NEW MEXICO
Public Education Department

HS: ALGEBRA- REASONING WITH EQUATIONS & INEQUALITIES

Cluster Statement: A: Understand solving equations as a process of reasoning and explain the reasoning.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	SMP 6: Students attend to precision when determining true solutions (not extraneous).	• Determine the domain of a rational function. • Determine the domain of a radical function. • Solve radical equations in one variable. • Solve rational equations in one variable. • Explain and give examples how extraneous solutions may arise when solving rational and radical equations. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Apply, Analyze
Previous Learning Connections • In Algebra 1, students solved linear and quadratic equations.	**Current Learning Connections** • In this course, students will extend their solving skills learned in Algebra 1 to rational and radical equations. Students will also relate the solving of other nonlinear equations learned in this course to solving rational and radical equations.	**Future Learning Connections** • In future math classes students will solve more challenging nonlinear equations, including trigonometric equations.

Clarification Statement
- This cluster builds on the framework of solving equations and extends it to rational and radical equations (and the knowledge of extraneous solutions).
- Equations are solved as a process of reasoning using properties of operations and equality, which can justify each step of the process. Students solve simple rational and radical equations using a variety of methods and explain why and where in the solution process the extraneous solution arose.

Common Misconceptions
- Students may struggle identifying when there is an extraneous solution.
- Struggle with expression versus equation.
- When students multiply or divide both sides of an inequality by a negative value, they forget to reverse the inequality symbol.
- Students may sometimes forget to consider the cases when the LCD is positive and negative when solving a rational inequality algebraically,

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying solving equations and explaining each step because students may need to justify the inverse operation used in each step with viable arguments. Students may practice expressing their mathematical thinking verbally and symbolically.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work with reasoning and solving one-variable equations because students need to understand each step of solving one-variable equations and explain the reason for each step. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with understanding and explaining the reasoning of solving equations benefit when learning experiences include ways to recruit interest such as providing choices in their strategies of solving equations and in their reasoning because students make connections of their prior knowledge of solving equations in different problems. By showing a different order of applying the inverse operations to the equations, students gain new skills and knowledge of solving complex equations and deeper understanding of solving equations.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with explaining reasoning of each step of solving equations benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities because students engage in meaningful discourse to construct viable arguments with the support of the cooperative learning group. Students justify and make connections with reasoning of other strategies used by other learners in the cooperative learning groups.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with explaining the reasoning of each step of solving equations and constructing viable argument benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because students build their reasoning and viable argument using their prior knowledge of solving one-step or two-step equations. Students connect their understanding of inverse operation to justify each step of solving equations.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with explaining the reasoning of solving equations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because students justify solving the equations in multiple ways and communicate their mathematical thinking verbally and symbolically. By presenting their mathematical thinking in multiple ways, students make connections of conceptual knowledge and gain fluency in procedural knowledge.

Internalize

Comprehension: *How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with explaining the reasoning of solving equations with viable arguments benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply the knowledge of solving one-variable equations to solving literal equations. Students identify the patterns of solving equations and make generalization of solving and rearranging equations.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on explaining the reason of each step of solving equations by critiquing student approaches/solutions to make connections through a short mini-lesson because students need to understand why the specific inverse operation is used and develop the viable argument using properties of equality.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit explaining the steps of solving equations by offering opportunities to understand and explore different strategies because students need to understand why some steps are interchangeable when solving the equations. Students need to explain the order of applying the inverse operations and how that relates to the order of operation of the equations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying solving complex equations and explaining the steps because students may deepen their understanding of inverse operation, such as logarithm as the inverse operation of exponent. Students explore strategies of solving equations with complex operations and justify their reason in cooperative learning groups.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying understanding solving equations as a process of reasoning and explaining the reasoning the use of mathematical representations within the classroom is critical because linguistically and culturally speaking, experiences of students provide different and varied types of representations for solving mathematical problems varies on different types of exposures they have in their home and communities. Students' abilities in explaining mathematical reasoning varies on their language preferences and representations.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: An Extraneous Solution Illustrative Mathematics

Megan is working solving the equation

$$\frac{2}{x^2-1} - \frac{1}{x-1} = \frac{1}{x+1}.$$

She says

If I clear the denominators I find that the only solution is $x = 1$ but when I substitute in $x = 1$ the equation does not make any sense.

a. Is Megan's work correct?

b. Why does Megan's method produce an x value that does not solve the equation?

IM Commentary
The goal of this task is to examine how extraneous solutions can arise when solving rational equations. The task presents an operation, "clearing denominators," which appears to lead to a contradiction. To resolve the contradiction, we examine more carefully what is happening when we clear denominators (MP6). One way to describe the process is that we find a common denominator for both sides and set the numerators equal to each other. This gives solutions to the original equation provided the solutions are in the domain of the rational functions on both sides, that is, provided they are not zeros of one or more of the denominators. In this case, the solution x=1 makes the numerators equal to one another but also makes the denominators of two of the expressions zero, and so x=1 is an extraneous solution.

Another way to think about the process is to connect it to the familiar process students have learned from solving linear equations of multiplying both sides by a non-zero constant to get an equivalent equation. In this case, clearing denominators amounts to multiplying both sides by x2−1. The problem is that when x=1 or x=−1 this is multiplying both sides by zero. So, the operation only produces an equivalent equation if you stay away from those two values, and consider them separately at the end of the process. Students may also experiment with graphs of the functions on the left hand and right-hand side of the equation to see that they are never equal. This confirms Megan's work which shows that if the two expressions are equal to one another then x=1, but this is not possible.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on understanding solving equations as a process of reasoning and explaining the reasoning, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, some students learned differently when it comes to expressing their ideas linguistically speaking they learned different languages. Learning about the different structures for the process and explaining it across the languages in the classroom can lead to a more robust understanding of solving equations for all students by making connections to the different structures of understanding solving equations in other languages.	This lesson uses right triangle trigonometry and a rational function to explore the percent of your visual field that is occupied by the area of a television. Sofa Away From Me: A Lesson by Mathalicious