NEW MEXICO
Public Education Department

HS: ALGEBRA- REASONING WITH EQUATIONS & INEQUALITIES

Cluster Statement: D: Represent and solve equations and inequalities graphically.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.REI.D.11: Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* *Note: Algebra 1 focuses on linear and exponential. In Algebra 2 the focus is on combining polynomial, rational, radical, absolute value, and exponential functions.*	SMP 3: Students can construct viable arguments by explaining how the x-coordinate of a solution to the system *y = f(x)* and *y = g(x)* solves *f(x) = g(x).* SMP 5: Students can use tools by finding solution(s) of system of equations from graph or tables. SMP 7: Students look for and make use of structure by explaining in their own words how and when a solution is given as a point (x, y) versus a value (x = a).	• Recognize what the solution *y = f(x)* and *y = g(x)* means on a graph. • Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. • Find approximate solutions for the system $y = f(x)$ and $y = g(x)$ using graphs and tables. • Find successive approximations and use them to solve the system $y = f(x)$ and $y = g(x)$. • Use paper/pencil or technology to produce a table of values. • Explain what x-coordinate of a common ordered pair represents in the context of the problem. **Webb's Depth of Knowledge:** 1-3 **Bloom's Taxonomy:** Understand, Apply, Analyze, Evaluate
Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
• Connect to the work of Algebra 1 in this cluster around linear and exponential. **(HSA.REI.D)**	• Connect this cluster across all of Algebra 2, particularly when each new function is presented.	• Connect this cluster to future work around determining specific solutions to new functions (i.e. zeros). Also, in Calculus, when students discuss the area between two curves and volume with rotation.

Clarification Statement

HSA.REI.D.11: Just as the **algebraic** work with **equations** can be reduced to a series of algebraic moves unsupported by reasoning, so can the **graphical visualization of solutions**. The simple idea that an equation f(x) = g(x) can be **solved (approximately)** by graphing y = f(x) and y = g(x) and finding the **intersection points** involves a number of pieces of conceptual understanding. [This method] seeks to convert an equation in one variable, f(x) = g(x), to a s**ystem of equations** in two **variables**, y = f(x) and y = g(x), by introducing a second variable y and **setting it equal** to each side of the equation. If x is a solution to the original equation, then f(x) and g(x) are equal, and thus **(x, y)** is a solution to the new system.

Common Misconceptions

- Students often interpret the solutions to an equation or graphical representation of an equation as only integer values.
- Students may believe an estimate of a value between two integer points is sufficient, but the standard states that students should find successive approximations to approximate the solution.
- Students believe the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying represent and solve equations and inequalities graphically because different representations within the same problem, students make the connection between the graph, table, word problem, and equations. When two expressions are equal to each other, the variable equal to a numerical value is the solution algebraically and graphically.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 6.EE.B.5: This standard provides a foundation for work to represent and solve equations and inequalities graphically because substituting numerical values into an equation to determine if the equation is true, the student will comprehend that the answer is a solution. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: *How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features give an example connected to this standard such as the size of text, images, graphs, tables, or other visual content; contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print material because teachers and learners should work together to attain the best match of features to learning needs.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because students persist on finding the solutions of equations and inequalities using the graphs constructed. Students check and interpret the solutions in the context of the problem to make sense of their mathematical thinking.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate because students understand the multiple representations of the solutions and make connections to the graphs that represents the equations and inequalities. Students connect the multiple representations to make sense of the meaning of the solution in the context of the problems.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because students use multiple ways, including graphing calculators and graph paper, to construct the graphic representations of the equations and inequalities. Students compare and verify the solutions from different representations to defend the solutions.

Internalize

Comprehension: *How will the learning for students' support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?*

- For example, learners engaging with representing and solving equations and inequalities graphically benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing explicit, supported opportunities to generalize learning to new situations because students apply their interpretation and knowledge of the solutions to new problems in the context of the situation. Students also extend their knowledge of solving equations and inequalities graphically to solving equations and inequalities algebraically.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on represent and solve equations and inequalities graphically by critiquing student approaches/solutions to make connections through a short mini-lesson because connections between solution (algebraically) and intersection (graphically) are equivalent. When students compare answers graphically and algebraically, intersections are the solution.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit to represent and solve equations and inequalities graphically by offering opportunities to understand and explore different strategies because interpreting solution using the different representations allows the students to visualize the answer that was only written as a system of equations or two expressions equal to each other. Students can check their work graphically to confirm their answer.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying represent and solve equations and inequalities graphically because different types of equations (logarithmic, exponential, trigonometric, etc) can use the graphing method to find solutions to word problems or algebraically.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Using and Connecting Mathematical Representations: The standard for mathematical practice, use appropriate tools strategically, provides a strong foundation to validate and bridge for students. Mathematical representations are mathematical tools. The linguistic and cultural experiences of students provide different and varied types of representations for solving mathematical problems. By explicitly encouraging students to use multiple mathematical representations students can draw on their "mathematical, social, and cultural competence". By valuing these representations and discussing them we can connect student representations to the representations of school mathematics and build a bridge for students to position them as competent and capable mathematicians. For example, when studying represent and solve equations and inequalities graphically the use of mathematical representations within the classroom is critical because students given a situation in two variables and they must find the value of one variable given the value of the other, create an equation to represent the situation, use technology to create a graph, and interpret each representation. Understanding how lines and tables represent solution sets of linear relationships will help students make sense of graphs of and solutions to linear inequalities, and later, to make sense of solutions to systems of linear equations in their Algebra 1 class.

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Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: SAT

$$x+1=\frac{2}{x+1}$$

Question 1474935 Answers

In the equation above, which of the following is a possible value of $x+1$?

- A. $1-\sqrt{2}$
- B. $\sqrt{2}$
- C. 2
- D. 4

[How Many Solutions?]

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on representing and solving equations and inequalities graphically, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, bringing in the different languages spoken in the home and connecting it to the tools available to translate different languages, i.e. Google translate, closed captions on televisions, etc. make connections that show that in the culture of mathematics, tools are used to translate mathematics and help us make sense of what we are seeing.	Students will model projectile motion in both function and parametric graphing. This was designed as an in-class modeling activity to be used prior to actually launching air-powered projectile rockets. A set of data is given in a spreadsheet and students create model functions using a variety of methods: vertex form (then using grab and move to fit the curve to the data points), standard form (using matrices), and quadratic regression. [Rocket Simulation Activity]