NEW MEXICO
Public Education Department

## HS: FUNCTIONS- BUILDING FUNCTIONS

**Cluster Statement:** B: Build new functions from existing functions.

| **Standard Text** | **Standard for Mathematical Practices** | **Students who demonstrate understanding can:** |
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| **HSF.BF.B.3**<br>**Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.**<br><br>*Note: Algebra 1 focuses on linear, exponential, quadratic, and absolute value. Algebra 2 includes simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.* | **Standard for Mathematical Practices**<br><br>SMP 5: Students can use tools by using graphing calculators or technology to experiment with parent functions and the results when different transformations are applied.<br><br>SMP 8: Students look for and express regularity in repeated reasoning by exploring different expressions for transformations of f(x) and generalizing the effects. | • Identify vertical transformations from a function or a graph.<br>• Identify horizontal transformations from a function or a graph.<br>• Identify a shrink or a stretch from a function or a graph.<br>• Write the results from such transformations.<br>• Recognize odd and even functions.<br>• Identify transformations of a function on a graph.<br>• Describe the effects of transformations on parent functions.<br><br>**Webb's Depth Of Knowledge:** 1-2<br><br>**Bloom's Taxonomy:** Understand, Apply and Analyze |
| **Standard Text**<br><br>HSF.BF.B.4.A<br>Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 $x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1.*<br><br>*Note: Algebra 1 focuses on linear, exponential, quadratic, and absolute value. Algebra 2 includes simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.* | **Standard for Mathematical Practices**<br><br>SMP.6 Students can attend to precision by understanding that some functions do not have an inverse unless there is some sort of restriction on the domain.<br><br>SMP 7: Students can look for and make use of structure by recognizing that the ordered pair (x, y) is reversed for a function's inverse. | **Students who demonstrate understanding can:**<br>• Write the inverse of a function.<br>• Determine restrictions on the domain to allow for an inverse to exist.<br>• Relate using an inverse as an operation that undoes another operation.<br>• Solve an equation of the form f(x)=c for a function f that has an inverse and write an expression for the inverse.<br><br>**Webb's Depth of Knowledge:** 1-2 |

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| | | **Bloom's Taxonomy:** Understand, Apply and Analyze |
| **Previous Learning Connections**<br>• Connect to the work in Algebra 1 with linear, exponential, quadratic, and absolute value functions for this cluster. **(HSF.BF.B)** | **Current Learning Connections**<br>• Connect to graphing functional relationships. **(HSF.IF.4)**<br>• Connect to trigonometric functions. **(HS.F-TF.B)** | **Future Learning Connections**<br>• Connecting to understanding the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. **(HS.F-BF.B.5)** |

**Clarification Statement**

HSF.BF.B.3: Students should describe the effect of **stretches, shrinkages**, **vertical and horizontal transformations** on **functions**. They should be able to find the value of the **transformation** when given a **graph** and be able to explain effects of transformations using technology. Students should know that adding a **constant** k to a function will change the graph of the function depending not only on the value of the constant, but on where it is inserted as well. If $y = f(x)$ is changed to $y = f(x) + k$, the curve will **shift** vertically (up for $k > 0$, down if $k < 0$). Adding k to x such that $y = f(x + k)$ will shift the curve horizontally (left for $k > 0$, right for $k < 0$). Multiplying f(x) by a constant k stretches ($k > 1$) or **squishes** ($0 < k < 1$) the graph vertically. If $k < 0$, the graph is also **flipped** over the **x-axis**. Multiplying x by k stretches ($k > 0$) or squishes ($k < 0$) the graph horizontally.

HSF.BF.B.4: Students should be able to find the **inverse** of **functions** and recognize that other functions may not have an inverse unless there are **restrictions** placed on the **domain**. If $f(x) = y$ is a function, the inverse function can be found by switching the place of x and y ($f(y) = x$), and then solving for y so that $f\text{-}1(x) = y$. For instance, if the function f(x) is $y = 2x^3$, then the inverse function $f^{-1}(x)$ consists of switching the places of x and y ($x = 2y^3$) and then solving for y.

