B.6: These standards provide a foundation for work with understanding the relationship between zeros and factors of polynomials because students will be producing equivalent forms of polynomials to reveal properties of the expression (in this case the factored form will reveal zeros; the Remainder Theorem can be used instead of long division to check if a factor is a zero of the expression). . If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with APR.B benefit when learning experiences include ways to recruit interest such as providing contextualized examples to their lives because finding the zeros of a polynomial can be bridged in context to a socially or culturally relevant prompt.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because the learner can use specific supports (eg. graph to check zeros, substitution to check zeros) to encourage self-awareness.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because finding and checking zeros of polynomials can be found by factoring/ using Remainder Theorem and checked graphically.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because polynomials can be factored using different strategies depending on the structure of the polynomial.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with understanding the relationship between zeros and factors of polynomials benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because the learner can link skills learned with factoring quadratics to unfamiliar polynomials of higher degree.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on understanding the relationship between zeros and factors of polynomials by critiquing student approaches/solutions to make connections through a short mini-lesson because zeros of polynomials must match the graph of the polynomial. By critiquing other students work, the learner can immediately make connections to the correctness of the work by observing a graph.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on understanding the relationship between zeros and factors of polynomials by offering opportunities to understand and explore different strategies because investigating graphs to identify zeros and using area models to factor polynomials can offer structure to the student as a starting point.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying understanding the relationship between zeros and factors of polynomials because some learners may be ready to factor more difficult polynomials.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

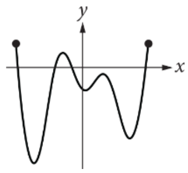
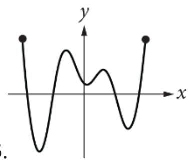
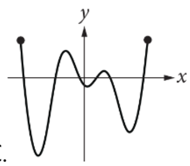
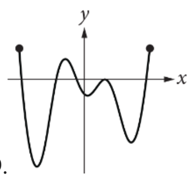
Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding the relationship between zeros and factors of polynomials the use of mathematical representations within the classroom is critical because graphing technology and area models can be used to factor polynomials and check the zeros of those factors. Mathematics can be designed in a context to connect home culture or interests in a way that a polynomial function could represent a quantity where its solution(s) could represent a critical value within the context. For example, a cubic function could represent the profit of a fundraiser given the cost of a ticket for the fundraiser. The zero(s) of the function would represent the break-even point. The context of the fundraiser could be framed around a cultural interest. A graph could be used to show a representation of the function to support the learner in bridging different representations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

Source: SAT https://satsuitequestionbank.collegeboard.org/results

20079	■■□	Passport to Advanced Math	Passport to Advanced Mathematics	Nonlinear functions	3. For a factorable or factored polynomial or simple rational function, b. understand and use the fact that for the graph of y = f(x), the solutions to f(x) = 0 correspond to x-intercepts of the graph and f(0) corresponds to the y-intercept of the graph; interpret these key features in terms of a context;	Calculator

If the function f has five distinct zeros, which of the following could represent the complete graph of f in the xy-plane?

- A. 
- B. 
- C. 
- D. 

Rationale

- Choice D is correct. A zero of a function corresponds to an x-intercept of the graph of the function in the xy-plane. Therefore, the complete graph of the function f, which has five distinct zeros, must have five x-intercepts. Only the graph in choice D has five x-intercepts, and therefore, this is the only one of the given graphs that could be the complete graph of *f* in the xy-plane.
- Choices A, B, and C are incorrect. The number of x-intercepts of each of these graphs is not equal to five; therefore, none of these graphs could be the complete graph of f, which has five distinct zeros.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding the relationship between zeros and factors of polynomials, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, students practice using mathematical tools to solve new problems. Students learn to identify what tools are at their disposal when connecting the zeros of a polynomial to its factors. This practices them for life outside the math classroom by providing skills that are lifelong.	Science: This approach is extended to a spherical body rolling on a curved path. Assuming that a curved path can be approximated by a sequence of many very short inclines, the problem is approached as a body rolling on this sequence of inclines, solving each with the work-energy theorem. Defining the curved path as a differentiable function, the slope of each incline is obtained through the function Teach Engineering Roller Coaster - Spherical Body rolling Social Studies: Not much is really known about the Pythagoreans or their rather mysterious founder, Pythagoras. Several different accounts of the Pythagoreans have come down to us from antiquity. Plato and Aristotle both reference the Pythagoreans throughout their philosophical writings. Even still, the true nature of the "cult of Pythagoras" is often shrouded in mystery. Pythagorious and his followers were a mystical society that placed great importance on the mathematical relations of the universe. There is no denying that they contributed greatly to the

	area of mathematics and philosophy. One needs only to reflect on the Pythagorean theorem, a mathematical principle said to have been discovered by Pythagoras himself, to appreciate the profound impact they had on the development of scientific thought. https://classicalwisdom.com/philosophy/cult-of-pythagoras/