NEW MEXICO
Public Education Department

HS: FUNCTIONS- INTERPRETING FUNCTIONS

Cluster Statement: C: Analyze functions using different representations.

Widely Applicable as Prerequisite for a Range of College Majors, Postsecondary Programs and Careers

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.IF.C.7 **Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.** • **HSF.IF.C.7.B: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.** • **HSF.IF.C.7.C: Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.** • **HSF.IF.C.7.E: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.** *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined. Algebra 2 focuses on using key features to guide selection of appropriate type of model function.*	SMP 4: Students can model with mathematics by focusing on using key features to guide the selection of an appropriate type of function to model a context. SMP 7: Students can look for and make use of structure by recognizing key symbolic and graphical features to identify the type of a function.	• Graph exponential, logarithmic, and trigonometric functions. • Describe key features of exponential, logarithmic, and trigonometric functions. • Graph functions expressed symbolically showing key features of the graph by hand in simple cases and with technology for more complicated cases. • Compare and contrast functions. **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. • HSF.IF.C.8.A: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. • HSF.IF.C.8.B: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay. *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined. Algebra 2 focuses on using key features to guide selection of appropriate type of model function.*	SMP 4: Students can model with mathematics by interpreting zeros, intervals where the function is increasing or decreasing, extrema and symmetry within a context. SMP 7: Students can look for and make use of structure by using rearranging functions to highlight key features.	• Rewrite a function to find and highlight key features. • Factor a quadratic expression to find zeros, extrema and symmetry • Interpret the meaning of zeros, extrema and symmetry within the context of a problem. • Complete the square for a quadratic function to reveal its key features. • Interpret the key features of a quadratic expression in terms of the context it represents. • Use properties of exponents to relate parts of an exponential function to its context (e.g., describe the initial value, growth/decay rate or factor and the growth period). • Identify how key features of an exponential function relate to characteristics in a real-world context. • Classify real-world problems as an exponential growth or decay. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSF.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* *Note: Algebra 1 focuses on linear, exponential, quadratic, absolute value, step, and piecewise defined. Algebra 2 focuses on using key*	SMP 5: Students can use tools by working flexibly with multiple representations. SMP 7: Students can look for and make use of structure by comparing the similarities and differences of functions.	• Make comparisons between functions in different forms using their knowledge of key features.

features to guide selection of appropriate type of model function.		**Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy:** Understand, Apply and Analyze
Previous Learning Connections • Connect to the work in Algebra 1 with this cluster which focused on linear, exponential, quadratic, absolute value, step, and piecewise defined by supporting Algebra 2 students to focus on using key features to guide selection of an appropriate type of model function.	**Current Learning Connections** • Connect to writing linear, quadratic, and exponential functions to describe relationships between quantities. **(HSA.CED.1-3)** • Connect to analyzing transformations of parent functions for linear, quadratic, and exponential functions. **(HSF.BF.3)**	**Future Learning Connections** • Connect to graphing all parent functions by hand and using technology and identifying their key features. **(HSF.IF.7)** • Connect to factoring to complete the square with quadratic functions with complex zeros. **(HSN.CN.7)**

Clarification Statement

HSF.IF.C.7: Students should be able to describe the significant features of different **functions graphically** and **algebraically**. Students should be able to use the **significant features** to **sketch** the graph of the function. Students should graph **linear** and **quadratic functions** and show **intercepts**, **maxima**, and **minima.** Students should know the **slope-intercept form** of linear functions, y = mx + b, and how to extract enough information from the equation to be able to draw it. When graphing **roots**, remember that for $\sqrt[n]{x}$, if n is **even,** the domain includes all **positive integers**. Otherwise, **negative** values are included as well. When graphing roots of the for $y = a\sqrt{x} + b$, remember the **y-intercept** is b. Students should remember that roots are **fractional exponents.** Students should know to look at the **highest degree of the polynomial** and its **coefficient,** ax^n. If n is even, the function will extend either **up** or **down** on both ends (as x goes to **positive** or **negative infinity**). If n is **odd**, they'll go in **opposite directions**. If a is positive, the even powered functions will go up and the odd powered functions will start down and go up. If a is negative, the even powered functions will go down, and the odd powered functions will start up and go down.

HSF.IF.C.8: Students should be able to rewrite quadratic and **exponential functions** in different ways to find key features of the expression and interpret those key features in terms of the context they represent. Students should be able to find the **x-intercepts** of a quadratic function using both **factoring** and **completing the square**.

HSF.IF.C.9: Students should be able to compare two given functions (linear, exponential, quadratic) whether that be as a function or **equation**, in a **table,** in a **graph,** or by **verbal description**. Students should start by knowing the difference between linear, quadratic and exponential functions, and be able to identify them by equation and by graph. Students should be able to compare two functions even when they're both represented differently. To do this successfully, they must be able to **translate** between an equation, a graph, words, and a table of values, and understand how certain aspects of one **representation** impact the rest.

