NEW MEXICO
Public Education Department

HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: B: Interpret functions that arise in applications in terms of the context.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.B.4 **For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship**. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. *Note: Algebra 1 focuses on linear, exponential, and quadratic. In Algebra 2 emphasize selection of appropriate models.*	MP1: Students can make sense of problems and persevere in solving them by thinking through the meaning of the key features in graphs as relative to a given context. MP.4: Students can model with mathematics by creating an approximate graph that could model a given context.	• Identify intercepts of a function. • identify intervals where the function is increasing. • Identify intervals where the function is decreasing. • Identify intervals where the function is positive. • Identify intervals where the function is negative. • Identify relative maximums of a function. • Identify relative minimums of a function. • Identify symmetries in the functions. • Identify end behavior of the functions. • Sketch graphs given a list of key features or a verbal model. • Sketch functions that model key feature behavior. • Label intercepts and intervals of a graph. • Interpret where the function is increasing, decreasing, positive, or negative. • Interpret relative maximums and minimums. • Interpret various symmetries, end behaviors, and periodicity.
		Webb's Depth Of Knowledge: 1-2
		Bloom's Taxonomy: Understand, Apply and Analyze

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *Note: Algebra 1 focuses on linear, exponential, and quadratic. In Algebra 2 emphasize selection of appropriate models.*	SMP 3: Students can construct viable arguments by explaining why the domain of a given context is discrete or continuous. SMP 4: Students will model with mathematics by connecting a function to the context it represents using quantities.	• Make connections between a graph of a function and its domain. • Make connections between the graph of a function and the context it describes. • Identify when the domain of a given context is discrete or continuous and explain why. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Standard Text HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *Note: Algebra 1 focuses on linear, exponential, and quadratic. In Algebra 2 emphasize selection of appropriate models.*	**Standard for Mathematical Practices** SMP 4: Students can model with mathematics by interpreting the average rate of change within the context of a problem. SMP 5: Students can use tools by using tables and graphs to determine the average rate of change over a specified interval.	**Students who demonstrate understanding can:** • Calculate the average rate of change of a function over a specified interval presented symbolically. • Calculate the average rate of change of a function over a specified interval presented in a table. • Interpret the average rate of change of a function over a specified interval presented symbolically for a given context. • Interpret the average rate of change of a function over a specified interval presented in a table for a given context. • Estimate the rate of change of a function from a graph. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze

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Previous Learning Connections	Current Learning Connections	Future Learning Connections
• Connect to the work of Algebra 1 within this cluster around linear, quadratic, and exponential. **(HSF.IF.B)**	• Connect to discovering features of families of functions. **(HSF.IF.7)** • Connect to finding key features of the entire family of functions. **(HSF.IF.4)**	• Connect to the work with trigonometric functions. **(HST.TF.B)**

Clarification Statement

HSF.IF.B.4: Students interpret the **key features** of the different **functions** listed in the standard. When given a **table** or **graph** of a function that models a **real-life situation**, explain the meaning of the characteristics of the table or graph in the **context** of the problem.

Key features of a **linear function** are **slope** and **intercepts**, of a **quadratic function** are **intervals of increase/decrease**, **positive/negative**, **maximum/minimum**, **symmetry**, and **intercepts**, of an **exponential function** include **y-intercept** and **increasing/decreasing intervals** and of an **absolute value** include **y-intercept**, **minimum or maximum**, **increasing or decreasing intervals**, and **symmetry**.

HSF.IF.B.5: Students should focus their attention on possible **input and output values**, framing them as the **domain and range of a function**. When given a description of a function that represents a situation, the students should determine reasonable domain and range. Students relate the domain of a function to its graph and, where applicable, to the **quantitative relationship** it describes. Students need to explain the reasonableness of a domain for a given context.

Students should understand that the domain of a function is the set of all possible inputs and the range is the set of all possible outputs. Also looking at if a function is **continuous** (time, amount of liquid filling a container) or discrete (number of people or things) and connecting back to number classifications

HSF.IF.B.6: Students will calculate and interpret the **average rate of change** of a linear, quadratic, **piecewise linea**r (to include absolute value), and exponential function (presented **symbolically** or as a table) over a specified **interval.** Students will **estimate** the rate of change from a graph. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect **data** from **experiments** or **simulations** (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.

Common Misconceptions

Students may confuse scatter plots and correlations.

Students may focus on the y values of the graph instead of the x values of the interval, when identifying key features of a graph.

Students may have difficulty understanding domain.

Students may confuse independent and dependent variables.

