NEW MEXICO
Public Education Department

HS: NUMBER AND QUANTITY- THE COMPLEX NUMBER SYSTEM

Cluster Statement: A: Perform arithmetic operations with complex numbers.

Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSN.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	SMP.1 Students can make sense of problems and persevere in solving them. Students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. SMP.2 Students can reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations. SMP.6 Students attend to precision by communicating precisely to others. They use clear and precise definitions in discussion with others and in their own reasoning.	• Identify the real number and the imaginary number of a complex number • Define an imaginary number (i.e. $i^2 = -1$). • Define complex numbers. • Find the complex conjugate. • Describe complex numbers in terms of their real and imaginary parts. **Webb's Depth Of Knowledge:** 1-2 **Bloom's Taxonomy:** understand
Standard Text	**Standard for Mathematical Practices**	**Students who demonstrate understanding can:**
HSN.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	SMP.1 Students can make sense of problems and persevere in solving them. Students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. SMP.2 Students can reason abstractly and quantitatively.	• Recognize that $i^2 = -1$ • Use the properties of operations to add and subtract complex numbers • Use the distributive property and the relation $i^2 = -1$ to multiply complex numbers. • Apply the commutative, associative, and distributive properties to complex numbers in order to add, subtract, and multiply. **Webb's Depth of Knowledge:** 1-2

	Students make sense of quantities and their relationships in problem situations. SMP.6 Students attend to precision by communicating precisely to others. They use clear and precise definitions in discussion with others and in their own reasoning.	**Bloom's Taxonomy:** understand, apply
Previous Learning Connections In Algebra 1, students solved quadratic equations using a variety of methods. Their solutions however were limited to real solutions.	**Current Learning Connections** Students will learn to solve quadratic and higher-order polynomial equations that have complex answers as those found within this cluster.	**Future Learning Connections** Students will relate this knowledge of complex numbers to solving rational equations, trigonometric equations and trigonometric form in subsequent math courses (Pre-Calculus, AP Calculus, College Algebra, etc).

Clarification Statement

Complex numbers expand the number system to include square roots of negative numbers and allows applications of complex numbers to electronics. Students use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.

Common Misconceptions

Since most variables are letters and symbols, students may confuse i as a variable.

Students may try to simplify a complex number by combining the real part and the imaginary part.

Multi-Layered System of Supports (MLSS)/Suggested Instructional Strategies

Pre-Teach

Pre-teach (targeted): *What pre-teaching will prepare students to productively struggle with the mathematics for this cluster within your HQIM?*

- For example, some learners may benefit from targeted pre-teaching that introduces new representations (e.g., number lines) when studying to perform arithmetic operations with complex numbers because students no longer will be using real numbers on both axes in their graphs. The y-axis will be used for the imaginary numbers.

Pre-teach (intensive): *What critical understandings will prepare students to access the mathematics for this cluster?*

- 8.NS.A.1: This standard provides a foundation for work with perform arithmetic operations with complex numbers because all numbers are classified as rational or irrational and the real number part of the complex numbers is still either. If students have unfinished learning within this standard, based on assessment data, consider ways to provide intensive pre-teaching support prior to the start of the unit to ensure students are ready to access grade level instruction and assignments.

Core Instruction

Access
Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with <perform operations with complex numbers> benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as graphs, tables, videos, and symbols using different color markers/pencils to emphasis the real and imaginary numbers because students will be introduced to imaginary numbers and need the variety of representation. Students will recognize and make the connection that the operations properties on the real and imaginary numbers will be similar.

Build
Effort and Perception: How will the learning for students provide multiple formats to reduce barriers to learning, such as providing the same information through different modalities (e.g., through vision, hearing, or touch) and providing information in a format that will allow for adjustability by the user?

- For example, learners engaging with <perform operations with complex numbers> benefit when learning experiences ensure information is accessible to learners with sensory and perceptual disabilities, but also easier to access and comprehend for many others such as displaying information in a flexible format to vary perceptual features such as graphs, tables, videos, and symbols using different color markers/pencils to emphasis the real and imaginary numbers because students will be introduced to imaginary numbers and need the variety of representation. Students will recognize and make the connection that the operations properties on the real and imaginary numbers will be similar.

Language and Symbols: How will the learning for students provide alternative representations to ensure accessibility, clarity and comprehensibility for all learners? (e.g., a graph illustrating the relationship between two variables may be informative to one learner and inaccessible or puzzling to another; picture or image may carry very different meanings for learners from differing cultural or familial backgrounds)

- For example, learners engaging with <performs arithmetic operations with complex numbers> benefit when learning experiences attend to the linguistic and nonlinguistic representations of mathematics to ensure clarity can comprehensibility for all learners such as <highlighting structural relations or make them more explicit> because <the imaginary number is introduced and students identify the real and imaginary numbers>.

Expression and Communication: How will the learning provide multiple modalities for students to easily express knowledge, ideas, and concepts in the learning environment?

- For example, learners engaging with <perform arithmetic operations on complex numbers> benefit when learning experiences attend to the multiple ways students can express knowledge, ideas, and concepts such as <providing multiple examples of ways to solve a problem (i.e. examples that demonstrate the same outcomes but use

differing approaches, strategies, skills, etc.)> because <students get a choice on the strategies that they can make a connection to and have the same outcome>.

