NEW MEXICO
Public Education Department

HS: FUNCTIONS- TRIGONOMETRIC FUNCTIONS

Cluster Statement: A: Extend the domain of trigonometric functions using the unit circle.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Find the measures of an angle in standard position and the reference angle. • Find arc length using radian measure on the unit circle. • Convert between degrees and radians. **Webb's Depth of Knowledge:** 1-2 **Bloom's Taxonomy: understand, apply**
HSF.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	SMP.2: Students reason abstractly and quantitatively by understanding the relationship between angle measures in radians and the quadrant that the terminal side of the reference angle lies. SMP.5: Students use appropriate tools strategically by using calculators with both degrees and radians and setting the calculator for the correct unit of measure.	
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:** • Identify the relationship between the unit circle and the coordinate plane. **Webb's Depth of Knowledge:** 1-2
HSF.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	SMP 6. Students attend to precision by understanding when to give a rounded answer (typically in degrees) vs. an actual answer (typically in radians) SMP 8 Students look for and express regularity in repeated reasoning by using their knowledge of special right triangles and relate it to the unit circle.	

		Bloom's Taxonomy: understand
Previous Learning Connections In Geometry, students learned the relationship of trigonometric ratios and special right triangles (which leads to evaluating on the unit circle). Students also learned about radian measure and how to convert to degree measure.	**Current Learning Connections** Students will use this knowledge of trigonometric functions (sine and cosine) and apply transformations and identify key features of these trigonometric functions. They will also be able to evaluate trig ratios on a coordinate plane (not on the unit circle).	**Future Learning Connections** In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios and functions (tangent, cosecant, secant and cotangent) and inverse trigonometric functions.

Clarification Statement

Students will be able to extend their knowledge of circle and trigonometric ratios (sine and cosine) to arc length, evaluating using a unit circle, and graphing trigonometric functions (sine and cosine).

Common Misconceptions

Students may confuse the direction of positive and negative angle measures, writing the wrong sign with the measure.

Students may mix-up radian and degree measures

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that uses images/resources (especially those being used the first time)when studying extending the domain of trigonometric functions using the unit circle because this is the first time many students will be working with a unit circle so providing that visual at the very beginning and explaining its purpose can be helpful in later parts of the lessons.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- F.IF.A1: This standard provides a foundation for work with extending the domain of trigonometric functions using the unit circle because this is the standard where students gain conceptual understanding of domain and range as sets of inputs and outputs for a given function. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as offering alternatives for auditory information because some students may have a hard time understanding the cyclic nature of these functions when described only in words or with numbers. Consider displaying a rotating wheel or other circular object to show why values repeat over time. This tangible object may help solidify the concept of periodicity.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to student's attention and affect to support sustained effort and concentration such as creating cooperative learning groups with clear goals, roles, and responsibilities

because students tend to struggle in the details (reducing a fraction, applying the correct sign in each quadrant, misidentified reference angle, etc). By setting up a cooperative group where each student has a role, the group knows who to look to for support at each step and provides structure to how groups work so that no group is sitting "stuck."

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as making connections to previously learned structures because understand the concept of equivalence from work with prior functions, we can extend that knowledge to this family of functions. Showing students that trigonometric functions evaluate the same whether in radian or degree can help students to check their work and can take some of the mystery out of these functions once students see they evaluate numerically and create graphs just like any other family of functions we have worked with.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing virtual or concrete mathematics (e.g. a unit circle with reference angles marked and/or degree and radian measures shown) because it is easy for students to get stuck in the details and not see the larger concept. Providing a visual that allows students to quickly identify equivalent angle measurements or reference angles in a

given quadrant can free their minds to focus on how the domain of these functions is not limited to a specified set of values.

