NEW MEXICO
Public Education Department

HS: FUNCTIONS- TRIGONOMETRIC FUNCTIONS

Cluster Statement: C: Prove and apply trigonometric identities.

Standard Text	Standard for Mathematical Practices	Students who demonstrate understanding can:
HSF.TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	SMP 3 Students can construct viable arguments and critique the reasoning of others when proving the Pythagorean Identity. SMP 5. Students use appropriate tools strategically by selecting an appropriate method to calculate trig ratios in all quadrants. SMP 6. Students attend to precision when determining whether a trigonometric ratio should be positive or negative based on the information given (such as quadrant or other restrictions).	• Use the concepts from the Pythagorean identity to calculate trigonometric ratios in any quadrant on the coordinate plane **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** understand, apply

Previous Learning Connections	Current Learning Connections	Future Learning Connections
In Geometry, students learned the relationship of trigonometric ratios. In 8th grade, Algebra 1, and Geometry, students also learned the Pythagorean Theorem and how to graph on a coordinate plane.	Students will use their knowledge of the unit circle and relate that to determine trigonometric ratios not on the unit circle.	In Precalculus and Calculus courses, students will connect this learning cluster to other trigonometric ratios (tangent, cosecant, secant and cotangent).

Clarification Statement
Students will make connections between their knowledge of the Pythagorean Theorem, trigonometric ratios, the unit circle and coordinate plane.

Common Misconceptions
Students may struggle to explain how the identities frame responses.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that provides additional time for confusion to happen with new mathematical ideas when studying proving and applying trigonometric identities because algebraic proofs can be very challenging for students and we want to confront that fact by providing extra supports and time for students to engage with the material, whether their work is exactly correct or not. The extra time experiencing the material will build deeper understanding.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.GB.7: This standard provides a foundation for work with proving and applying trigonometric identities because students used the visual structure of right triangles to apply the Pythagorean theorem. This process can be thought of as a concrete version of the algebraic process this cluster calls for. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access

Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features because trigonometric identities may appear in a variety of rearranged formats. Highlighting these different formats can help students to see the identities in proofs, even when they are not shown in one specific way.

Build

Effort and Persistence: How will the learning for students provide options for sustaining effort and persistence?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences attend to students attention and affect to support sustained effort and concentration such as providing feedback that emphasizes effort, improvement, and achieving a standard rather than on relative performance because students may not be successful and completing every single proof and our focus should be on engaging with the proof. Focusing feedback on the step's students did will encourage students to continue trying rather than focus on completing every single proof perfectly.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as highlighting structural relations or make them more explicit because some students may struggle to see the identities in rearranged formats. Provide students with, or have students create, a sheet that shows common trigonometric identities re-written in a variety of ways, so they have a quick reference as they move through proofs.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as providing different approaches to motivate, guide, feedback or inform students of progress towards fluency because students may start these proofs in a variety of ways. Showing different approaches, even those that are incorrect or incomplete, serve as opportunities for students to discuss the mathematics behind the approaches which may spark ideas and/or deepen conceptual understanding of the process.

Internalize

Executive Functions: How will the learning for students support the development of executive functions to allow them to take advantage of their environment?

- For example, learners engaging with proving and applying trigonometric identities benefit when learning experiences provide opportunities for students to set goals; formulate plans; use tool and processes to support organization and memory; and analyze their growth in learning and how to build from it such as asking questions to guide self-monitoring and reflection because students frequently feel stuck either starting or through the process of a proof using trigonometric identities. Providing prompting questions like "is everything in terms of sin and cos?", "do you see any version of the Pythagorean identity?", etc. can help students find ways to progress through a proof when they feel stuck.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on proving and applying trigonometric identities by critiquing student approaches/solutions to make connections through a short mini-lesson because students frequently are able to take certain steps in a proof but may find themselves feeling stuck. Whether this is because they have made an error or just cannot see the next step, showing these approaches to their peers can help students make connections between the work they did and that of others, as well as critique steps that are different.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit proving and applying trigonometric identities by confronting student misconceptions because students may expect that each proof will progress in the same format, or that identities will always appear in the same way. Every proof is different and showing students that identities can be applied in a variety of ways may help them feel freed from looking for specific instances of the identities. ...

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to understand concepts more quickly and explore them in greater depth than other students. when studying proving and applying trigonometric identities because when students see the patterns and process clearly, we should allow them to challenge themselves at their own pace to try increasingly more challenging proofs.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Goal Setting: Setting challenging but attainable goals with students can communicate the belief and expectation that all students can engage with interesting and rigorous mathematical content and achieve in mathematics. Unfortunately, the reverse is also true, when students encounter low expectations through their interactions with adults and the media, they may see little reason to persist in mathematics, which can create a vicious cycle of low expectations and low achievement. For example, when studying proving and applying trigonometric identities goal setting is critical because these algebraic proofs can be very challenging for students and if they do not have a productive mind frame as a starting point, they will struggle to make any progress. Encouraging students to set a goal to simply start these proofs by choosing to work from left to right or right to left or a goal to accurately rewrite an expression in terms of sin/cos can give students the support they need in engaging with difficult material. The importance is not that they can complete every proof, but rather that they have a goal they can achieve, and they work mathematically toward that goal.

Standards Aligned Instructionally Embedded Formative Assessment Resources:	
http://tasks.illustrativemathematics.org/content-standards/HSF/TF/C/8/tasks/1835 This type of assessment question requires students to take a given ratio for a trigonometric function and use it to exactly state the value of two other trigonometric functions. This will require knowledge of the trigonometric functions as ratios of side lengths and possibly trigonometric identities. Students will engage with SMP 7 as they use the structure of the ratio to determine the remaining trigonometric values. Additional Assessment: https://www.map.mathshell.org/lessons.php?unit=9255&collection=8&redir=1	
Relevance to families and communities: During a unit focused on proving and applying trigonometric identities, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, for example, connect the process of proofs with the concept of replacing equivalent items can allow students to relate to the process. Students may have experienced this in recipes (replacing butter with oil), in making purchases (choosing one brand over another), etc. Use this as an opportunity to talk about equivalence. In some instances, the outcome may be changed by replacement, but in this mathematical process, equivalence is preserved.	**Cross-Curricular Connections:** Economics – Substitution and utility when making purchases