**Common Misconceptions**

Students often have difficulty determining the direction of the horizontal shifts.

Students often confuse the notation for the inverse and negative numbers.

**Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies**

**Pre-Teach**

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that rehearses prior learning when studying building new functions from existing functions because in prior lessons and grade levels, students have been introduced to many aspects and content of functions.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- HS.F-IF.A.1: This standard provides a foundation for work with building new functions from existing functions because this prerequisite has students understand functions based on domain and range. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

**Core Instruction**

**Access**

Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for visual information for all images, graphics, video, or animations; touch equivalents (tactile graphics or objects of reference) for key visuals that represent concepts; objects and spatial models to convey perspective or interaction; auditory cues for key concepts and transitions in visual information because offering alternatives provides more flexibility for students to obtain the necessary information and skills needed to successfully complete the tasks.

**Build**

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as displaying the learning goals in multiple ways because this supports students to build individual skills in self-determination and pushes them to not give up on the content and will improve their potential for learning the content.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as embedding support for vocabulary and symbols within the text (e.g., hyperlinks or footnotes to definitions, explanations, illustrations, previous coverage, translations) because embedding the supports provides accessibility for all which can help in achieving to link or associate alternate representations of meaning for students.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as solving problems using a variety of strategies because when students are presented with various strategies they are able to explore the variety of methods used to solve problems and come up with solutions.

**Internalize**

Self-Regulation: How will the design of the learning strategically support students to effectively cope and engage with the environment?

- For example, learners engaging with building new functions from existing functions benefit when learning experiences set personal goals that increase ownership of learning goals and support healthy responses and interactions (e.g., learning from mistakes), such as supporting students with metacognitive approaches to frustration when working on mathematics because allowing students to become aware and understand their own thought processes allows for students to overcome those frustrations that they are experiencing in mathematics and for them to move beyond them to overcome them.

**Re-teach**

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on building new functions from existing functions by revisiting student thinking through a short mini-lesson because students will be able to activate prior learning on functions and make the connection between the content.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit building new functions from existing functions by offering opportunities to understand and explore different strategies because providing students with various strategies allows for further depth in understanding and further delving into the depth of the content.

**Extension**

*What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?*

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying building new functions from existing functions because when students are able to draw connections between various topics, the learning potential is increased and prior knowledge activation is improved upon and built upon.

**Culturally and Linguistically Responsive Instruction:**
**Validate/Affirm:** How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
**Build/Bridge:** How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of

low expectations and low achievement. For example, when studying building new functions from existing functions goal setting is critical because it allows students to take ownership of the content and what the expectations for learning are as they are clearly identified while making a meaningful connection between the learning and daily lives.

**Standards Aligned Instructionally Embedded Formative Assessment Resources:**

http://tasks.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/742

This type of assessment question requires students to apply vertical and horizontal translations as well as a reflection to a given graph. Further, students are asked to identify the location of specific coordinates on the new graphs. This will engage students with SMP 7 as they use the structure of the graph, the expression of the transformation and/or a table of values to create new graphs and identify the imaged points.

https://www.engageny.org/resource/algebra-i-module-3-topic-c-lesson-17

**Relevance to families and communities:**

During a unit focused on building new functions from existing functions, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, learning about the characteristics of building functions from existing functions allows for making connections to building in any concept which can be done at home such as working on a home project that requires building on to something that already exists. Students can make connections to prior learning (something that already exists) and build onto that knowledge.

**Cross-Curricular Connections:**

Science: The equation for velocity, $M(v) = 6v^2$, is one where the variable, v, has directions. Therefore, an inverse function of M(v) cannot give back both a positive and negative velocity. Consider providing a connection for students to consider how they will handle this situation.