Internalize
Comprehension: How will the learning for students support transforming accessible information into usable knowledge, knowledge that is accessible for future learning and decision-making?

- For example, learners engaging with <perform arithmetic operations with complex numbers> benefit when learning experiences attend to students by intentionally building connections to prior understandings and experiences; relating important information to the learning goals; providing a process for meaning making of new learning; and, applying learning to new contexts such as providing options for organizing and possible approaches (tables and representations for processing mathematical operations) because < vertically alignment of the real and complex numbers helps organize the student's work and keep the numbers separated. Boxes help the students multiply binomials and trinomials. Multiple examples give student choices on which method to use.

Re-teach

Re-teach (targeted): *What formative assessment data (e.g., tasks, exit tickets, observations) will help identify content needing to be revisiting during a unit?*

- For example, students may benefit from re-engaging with content during a unit on performing arithmetic operations with complex numbers by revisiting student thinking through a short mini-lesson because one of the students' misconception is that the i is another variable. Check for misconceptions using aggressive monitoring.

Re-teach (intensive): *What assessment data will help identify content needing to be revisited for intensive interventions?*

- For example, some students may benefit from intensive extra time during and after a unit perform arithmetic operations with complex numbers by offering opportunities to understand and explore different strategies because students may make the connection between the properties of equations and the procedures within the complex number operations.

Extension

What type of extension will offer additional challenges to 'broaden' your student's knowledge of the mathematics developed within your HQIM?

- For example, some learners may benefit from an extension such as the opportunity to explore links between various topics when studying to perform arithmetic operations with complex numbers because students explore how the operations will be used in later lessons by watching a short video.

Culturally and Linguistically Responsive Instruction:
Validate/Affirm: How can you design your mathematics classroom to intentionally and purposefully legitimize the home culture and languages of students and reverse the negative stereotypes regarding the mathematical abilities of students of marginalized cultures and languages?
Build/Bridge: How can you create connections between the cultural and linguistic behaviors of your students' home culture and language the culture and language of school mathematics to support students in creating mathematical identities as capable mathematicians that can use mathematics within school and society?

Eliciting and Using Evidence of Student Thinking: Eliciting and using student thinking can promote a classroom culture in which mistakes or errors are viewed as opportunities for learning. When student thinking is at the center of classroom activity, "it is more likely that students who have felt evaluated or judged in their past mathematical experiences will make meaningful contributions to the classroom over time." For example, when studying, performing arithmetic operations with complex numbers eliciting and using student thinking is critical because making mistakes and finding the error, students make adjustments and begin asking questions without any repercussions. They are comfortable and know mistakes are allowed and corrections can be made. Mistakes allow students to try instead of leaving questions blank. Challenging questions can also lead to critical thinking and when a task is complete, whether it's right or wrong, students feel the ownership of learning.

Standards Aligned Instructionally Embedded Formative Assessment Resources:

CollegeBoard | Question ID 5344950

Assessment	Test	Cross-Test and Subscore	Difficulty	Primary Dimension	Secondary Dimension	Tertiary Dimension	Calculator
SAT	Math	Additional Topics in Math	Hard	Additional Topics in Math	Complex numbers	1. Apply knowledge and understanding of the complex number system to add, subtract, multiply, and divide with complex numbers and solve problems.	No Calculator

$i^2+(-i)^2$

In the complex number system, what is the value of the given expression? (Note: $i=\sqrt{-1}$)

Question Difficulty: Hard

A. -2
B. 0
C. 2
D. $2i$

Choice A is correct. The power of a product property states that $(xy)^a=x^a y^a$. Using this property, the second term of the given expression can be rewritten as $(-1\times i)^2=(-1)^2 i^2$, or i^2. Substituting i^2 in place of $(-i)^2$ in the given expression yields i^2+i^2, or $2i^2$. Since $i=\sqrt{-1}$, $i^2=-1$ and $2i^2=2(-1)$, or -2.

Choice B is incorrect and may result from rewriting $(-i)^2$ as $-i^2$ instead of i^2. Choice C is incorrect and may result from rewriting i^2 as 1 instead of -1. Choice D is incorrect and may result from rewriting i^2 as i instead of -1.

This type of assessment question requires students to simplify an expression using powers of the imaginary unit. Students will engage in SMP 6 as they must attend to the sign of their answers carefully.

Relevance to families and communities:	**Cross-Curricular Connections:**
During a unit focused on performing arithmetic operations with complex numbers, consider options for learning from your families and communities the cultural and linguistic ways this mathematics exists outside of school to create stronger home to school connections for students, learning about the history of the complex number systems and how the complex numbers originated and used in other countries. Different families can contribute small history pieces and will eventually turn into a big presentation to the class by the student.	**Science -** Science and Electrical Engineering use complex numbers, especially when dealing with light and radio wave. http://faculty.wcas.northwestern.edu/~infocom/Ideas/electric.html **History -** The ancient Greeks once believed that all numbers were rational numbers; that is, that every number could be expressed as the ratio of two integers, and they were very disturbed when it was demonstrated that the measure of the hypotenuse of an isosceles right triangle, having arms of unit measure, was not a rational number. http://mathforum.org/library/drmath/view/55747.html