

## HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

**Cluster Statement:** A: Interpret the structure of expressions.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers.

*Note, the A-SSE domain is especially important in the high school content standards overall as a widely applicable prerequisite.*

<p><b>Standard Text</b></p> <p><b>HSA.SSE.A.1: Interpret expressions that represent a quantity in terms of its context.</b></p> <ul style="list-style-type: none"> <li><b>HSA.SSE.A.1.A: Interpret parts of an expression, such as terms, factors, and coefficients.</b></li> <li><b>HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</b></li> </ul> <p><i>Note: Algebra 2 focuses polynomial and rational.</i></p>	<p><b>Standard for Mathematical Practices</b></p> <p>SMP 4: Students can model with mathematics by identifying the meaning of the terms, factors, and coefficients of polynomial and rational expressions in context.</p> <p>SMP 7: Students can look for and make use of structure in expressions by seeing how the structure of an algebraic expression reveals properties of the function it defines.</p>	<p><b>Students who demonstrate understanding can:</b></p> <ul style="list-style-type: none"> <li>Identify how parts of an expression relate to a real-world situation.</li> <li>Interpret how parts of an expression relate to a real-world situation.</li> <li>Interpret algebraic expressions that describe real-world scenarios, including parts within an expression and using grouping strategies to interpret expressions.</li> </ul> <p><b>Webb’s Depth of Knowledge:</b> 1-2</p> <p><b>Bloom’s Taxonomy:</b> Remember, Understand, Analyze</p>
<p><b>Standard Text</b></p> <p><b>HSA.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</b></p> <p><i>Note: Algebra 2 focuses polynomial and rational.</i></p>	<p><b>Standard for Mathematical Practices</b></p> <p>SMP 3: Students can construct viable arguments by explaining whether expressions are equivalent using mathematical justifications.</p> <p>SMP 8: Students look for and express regularity in repeated reasoning by classifying expressions.</p>	<p><b>Students who demonstrate understanding can:</b></p> <ul style="list-style-type: none"> <li>Identify patterns of factoring.</li> <li>Factor a polynomial or rational expression.</li> <li>Classify expressions by method of factoring.</li> <li>Apply different algebraic properties to an expression to produce an equivalent form.</li> </ul> <p><b>Webb’s Depth of Knowledge:</b> 1-2</p> <p><b>Bloom’s Taxonomy:</b> Remember, Understand, Apply</p>

<p><b><u>Previous Learning Connections</u></b></p> <ul style="list-style-type: none"> <li>• Connect to the work with linear, quadratic, and exponential expressions in Algebra 1 (HSA.SSE.A)</li> <li>• Connect to rewriting quadratic functions to find specific key features. (HSA.SSE.B.3)</li> </ul>	<p><b><u>Current Learning Connections</u></b></p> <ul style="list-style-type: none"> <li>• Connect to rewriting formulas to highlight quantities of interest. (HSA.CED.4)</li> </ul>	<p><b><u>Future Learning Connections</u></b></p> <ul style="list-style-type: none"> <li>• Connect to work with expressions of all function types.</li> </ul>
<p><b>Clarification Statement</b></p> <ul style="list-style-type: none"> <li>• HSA.SEE.A.1: Algebra 1 emphasized linear, exponential and quadratic expressions. The work of Algebra 2 is to generalize that work to polynomial and rational expressions by examining real-world situations that can be modeled by algebraic expressions and explaining how parts of the expression describe different aspects of the situation.</li> <li>• HSA.SEE.A.2: Seeing structure in expressions entails a dynamic view of an algebraic expression, in which potential rearrangements and manipulations are ever present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which not. Emphasize that there are many algebraic properties that can be used to write equivalent forms of an expression. Complex, linear and non-linear equations need to be addressed.</li> </ul>		
<p><b>Common Misconceptions</b></p> <ul style="list-style-type: none"> <li>• Students may confuse equations with expressions. The focus in this cluster is on analyzing expressions.</li> <li>• Students may confuse the order of operations when they simplify an expression.</li> <li>• Students may not have a conceptual basis for patterns and therefore struggle to recognize and apply them to new situations.</li> </ul>		
<p><b>Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies</b></p> <p><b>Pre-Teach</b></p> <p>Pre-teach (targeted): <i>What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?</i></p> <ul style="list-style-type: none"> <li>• For example, some learners may benefit from targeted pre-teaching that rehearses new mathematical language when studying to interpret the structure of expressions because with the new vocabulary terms, it can get confusing. Students need to differentiate between expression and equation.</li> </ul> <p>Pre-teach (intensive): <i>What critical understandings will prepare students to access the mathematics for this cluster?</i></p> <ul style="list-style-type: none"> <li>• 5.OA.A.2: This standard provides a foundation for work with interpreting the structure of expressions because students write out the numerical expression without the calculation. Students become comfortable with using the vocabulary words: difference, greater than, multiple, etc. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.</li> <li>• 6.EE.A.4: This standard provides a foundation for work with interpreting the structure of expressions because being able to tell if two expressions are equivalent is the building blocks for being able to construct and deconstruct expressions to use their structure. Being able to tell if what you have done to an expression essentially changes it or not leads to the understanding of how to use these changes to manipulate the expressions and equations to better understand their structure. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.</li> </ul>		