Common Misconceptions

Students may have difficulty identifying the key features needed to sketch the graphs or identifying those features algebraically.

Students may have difficulty with contextualizing and decontextualizing expressions.

Students will often confuse functions given in a table as a representation of a finite set of numbers rather than a subset of the entire function. They also may have difficulty with the abstractness of determining what is happening with a function over intervals of the domain that they cannot see.

Students may not distinguish between the different type of logarithms, i.e., natural logs, when using calculator

Students may struggle with applying translations, stretches, compressions, and reflections to a parent function.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying analyze functions using different representations because connections between the different representations, students need exposure to the different family functions (linear, quadratic, polynomial, etc.)

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- This standard 8.F.A.1 provides a foundation for work with analyzing functions using different representations because order pairs of numbers have a relationship and the point is represented on the graph contextually. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Physical Action: How will the learning for students provide a variety of methods for navigation to support access?

- For example, learners engaging with analyze functions using different representations benefit when learning experiences ensure information is accessible to learners through a variety of methods for navigation, such as varying methods for response and navigation by providing alternatives to <requirements for rate, timing, speed, and range of motor action with instructional materials, physical manipulatives, and technologies; physically responding or indicating selections; physically interacting with materials by hand, voice, single switch, joystick, keyboard, or adapted keyboard> because real-world hands-on activities require communication and collaboration. Integrate technology to collect data and graph different functions. Technology can be used to graph, calculate regressions and analyze different features of the graph.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to students' attention and affect to support sustained effort and concentration such as constructing communities of learners engaged in common interests or activities because different ideas from multiple team members can create collaboration and peer tutoring. Immediate feedback can also open discussions/ideas to further the project.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with analyze functions using different representations benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as allowing for flexibility and easy access to multiple representations of notation where appropriate (e.g., formulas, word problems, graphs) because making the connection verbally, graphically, numerically and algebraically, students explain the context of the graphs paying attention to the key features. Highlight key words in word problems. interpret the graphs in context using the rate of change, making predictions, intercepts, maximum and minimum values and decreasing/increasing.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with analyzing functions using different representations benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing calculators, graphing calculators, geometric sketchpads, or pre-formatted graph paper because visually displayed on graph paper and using the graphing calculator to check their work, students become more confident and appreciate the different tools available to construct the graphs.

Internalize

Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with analyze functions using different representations benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as highlighting previously learned skills that can be used to solve unfamiliar problems because background knowledge using different hands-on strategies like paper folding, graphic organizers, technology, students make the connection with new skills.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on analyze functions using different representations by providing specific feedback to students on their work through a short mini lesson because immediate feedback provides support for learning. There are several family functions with different key features and interpretation.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit on analyzing functions using different representations by offering opportunities to understand and explore different strategies because explaining the context of the problem verbally, graphically and writing, students comprehend the different family functions/equations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as in-depth, self-directed exploration of self-selected topics when studying analyze functions using different representations because making a real-world connection with a choice to select what the learner is interested in will make a deeper connection to the mathematical concept and skill.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Posing Purposeful Questions: CLRI requires intentional planning around the questions posed in a mathematics classroom. It is critical to consider "who is being positioned as competent, and whose ideas are featured and privileged" within the classroom through both the types of questioning and who is being questioned. Mathematics classrooms traditionally ask short answer questions and reward students that can respond quickly and correctly. When questioning seeks to understand students' thinking by taking their ideas seriously and asking the community to build upon one another's ideas a greater sense of belonging in mathematics is created for students from marginalized cultures and languages. For example, when studying analyzing functions using different representations the pattern of questions within the classroom is critical because asking open ended questions allows the students to think, answer and have a reason for their answer. Ask probing questions that allow students to elaborate and clarify their different graphs and key representations. The explanation of the different family functions from linear to trigonometric functions include all types of learners from the low to the high so everyone feels included.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
http://tasks.illustrativemathematics.org/content-standards/HSF/IF/C/9/tasks/1279
This type of assessment question requires students to analyze function and a graph and compare the key features in context of a scenario. Students will engage with SMP 7 as they use the structure of both the equation and the graph to answer questions in context.
Additional assessment:
Analyzing Graphs
https://www.map.mathshell.org/lessons.php?unit=9245&collection=8

https://www.map.mathshell.org/lessons.php?unit=9240&collection=8

Relevance to families and communities:	Cross-Curricular Connections:
During a unit focused on analyzing functions using different representations, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, use community data and technology to graph, calculate and analyze the regression between the variables. Data and graphs can be used to make comparisons between the communities.	Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. Design a Navajo Rug