HS: ALGEBRA- SEEING STRUCTURE IN EXPRESSIONS

Cluster Statement: B: Write expressions in equivalent forms to solve problems.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSA.SSE.B.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	SMP 4: Students model with mathematics modeling real-life situations mathematically.	• Find the sums of finite geometric series; find the common ratio. • Use an infinite series as a model; apply a given formula for the sum of a finite geometric series by solving for the isolated variable. • Apply a given formula for the sum of a finite geometric series to solve for a coefficient. • Apply a given formula for the sum of a finite geometric series to justify real world scenarios.
		Webb's Depth of Knowledge: 1-2
		Bloom's Taxonomy: apply

<u>Previous Learning Connections</u>	**<u>Current Learning Connections</u>**	**<u>Future Learning Connections</u>**
• In Algebra I students have studied exponential growth and decay, so can identify first terms and common ratios. Students have written arithmetic and geometric sequences both recursively and explicitly. Students have also used arithmetic and geometric sequences to model situations.	• Students will transfer previous learning to geometric series.	• This is an important concept for Calculus when learning about Riemann sums, series, and sequences.

Clarification Statement

Introduce **geometric sequences**. Students need to identify the **common ratio**, nth **term**, and previous term. Students calculate the nth term substituting the common ratio and the first term. Students apply the formula for the sum of the finite **geometric series** by solving for an isolated variable or for a coefficient. Students model real-world applications and should explain in contextual situations.

Common Misconceptions

Geometric series are obtained through a series of additions or subtraction. When applying the sum formula for geometric series, students may subtract the values in the numerator before applying the exponent. Remind them that exponents are evaluated before addition and subtraction in the order of operation.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying about writing expressions in equivalent forms to solve problems because students will see how the prior knowledge and the new lesson will be connected. The formula for the finite geometric series will make more sense when the connection is made by reviewing the binomial expansion or multiplying two binomials.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 7.EE.A.1: This standard provides a foundation for work with writing expressions in equivalent forms because performing operations on binomials is critical. Students need to learn how to add, multiple, and subtract to derive the formula for finite geometric series. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: *How will the learning for students provide multiple options for recruiting student interest?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because to recruit all learners equally, it is critical to provide options that optimize what is relevant, valuable, and meaningful to the learner.

Build

Effort and Persistence: *How will the learning for students provide options for sustaining effort and persistence?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that is substantive and informative rather than comparative or competitive because there is not one best right way to write and equation to solve a problem and the nuance need to be highlighted so that students can improve their work over time.

Language and Symbols: *How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or* *puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity and comprehensibility for all learners such as embedding support for unfamiliar references within the text (e.g., domain specific notation, lesser known properties and theorems, idioms, academic language, figurative language, mathematical language, jargon, archaic language, colloquialism, and dialect) because students are often asked to interpret a scenario with information given in one form to answer a question and if they don't understand the context of what is happening, it makes it nearly impossible to complete the task.

Expression and Communication: *How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because the concept is to be able to look at the different forms, not the incidental calculations needed to do so.

Internalize

Self-Regulation: *How will the design of the learning strategically support students to effectively cope and engage with the environment?*

- For example, learners engaging with writing expressions in equivalent forms to solve problems benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because the path forward is not always clear when trying to find the best way to solve a problem, with the most obvious path not always being the most efficient path to the solution.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on writing expressions in equivalent forms by critiquing student approaches/solutions to make connections through a short mini-lesson because making connections using different strategies, students are able to communicate using mathematical terms and the more practice, they'll use the terms easily.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit writing expressions in equivalent forms by helping students move from specific answers to generalizations for certain types of problems because seeing the bigger picture to a detailed problem will address the conceptual understanding and students can analyze the formula to the context of the word problems.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying writing expressions in equivalent forms because explorations give opportunities for collaboration and thinking outside the box to make connections. Exploration allows the students to interact and learn from each other.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Building Procedural Fluency from Conceptual Understanding: Instruction should build from conceptual understanding to allow students opportunities to make meaning of mathematics before focusing on procedures. When new learning begins with procedures it privileges those with strong prior familiarity with school mathematics procedures for solving problems and does not allow learning to build for more methods for solving tasks that occur outside of school mathematics. For example, when studying to write expressions in equivalent forms to solve problems the types of mathematical tasks are critical because teachers need to be aware of the real-world problems that need to be used as tasks for students to solve problems. If teachers use tax and sales real -world solving problems different communities apply tax rates differently. Students' solutions would vary. Instruction should be culturally and linguistically appropriate and relevant to allow students engagement and gain their interest.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
Source: https://satsuitequestionbank.collegeboard.org/

Question ID 19944

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Passport to Advanced Math	■■■	Passport to Advanced Mathematics	Nonlinear functions	2. For a quadratic or exponential function, e. make connections between tabular, algebraic, and graphical representations of the function, by ii. identifying features of one representation given another representation, including maximum and minimum values of the function;	No Calculator

In the quadratic equation above, a is a nonzero constant. The graph of the equation in the xy-plane is a parabola with vertex (c, d). Which of the following is equal to d ?

A. $-9a$

B. $-8a$

C. $-5a$

D. $-2a$

Rationale

Choice A is correct. The parabola with equation $y = a(x-2)(x+4)$ crosses the x-axis at the points $(-4,0)$ and $(2,0)$. By symmetry, the x-coordinate of the vertex of the parabola is halfway between the x-coordinates of $(-4,0)$ and $(2,0)$. Thus, the x-coordinate of the vertex is $\frac{-4+2}{2} = -1$. This is the value of c. To find the y-coordinate of the vertex, substitute -1 for x in $y = a(x-2)(x+4)$:

$y = a(x-2)(x+4) = a(-1-2)(-1+4) = a(-3)(3) = -9a$

Therefore, the value of d is $-9a$.

Choice B is incorrect because the value of the constant term in the equation is not the y-coordinate of the vertex, unless there were no linear terms in the quadratic. Choice C is incorrect and may be the result of a sign error in finding the x-coordinate of the vertex. Choice D is incorrect because the negative of the coefficient of the linear term in the quadratic equation is not the y-coordinate of the vertex.

Relevance to families and communities: During a unit focused on writing expressions in equivalent forms to solve problems, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students. For example, learning about communities have the different tax rate is used in the home and community can a be a great way to connect schools' tasks with home tasks.	**Cross-Curricular Connections:** Art: In the realm of digital art, so many wonderful and playful genres exist that stimulate the imagination, but so few do it with the intricate style of fractal art. Fractal art is achieved through the mathematical calculations of fractal objects being visually displayed, with the use of self-similar transforms that are generated and manipulated with different assigned geometric properties to produce multiple variations of the shape in continually reducing patterns. Sounds extremely technical and not that artistic, true, but these equations create some of the most mesmerizing and inspiring artwork to emerge from the digital art arena. https://fractalfoundation.org/resources/what-are-fractals/ 35 Phenomenal Fractal Art Pictures Engineering: The Invention of Fractal Antennas Dr. Cohen built the first bona fide fractal element antenna in 1988. He is now one of the world's most innovative antenna designers, now with 26 years of professional experience, and 53 years of practical experience, stemming from his 'ham' antenna work over many years. Fractal Antennas website: Invention