Students may confuse shift with rate of change

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that analyzes common misconceptions when studying interpret functions that arise in applications in terms of the context because quantities in graphs and tables should be interpreted in context to the problem and domains should be within the context.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.F.A.1: This standard provides a foundation for work with interpret functions that arise in applications in terms of the context because students need to interpret the ordered pairs on the graph for analyzing or making predictions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Interest: How will the learning for students provide multiple options for recruiting student interest?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences include ways to recruit interest such as providing novel and relevant problems to make sense of complex ideas in creative ways because when given a function, students can explain the x and y variables. When given in an application context, students need to make a connection with their lives and explain the variables in the function.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as generating relevant examples with students that connect to their cultural background and interests because collaboration within a group of student can generate some effective communication skill and this creates peer mentoring for group members that need help.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting how complex terms, expressions, or equations are composed of simpler words or symbols by attending to the structure because interpreting an equation with the independent variable and dependent variables contextual to the problem will help the students interpret the graph and the average rate of change and other important characteristics of the graph.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as composing in multiple media such as text, speech, drawing, illustration, comics, storyboards, design, film, music, dance/movement, visual art, sculpture, or video using physical manipulatives (e.g., blocks, 3D models, base-ten blocks) because learners from different cultures and backgrounds need exposure using multiple tools to construct graphs and tables. Students make connections and interpret the functions when they are presented with different tools.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with interpret functions that arise in applications in terms of the context benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing templates, graphic organizers, concept maps to support note-taking because <students need to internalize and interpret the information from the sketch of a graph, table, verbal description or equation. Students explore multiple representation, and, in the end, all interpretation should have the same results.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- This standard 8.F.A.1 provides a foundation for work with interpret functions that arise in applications in terms of the context because students need to interpret the ordered pairs on the graph for analyzing or making predictions. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit interpret functions that arise in applications in terms of the context by addressing conceptual understanding because students need to make connections about the numbers they choose for their domain and range in context with the problem. They will need to interpret the characteristics of a graph.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying interpret functions that arise in applications in terms of the context because students can interpret their own graphs and explore how graphs can be integrated according their interest.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?

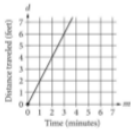
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Tasks: The type of mathematical tasks and instruction students receive provides the foundation for students' mathematical learning and their mathematical identity. Tasks and instruction that provide greater access to the mathematics and convey the creativity of mathematics by allowing for multiple solution strategies and development of the standards for mathematical practice lead to more students viewing themselves mathematically successful capable mathematicians than tasks and instruction which define success as memorizing and repeating a procedure demonstrated by the teacher. For example, when studying interpret functions that arise in applications in terms of the context the types of mathematical tasks are critical because clearly defined tasks set the routine for interaction and support for students. Interpreting and sketching key characteristics of graphs and tables, students make the connections graphically, verbally, table, and symbolically. Allowing students to explain, think out loud, making conjectures, and communicate with peers to come up with mathematical ideas.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

CollegeBoard | Question ID 1474415

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Heart of Algebra	Easy	Heart of Algebra	Linear functions	4. Make connections between verbal, tabular, algebraic, and graphical representations of a linear function, by c. determining how a graph is affected by a change to its equation.	No Calculator



The graph above shows the distance traveled d, in feet, by a product on a conveyor belt m minutes after the product is placed on the belt. Which of the following equations correctly relates d and m ?

Question Difficulty: Easy

A. $d = 2m$

B. $d = \frac{1}{2}m$

C. $d = m + 2$

D. $d = 2m + 2$

Choice A is correct. The line passes through the origin. Therefore, this is a relationship of the form $d = km$, where k is a constant representing the slope of the graph. To find the value of k, choose a point (m,d) on the graph of the line other than the origin and substitute the values of m and d into the equation. For example, if the point $(2,4)$ is chosen, then $4 = k(2)$, and $k = 2$. Therefore, the equation of the line is $d = 2m$.

Choice B is incorrect and may result from calculating the slope of the line as the change in time over the change in distance traveled instead of the change in distance traveled over the change in time. Choices C and D are incorrect because each of these equations represents a line with a d-intercept of 2. However, the graph shows a line with a d-intercept of 0.

SAT Item

This type of assessment question requires students to analyze a graph in context and create a linear equation to fit. Students will engage with SMP 7 as they use the structure of the line in the graph to write an equation.

Relevance to families and communities:

During a unit focused on interpret functions that arise in applications in terms of the context, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, families create their own story and make the connection with a table and graph. Key features should be included. Every family will have a different story.

Cross-Curricular Connections:

In this lesson, students use exponential decay and rational functions to understand why addicted patients seek more and stronger opioids to alleviate their pain. Students discuss the role that various parties played in creating the crisis and ways they can help to solve it.

House of Pain: A Lesson by Mathalicious