Internalize

- For example, learners engaging with extending the domain of trigonometric functions using the unit circle benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing scaffolds that connect new information to prior knowledge (e.g., word webs, half-full concept maps) because students will have previously worked with converting units and therefore relating radian measurements as just another unit will ease potential fear of this new concept. Further, students have familiarity with right triangle calculations which can be useful in calculating sin, cos and tan within the unit circle. Showing that the values of the trigonometric functions are not arbitrary but arise from concepts they are familiar with will help build connections between prior knowledge and new learning.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on extending the domain of trigonometric functions using the unit circle by providing specific feedback to students on their work through a short mini-lesson because students may have a fear of trigonometry that may affect their willingness to interact with the content. Providing student specific feedback, identifying things students are doing correct and/or effort students are putting forth can encourage students to continue engaging with the material. There are many opportunities for students to make minor errors and if this is the focus, students may shut down. Focused and encouraging feedback can help to counteract any feelings that arise as errors are made.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit extending the domain of trigonometric functions using the unit circle by addressing conceptual understanding because the goal of this cluster is understanding the basis of periodic functions, rather than procedural computations. Students may lack conceptual understanding of domain/range, radian measures of angles and/or the periodicity of functions. Each of these concepts should be explored so students can adequately express their understanding.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students when studying extending the domain of trigonometric functions using the

unit circle because once students understand the behavior of periodic functions they can extend the pattern on their own with little guidance from the teacher.

Culturally and Linguistically Responsive Instruction:

Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Facilitating Meaningful Mathematical Discourse: Mathematics discourse requires intentional planning to ensure all students feel comfortable to share, consider, build upon and critique the mathematical ideas under consideration. When student ideas serve as the basis for discussion we position them as knowers and doers of mathematics by using equitable talk moves students and attending to the ways students talk about who is and isn't capable of mathematics we can disrupt the negative images and stereotypes around mathematics of marginalized cultures and languages. "A discourse-based mathematics classroom provides stronger access for every student — those who have an immediate answer or approach to share, those who have begun to formulate a mathematical approach to a task but have not fully developed their thoughts, and those who may not have an approach but can provide feedback to others." For example, when studying extending the domain of trigonometric functions using the unit circle facilitating meaningful mathematical discourse is critical because some students will naturally see and understand the patterns of periodic functions quicker than others. Eliciting peer-to-peer explanations can build a cooperative environment in the classroom. Further, by providing sentence frames for students, all students can engage in mathematical conversations, even if they are not sure what the solution is. When students speak about mathematics, the engage meaningfully in the content and deepen their understanding.

Standards Aligned Instructionally Embedded Formative Assessment Resources:
SAT Item

CollegeBoard **Question ID 423225**

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	**Math**	**Additional Topics in Math**	**Hard**	**Additional Topics in Math**	**Circles**	**6. Convert between angle measures in degrees and radians.**	**No Calculator**

The number of radians in a 720-degree angle can be written as $a\pi$, where a is a constant. What is the value of a ?

Question Difficulty: Hard

The correct answer is 4. There are π radians in a $180°$ angle. An angle measure of $720°$ is 4 times greater than an angle measure of $180°$. Therefore, the number of radians in a $720°$ angle is 4π.

This type of assessment question requires students to translate an angle measurement from degrees to radians. Students may use a conversion factor to do so or may use the fact that pi radians is equivalent to 180 degrees,

and the given angle is 4 times 180 degrees. Students will engage with SMP 7 in either case as they use the structure of the measurement to convert to different unit of measurement.	
Relevance to families and communities: During a unit focused on extending the domain of trigonometric functions using the unit circle, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, periodicity refers to a pattern that repeats over time. Consider providing examples of numeric and non-numeric patterns that exist (springs bouncing, pendulums swinging, temperature fluctuations throughout the year, hunger growing and decreasing as you approach mealtime) and then ask students to provide some examples of their own.	**Cross-Curricular Connections:** Many of the Navajo rug designs you will discover by following the project will be good examples of symmetrical balance. Symmetrical balance is a type of visual balance where the overall composition is arranged to look like it is the same on both sides of the center of the design. In other words, it is a design which could be folded in half, and as the design folds, each part of the design would match up with its symmetrical counterpart on the opposite side of the center. The rug design on the right is symmetrical left-to-right. If a line was drawn vertically down the center of the rug, the arrangement of shapes and colors would appear to be exactly the opposite of each other on both sides of that line. Design a Navajo Rug