## Core Instruction

### *Access*

*Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as the size of text, images, graphs, tables, or other visual content, contrast between background and text or image; color used for information or emphasis; volume or rate of speech or sound; speed or timing of video, animation, sound, simulations, etc.; layout of visual or other elements; font used for print materials because in properly prepared digital materials, the display of the same information is very malleable and customizable. Such malleability provides options for increasing the perceptual clarity and salience of information for a wide range of learners and adjustments for preferences of others. While these customizations are difficult with print materials, they are commonly available automatically in digital materials, though it cannot be assumed that because it is digital it is accessible as many digital materials are equally inaccessible.

### *Build*

*Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that encourages perseverance, focuses on development of efficacy and self-awareness, and encourages the use of specific supports and strategies in the face of challenge because assessment is most productive for sustaining engagement when the feedback is relevant, constructive, accessible, consequential, and timely. But the type of feedback is also critical in helping learners to sustain the motivation and effort essential to learning. Mastery-oriented feedback is the type of feedback that guides learners toward mastery rather than a fixed notion of performance or compliance. It also emphasizes the role of effort and practice rather than “intelligence” or inherent “ability” as an important factor in guiding learners toward successful long-term habits and learning practices. These distinctions may be particularly important for learners whose disabilities have been interpreted, by either themselves or their caregivers, as permanently constraining and fixed.

*Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making explicit links between information provided in texts and any accompanying

representation of that information in illustrations, equations, charts, or diagrams because providing alternatives—especially illustrations, simulations, images or interactive graphics—can make the information in text more comprehensible for any learner and accessible for some who would find it completely inaccessible in text.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing scaffolds that can be gradually released with increasing independence and skills (e.g., embedded into digital programs) because fluency is also built through many opportunities for performance, be it in the form of an essay or a dramatic production. Performance helps learners because it allows them to synthesize their learning in personally relevant ways.

### **Internalize**

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with interpreting the structure of expressions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as addressing subject specific phobias and judgments of “natural” aptitude (e.g., “how can I improve on the areas I am struggling in?” rather than “I am not good at math”) because reminders, models, checklists, and so forth can assist learners in choosing and trying an adaptive strategy for managing and directing their emotional responses to external events (e.g., strategies for coping with anxiety-producing social settings or for reducing task-irrelevant distracters) or internal events (e.g., strategies for decreasing rumination on depressive or anxiety-producing ideation).

### **Re-teach**

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on interpreting the structure of expressions by examining tasks from a different perspective through a short mini-lesson because students learn differently. Auditory learners may need an explanation and some one-on-one explanations. Students may also learn from one another. Videos and group collaborations are also a great way to help students understand a lesson from a different perspective.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interprets the structure of expressions by confronting student misconceptions because one-on-one explanations of mistakes made will help the students make the connections to their mistakes. Students may also have their ah-ha moment by recognizing their own mistake.

### **Extension**

*What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?*

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying interpreting the structure of expressions because structuring of an expression is the foundation for creating expressions and equations in word problems. Students need to analyze the word problems and pick out the important phrases and create an expression/equation.

**Culturally and Linguistically Responsive Instruction:**

**Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

**Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion, we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the way students talk about who is and isn't capable of mathematics, we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying, interpreting the structure of expressions facilitating meaningful mathematical discourse is critical because teachers need to lead to some sort of discourse that will ensure that all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration of their own cultures and languages. Interpretation can be critical because students may vary depending on how they learned the previous concepts. Mathematics discourse must lead to digging deeper understanding on why, how and what without deviating from their cultures and beliefs. Answers from the questions may depend on how they perceived and interpreted base from the given activities.

**Standards Aligned Instructionally Embedded Formative Assessment Resources:**

Source: SAT

$$\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$$

If for all positive values of x, what is the value of  $\frac{a}{b}$  ?

**Relevance to families and communities:**

During a unit focused on interpreting the structure of expressions , consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, During a unit focused on counting, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the different

**Cross-Curricular Connections:**

Science:  
Earth's Place in the Universe  
Use mathematical or computational representations to predict the motion of orbiting objects in the solar system. This a connection because each of the terms in the equations model an aspect in the motion of orbiting objects.

[Let's go to Mars!](#)

structures for the number names across the languages in your classroom can lead to a more robust understanding of number for all students by making connections to the different structures of number-names in other languages.

This activity is designed for students familiar with advanced algebra concepts. In this lesson, students will:

- Use algebraic computations to determine the relative positions of Earth and Mars during which an optimal (low-energy) transfer of a spacecraft can occur.
- Combine this information with planetary-position data to determine the next launch opportunity to Mars.