NEW MEXICO
Public Education Department

HS: ALGEBRA- ARITHMETIC WITH POLYNOMIALS & RATIONAL EXPRESSIONS

Cluster Statement: D: Rewrite rational expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

<table>
<tr>
<td>Standard Text

HSA.APR.D.6: Rewrite simple rational expressions in different forms;
write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$,
where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.</td>
<td>Standard for Mathematical Practices

SMP 3: Students construct viable arguments and critique the reasoning of others by explaining the steps in creating equivalent forms of rational expressions, including identifying the quotient and the remainder as a fraction with the divisor as the denominator.

SMP 5: Students use appropriate tools strategically when recognizing which method is appropriate to use in a variety of situations.

SMP 7: Students look for and make use of structure when gaining procedural fluency and conceptual understanding of how and why to rewrite rational expressions as quotients and remainders.</td>
<td>Students who demonstrate understanding can:
• Divide polynomials using long division.
• Divide polynomials using synthetic division.
• Relate the algorithm of dividing multi-digit integers with polynomial long division.
• Perform partial fraction decomposition.
• Determine the quotient and remainder of rational expressions using inspection, long division, and/or a computer algebra system.

Webb’s Depth of Knowledge: 1-2

Bloom’s Taxonomy:
Understand, Apply</td>
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<td><u>Previous Learning Connections</u>
• Students are building on their knowledge of factors of quadratics learned in Algebra 1.</td>
<td><u>Current Learning Connections</u>
• Students will use the skills learned to factor and divide polynomials to simplify rational expressions.</td>
<td><u>Future Learning Connections</u>
• Students will be able to perform all operations with rational expressions.</td>
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Clarification Statement

Rational expressions can be rewritten using properties of fractions and elementary numerical algorithms.

Common Misconceptions

- Students may forget to write the polynomial in descending order.
- Students may not recognize a missing term in the divisor or dividend and forget to insert a zero for the missing term.
- Students might make errors in signs when doing synthetic division and synthetic substitution because values are added rather than subtracted as in long division. Remind them that terms are always added for synthetic substitution and synthetic division.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying rewriting rational expressions because arithmetic operation with polynomials and factoring will be used when rewriting rational expressions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- A.SSE.A.2: This standard provides a foundation for work with rewriting rational expressions because students use the structure of an expression to identify ways to rewrite it. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences include ways to recruit interest such as creating accepting and supportive classroom climate because peers might take different paths in rewriting rational expressions, all of which are acceptable methods.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because rewriting rational expressions may have multiple entry points depending on how the rational expression is structured.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because tools such as factoring, synthetic division, and operations with fractions might lend themselves to be efficient and effective in rewriting rational expressions.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because the learner might not know when a rational expression has been fully rewritten as a simplified form.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with rewriting rational expressions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because rewriting rational expressions involves being fluent in fraction operations which has been a traditional frustration with students.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on rewriting rational expressions by providing specific feedback to students on their work through a short mini lesson because an expression might not be fully simplified. Students might not have applied a full set of mathematical properties to rewrite a rational expression and may benefit from focused feedback on where to go next in their work.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit rewriting rational expressions by confronting student misconceptions because the student might rewrite a rational expression incorrectly and simplify a polynomial incorrectly (e.g., (x+ y)2 = x2+ y2) or might have factored a polynomial incorrectly.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying rewriting rational expressions because some students will be ready for more complex rational expressions with more complex terms than others. Pockets of students can be paired homogeneously by ability to work on more complex rational expressions to explore in greater depth.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

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Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying rewriting rational expressions, the use of mathematical representations within the classroom is critical because students are in fact rewriting simple rational expressions as equivalent representations. Students are asked to draw on their mathematical competence by simplifying rational expressions previously learned within this standard domain. For example, a student might have to factor a numerator and/or denominator to simplify a rational expression. Or a student might have to perform synthetic division to rewrite a rational expression as an equivalent representation. These skills could build a bridge for students to position them as competent and capable mathematicians and leverage further study of mathematics.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: SAT

Which of the following is equivalent to $\frac{4x^2+6x}{4x+2}$?

- A. x
- B. $x+4$
- C. $x-\frac{2}{4x+2}$
- D. $x+1-\frac{2}{4x+2}$

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on rewriting rational expressions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, learners look for and make use of structure and make sense of problems and persevere in solving them. As students rewrite rational expressions in equivalent forms, they are building confidence in taking what they know to apply to problem solving scenarios. These Mathematical Practices skills exist outside of school as the student builds their critical thinking skills through rewriting rational expressions.	Medicine and Analytical Chemistry: MRI and NMR Spectroscopy involves Fast Fourier Transformation that allows the creation of images from the "ringing" after the atoms are subjected to radio waves in strong magnetic fields. The Fourier series consists of terms of increasing orders. (An Algorithm for the Machine Calculation of Complex Fourier Series by James W. Cooley and John W. Tukey)