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Public Education Department

HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: C: Use polynomial identities to solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSA.APR.C.4: Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	SMP 7: Students look for and make use of structure when given a polynomial function in factored form. Students will be able to find the zeros, plot the zeros and then make a sketch of the graph of that is reflective of the function (and its other key features). SMP 8: Students look for and express regularity in repeated reasoning when connecting the representations of functions on the coordinate plane with their zeros and factored forms.	• Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc. • Prove polynomial identities by showing steps and providing reasons. • Illustrate how polynomial identities are used to determine numerical relationships. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply
<u>Previous Learning Connections</u> • Students are building on their knowledge of zeros and factors of quadratics learned in Algebra 1.	**<u>Current Learning Connections</u>** • Students are learning about factoring with polynomials of degrees higher than 2 (perfect cubes, quadratics, factor by grouping, etc). Students are also understanding that not all polynomials are factorable, but still can by divided by another polynomial. Students continue to build their understanding of how factored form relates to zeros on a graph. Later in the year, these skills are used in simplifying rational expressions.	**<u>Future Learning Connections</u>** • In 4th year math (Pre-Calculus, Calculus, and college level math) students will build on their factoring skills (with rational expressions and trigonometric expressions). Students will also determine zeros of trigonometric functions in subsequent math courses.

Clarification Statement
Students make systematic lists of all arrangements and count the number of unique subgroups. Students use prior knowledge of counting techniques to calculate the number of combinations.

Common Misconceptions
- There are no y-axis zeros.
- Easily get lost with the different coefficients and degrees.
- Multiplying the degrees.

- Students might incorrectly expand binomial expressions by choosing the wrong row of coefficients in Pascal's triangle. Remind students that for an exponent of n, choose row n of Pascal's triangle. Row n will be the row with the value n as the second entry.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., Pascal's Triangle, the Binomial Theorem) when studying the use of polynomial identities to solve problems because there are structures that exist to make expanding binomials more efficient and effective.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- A.SSE.A.2: This standard provides a foundation for work with the use of polynomial identities to solve problems because students have looked at the structure of an expression to identify ways to rewrite it. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences include ways to recruit interest such as providing time for self-reflection about the content and activities because the learner can reflect on how the Binomial Theorem connects to expanding polynomials of higher degrees.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as providing alternatives in the mathematics representations and scaffolds because the learner can use area models as an alternative to the expansion of binomials.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because area models, although not efficient for higher degree binomials, can still be used to expand binomials of higher degrees.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to the multiple ways' students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because the student can use the Binomial Theorem or area models but expand binomials.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with using polynomial identities to solve problems benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as incorporating explicit opportunities for review and practice because although the power of the binomial is a positive integer, the coefficients of the binomial terms may be positive or negative.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on the use of polynomial identities to solve problems by revisiting student thinking through a short mini-lesson because looking at other students' work can support all learners in understanding the structure of the Binomial Theorem. Learners listening to their peers explain their thinking can benefit all as student thinking is delivered in student friendly language.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on the use of polynomial identities to solve problems by offering opportunities to understand and explore different strategies because some students will insist on using area models or the FOIL method for expanding a binomial regardless of the power of the binomial. As the Binomial Theorem will be more effective and efficient to expand certain binomials, pockets of students are afforded the opportunity to explore and apply previous strategies.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various strategies when studying the use of polynomial identities to solve problems because some learners can investigate and explain when it would be more appropriate to use an area model, the FOIL method, or the Binomial Theorem when expanding binomials.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying the use of polynomial identities to solve problems the types of mathematical tasks are critical because, for example, students' familiarity of structure can support the expansion of binomials. When squaring a binomial, students have a working knowledge of area models and the FOIL method. As the power of a binomial grows, these methods break down and become messy. The student can then decide when it would be more efficient to use the Binomial Theorem and Pascal's Triangle to expand a binomial. Conceptual understanding of expanding binomials will lead to procedural fluency as the student decides what method works best for their learning style.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: Illustrative Mathematics

Felicia notices what appears to be an interesting pattern between powers of 11 and powers of x+1:

$$11^0 = 1 \qquad (x+1)^0 = 1$$
$$11^1 = 11 \qquad (x+1)_1 = x+1$$
$$11^2 = 121 \qquad (x+1)^2 = x^2+2x+1$$

- The digits of the number 11n are the same as the coefficients of the polynomial $(x+1)^n$. Is this always true?
- Does this pattern continue for n=3 and n=4?
- What is the answer to Felicia's question?

IM Commentary
This task has students combine polynomial arithmetic with pattern-matching. Students can expand powers of x+1 using either repeated multiplication (A-APR.1) or by the binomial theorem (A-APR.5), and then are asked to analyze the question of whether the similarity of coefficients with the digits of powers of 11 is a coincidence. Identifying patterns, as Felicia has done, is an important part of mathematics. In this case, there is a deep relationship between the numbers and polynomials that Felicia is investigating; on the other hand, further consideration shows that the pattern does not continue. It is important for students not only to identify patterns but also to look more deeply to understand whether or not the patterns are ''generalizable'' or true because of some essential mathematical structure.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on the use of polynomial identities to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, the student makes	In this activity, students relate the graph of a rational function to the graphs of the polynomial functions of its numerator and denominator. Students graph these polynomials one at a time and identify their y-intercepts and zeros. Asymtotes and Zeros of Rational Functions: Algebra 2

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use of structure and decides on a method to expand binomials that is effective and efficient and makes sense to them. The Standards for Mathematical Practice come alive as they use polynomial identities to solve problems as the bridge to perseverance and making use of structure and repeated reasoning.	